Fall, 2022

ME 323 – Mechanics of Materials

Lecture 25 – Energy methods (cont.)

Reading assignment: Ch.16 lecturebook

PURDUE UNIVERSITY | Mechanical Engineering

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Tradition and handcraft—combined with high-tech, where it outperforms handcraft

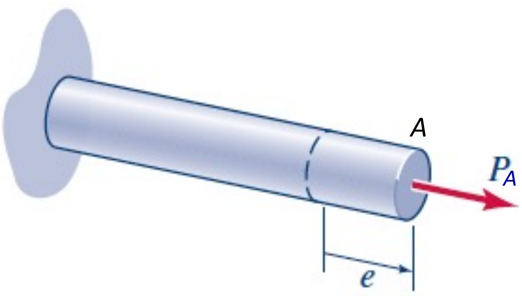
"The law of conservation of energy tells us we can't get something for nothing, but we refuse to believe it."

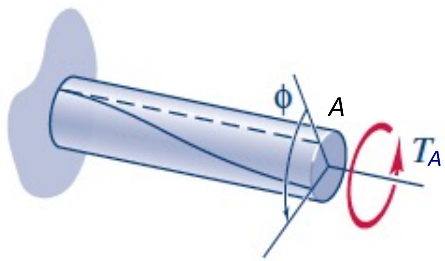
Isaac Asimov (1920-1992)

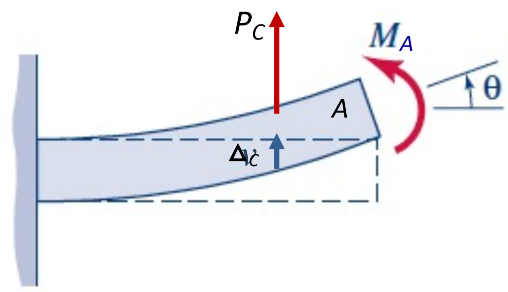
Video

Energy methods

Work and elastic strain energy







Work done by the force:

$$W_{\text{ext}} = \frac{1}{2} P_A e_A$$

Stored elastic strain energy:

$$U = \frac{1}{2} \int_0^L \frac{P(x)^2}{E(x)A(x)} \mathrm{d}x$$

$$U = \frac{P^2 L}{2EA}$$

Work done by the torque:

$$W_{\text{ext}} = \frac{1}{2} T_A \phi_A$$

Stored elastic strain energy:

$$U = \frac{1}{2} \int_0^L \frac{T(x)^2}{G(x)I_p(x)} \mathrm{d}x$$

$$U = \frac{T^2 L}{2GI_p}$$

Work done by the moment:

$$W_{\text{ext}} = \frac{1}{2} M_A \theta_A$$

$$W_{\text{ext}} = \frac{1}{2} P_C \Delta_C$$

Stored elastic strain energy:

$$U_\sigma = \frac{1}{2} \int_0^L \frac{M(x)^2}{E(x)I(x)} \mathrm{d}x$$

$$U_\tau = \frac{1}{2} \int_0^L \frac{f_s V(x)^2}{G(x)A(x)} \mathrm{d}x$$

$$U_\tau << U_\sigma$$

Energy methods - Castigliano's Second Theorem

Castigliano's Second Theorem (last lecture …)

Consider a <u>determinate</u> linearly elastic deformable body acting upon by N_P forces P_i , N_M moments M_i , and N_T torques T_i . Among all possible equilibrium configurations of the body, the actual configuration is the one for which:

$$\Delta_i = \frac{\partial U}{\partial P_i} \qquad i = 1, 2, ...N_P \qquad \text{(displacement – force)}$$

$$\theta_i = \frac{\partial U}{\partial M_i} \qquad i = 1, 2, ...N_M \qquad \text{(slope – bending moment)}$$

$$\phi_i = \frac{\partial U}{\partial T_i} \qquad i = 1, 2, ...N_T \qquad \text{(angle of rotation – torque)}$$

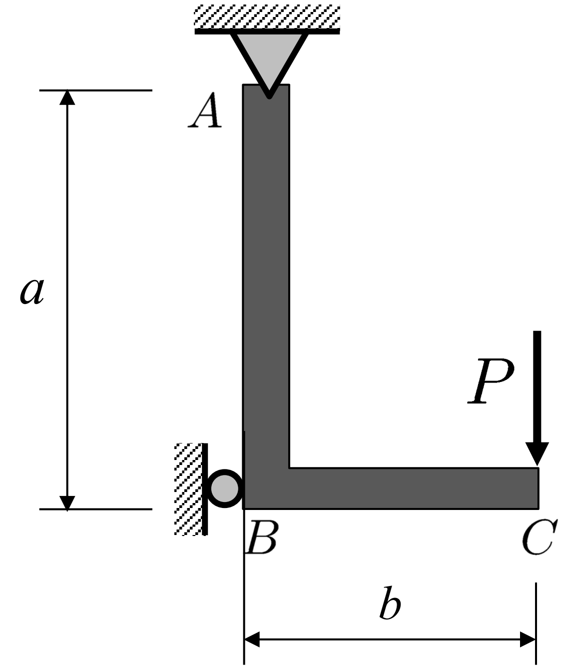
where generalized displacements ($\Delta_i,\ \theta_i,\ \phi_i$) correspond to and are in the direction of the load ($P_i,\ M_i,\ T_i$).

<u>Note</u>: some of these loads could be **dummy loads**, with value zero, that will facilitate the calculation of a generalized displacement at their point of application.

Energy methods

Problem 48:

Determine the vertical displacement at point C.
(Neglect shear effects)



Energy methods - Castigliano's Second Theorem

Castigliano's Second Theorem (final version)

Consider an indeterminate linearly elastic deformable body acting upon by N_P forces P_i, N_M moments M_i, and N_T torques T_i. Among all possible equilibrium configurations of the body, the actual configuration is the one for which:

$$\Delta_i = \frac{\partial U}{\partial P_i} \qquad i = 1, 2, \ldots N_P \qquad \text{(displacement – force)}$$

$$\theta_i = \frac{\partial U}{\partial M_i} \qquad i = 1, 2, \ldots N_M \qquad \text{(slope – bending moment)}$$

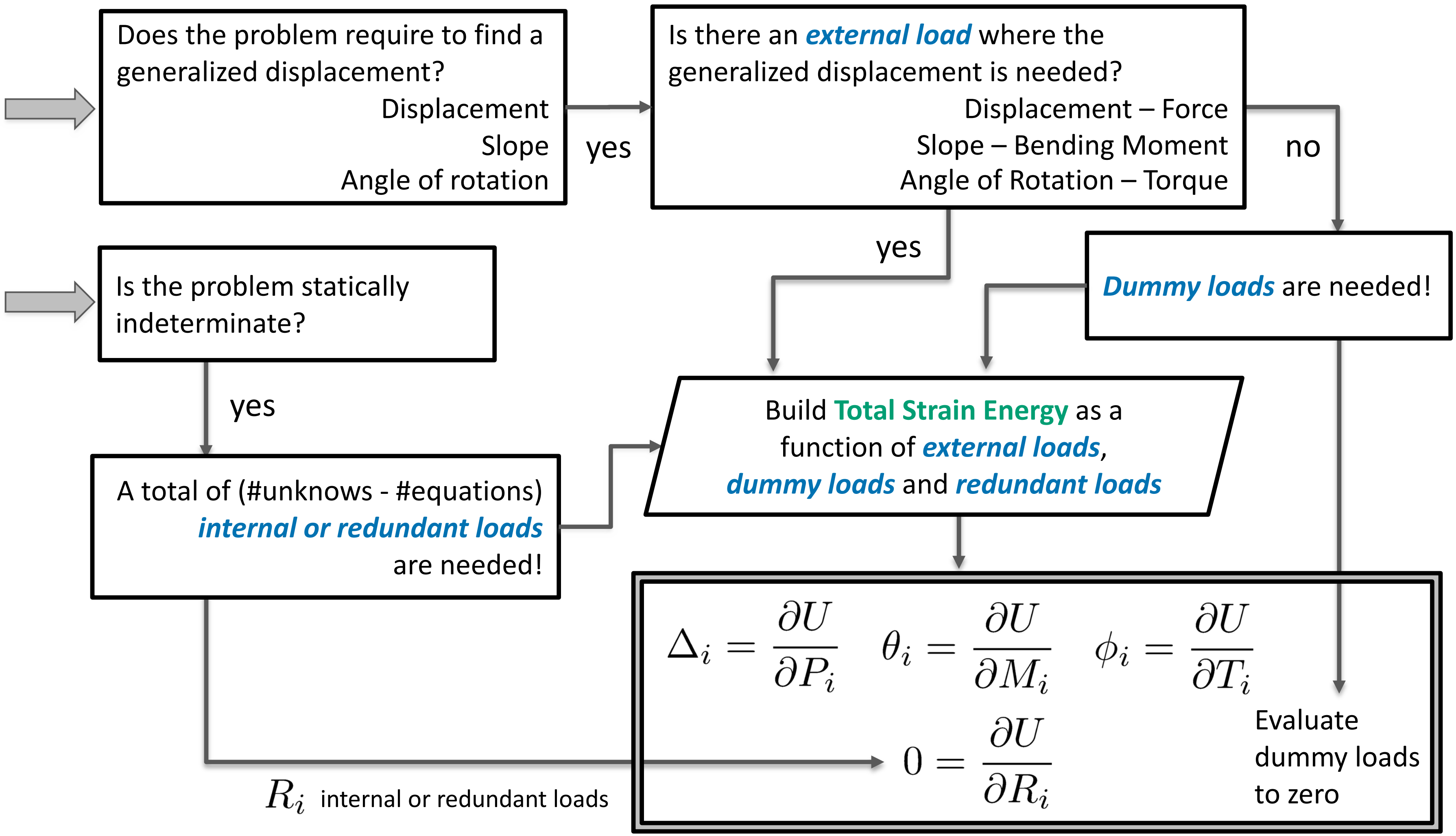
$$\phi_i = \frac{\partial U}{\partial T_i} \qquad i = 1, 2, \ldots N_T \qquad \text{(angle of rotation – torque)}$$

$$0 = \frac{\partial U}{\partial R_i} \qquad i = 1, 2, \ldots N_R \qquad \text{(redundant or internal load)}$$

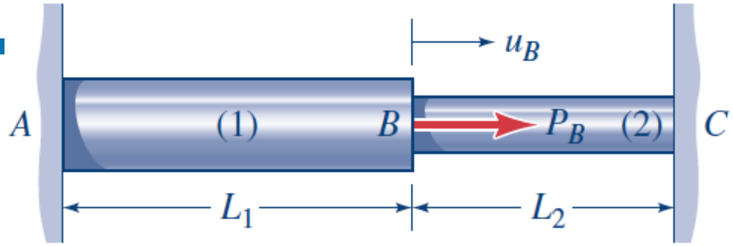
where generalized displacements (Δ_i, θ_i, ϕ_i) correspond to and are in the direction of the load (P_i, M_i, T_i), **and the redundant or internal load R_i (that do not do any external work)**.

Note: some of these loads could be **dummy loads**, with value zero, that will facilitate the calculation of a generalized displacement at their point of application.

Energy methods - Castigliano's Second Theorem



Energy methods



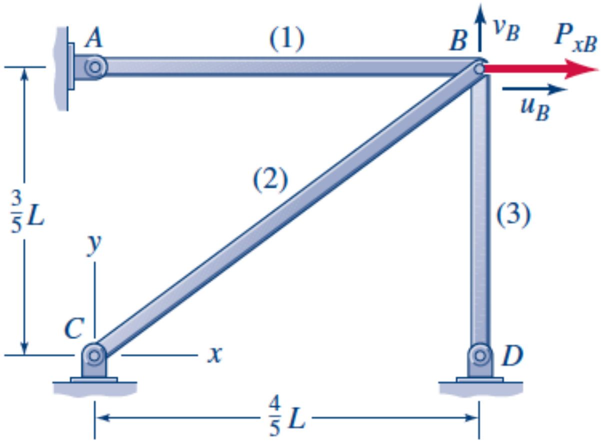
Problem 49 (Problem 7, again):

Determine the elongation at B.

Energy methods

Problem 50 (practice problem):

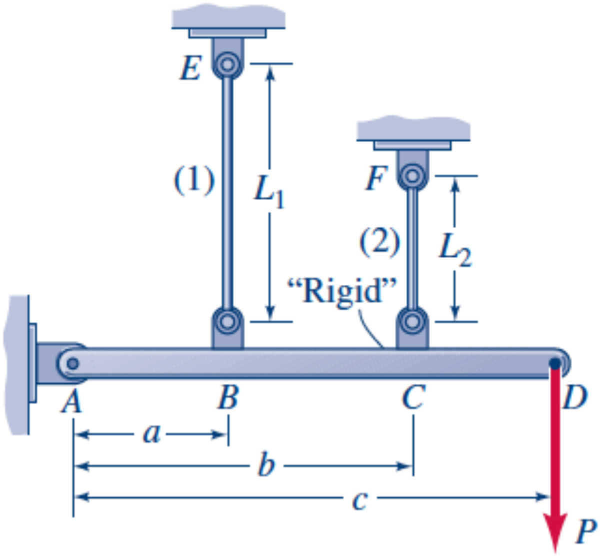
Determine the vertical and horizontal displacements at point B.



Energy methods

Problem 51 (Problem 9, again):

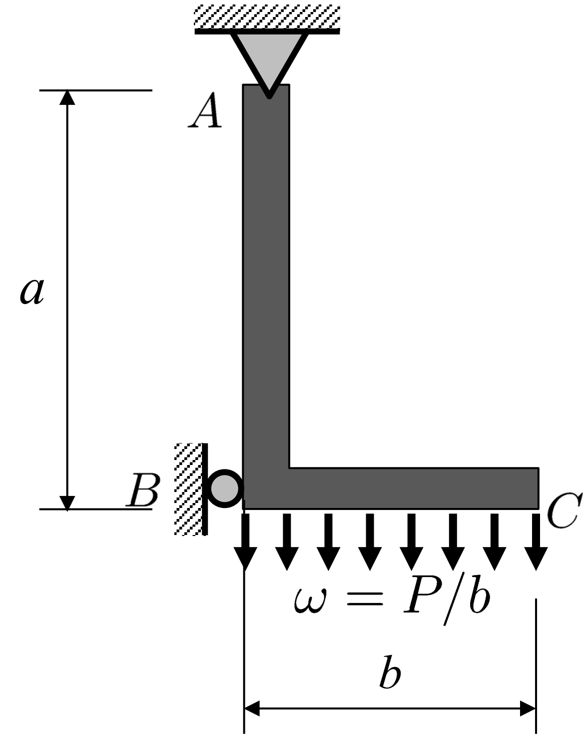
Determine the vertical displacement at D.



Energy methods

Problem 52 (practice problem):

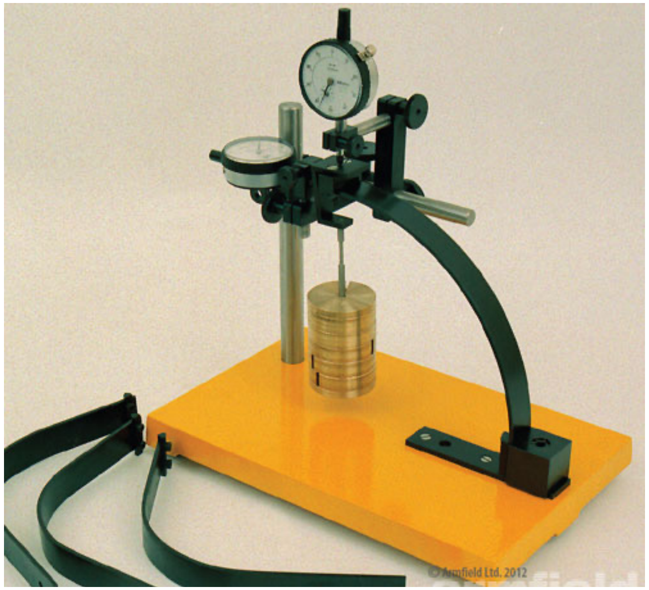
Determine the vertical displacement at point C.
(Neglect shear effects)



Energy methods

Problem 53 (practice problem):

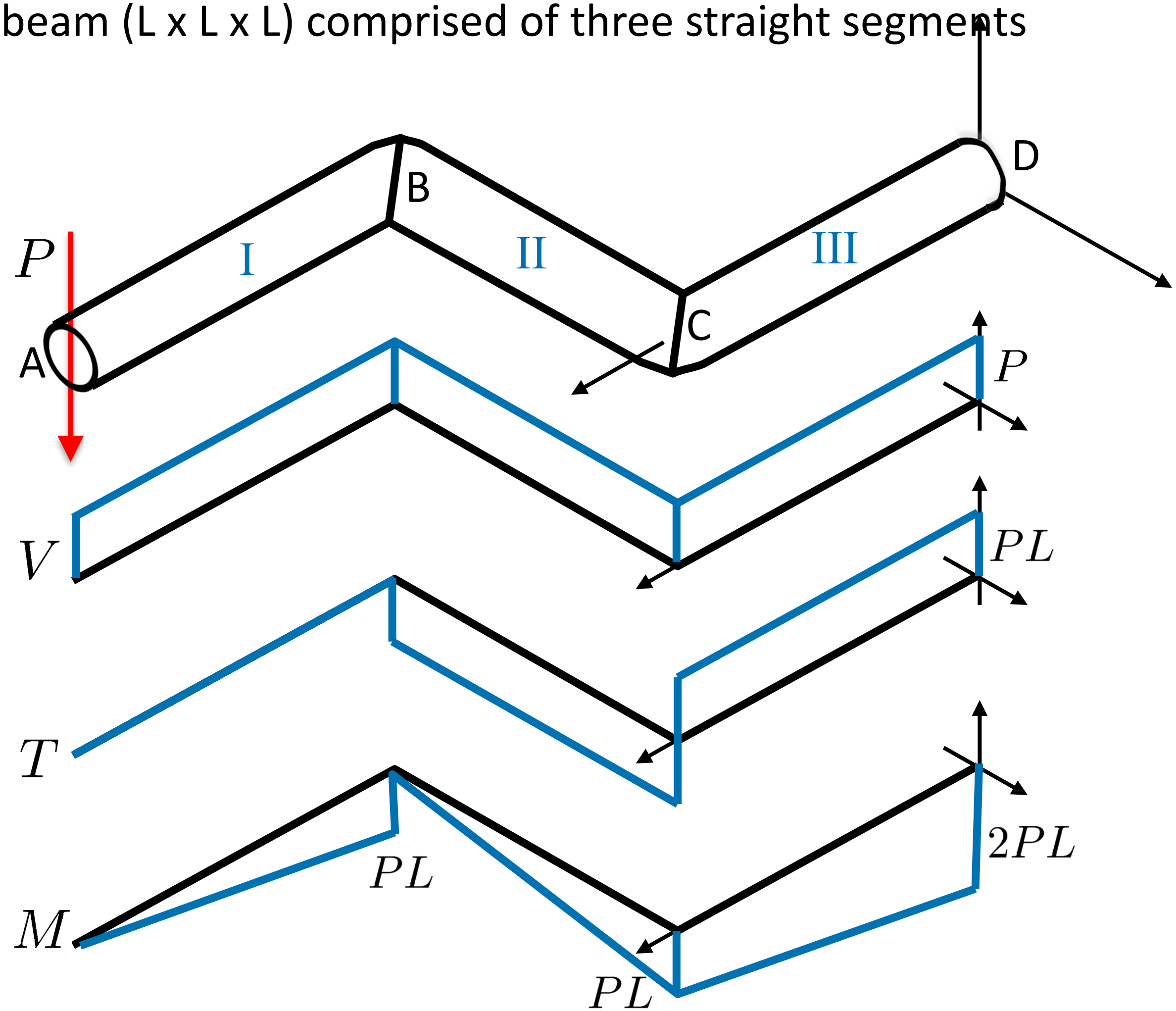
Curved bar apparatus: a vertical load is applied at the free end of the curved beam, where vertical and horizontal displacements are measured.



Energy methods

Problem 54 (practice problem):

Cantilever beam (L x L x L) comprised of three straight segments



Energy methods

Any questions?