Fall, 2022

ME 323 – Mechanics of Materials

Lecture 38 – Failure theories (cont.)

Reading assignment: Ch.15 lecturebook

PURDUE UNIVERSITY | Mechanical Engineering

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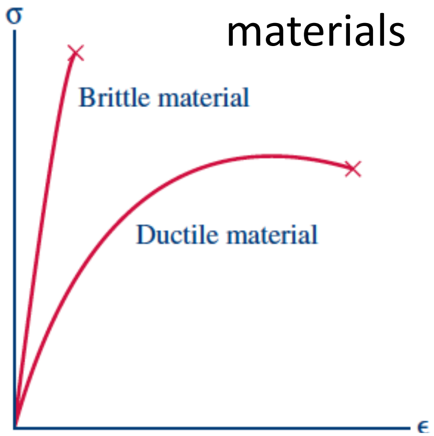
Failure theories

Material properties from stress-strain diagrams (Lecture 3)

- Tensile uniaxial test – two distinct failure mechanisms

Brittle

Brittle vs. ductile materials



Ductile
(yielding before fracture)

- What is the criterion if the solid is under combined loads and the state of stress is triaxial?

Failure theories – Ductile materials

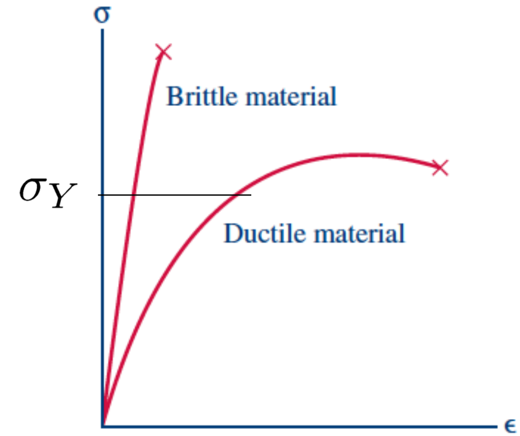
Failure criteria for ductile materials

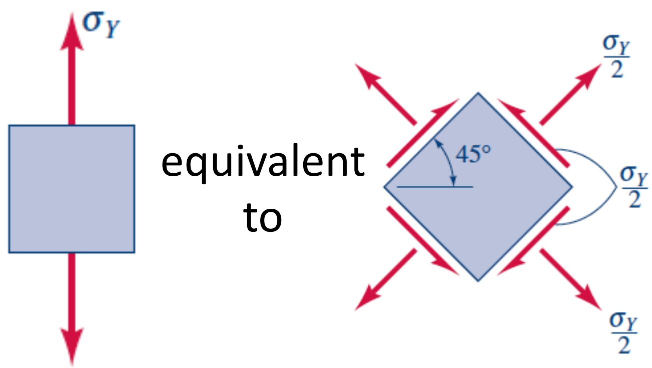
- Maximum-shear-stress theory

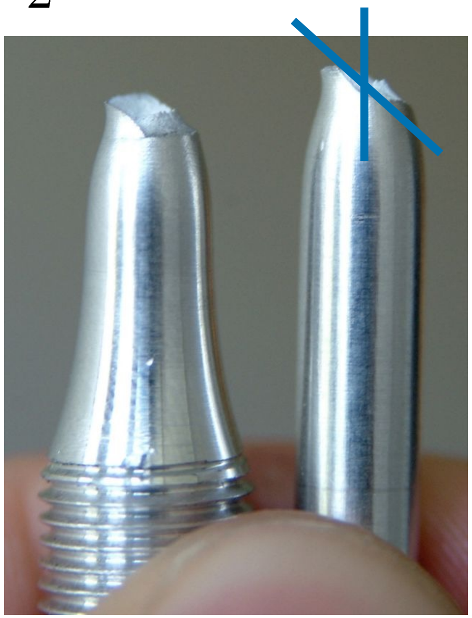
$$\tau_{\max}^{\text{abs}} = \frac{\sigma_Y}{2}$$

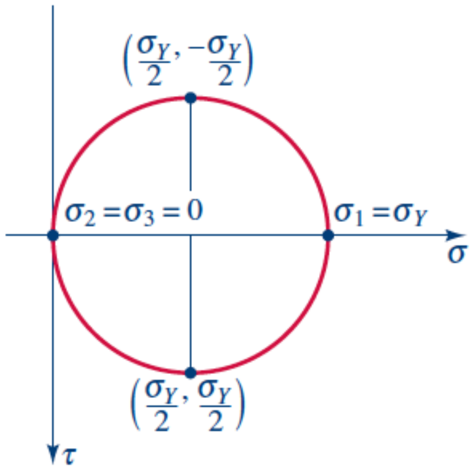
failure criterion
(where σ_Y is measured)

$$\tau_{\max}^{\text{abs}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma_Y}{2}$$









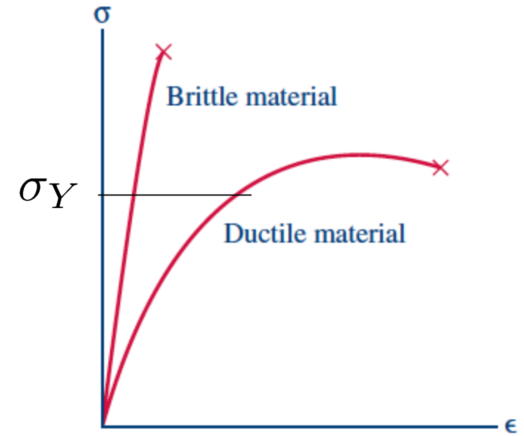
Failure theories – Ductile materials

Failure criteria for ductile materials

- Maximum-shear-stress theory

$$\tau_{\max}^{\text{abs}} = \frac{\sigma_Y}{2}$$ failure criterion (where σ_Y is measured)

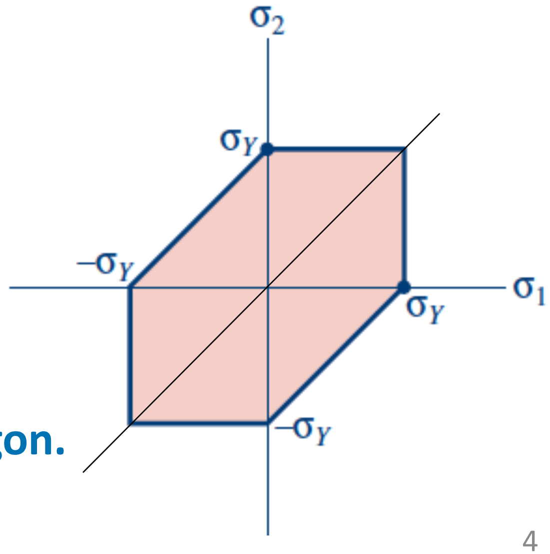
$$\tau_{\max}^{\text{abs}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma_Y}{2}$$



For plane stress, using $\sigma_3 = 0$ in this analysis!)

- $\sigma_1 > 0 \quad \sigma_2 > 0$
- $\sigma_1 < 0 \quad \sigma_2 < 0$
- $\sigma_1 > 0 \quad \sigma_2 < 0$

What are the conditions for each quadrant?



The failure envelope is a hexagon.
+ inside: no failure
+ outside: failure

Failure theories – Ductile materials

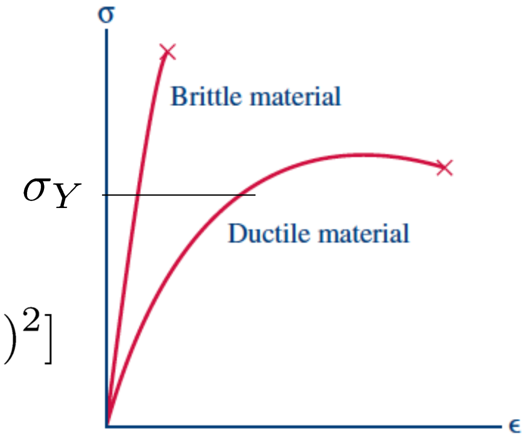
Failure criteria for ductile materials

- Maximum-distortion-energy theory

Total elastic energy = Change of volume + Distortion

Distortion energy:

$$\bar{u}_d = \frac{1}{12G}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2]$$



Distortion energy at yielding point: $(\bar{u}_d)_Y = \frac{1}{6G}\sigma_Y^2$

Failure criterion (for a triaxial state of stress)

$$\sigma_M = \frac{1}{\sqrt{2}}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2]^{1/2} = \sigma_Y$$

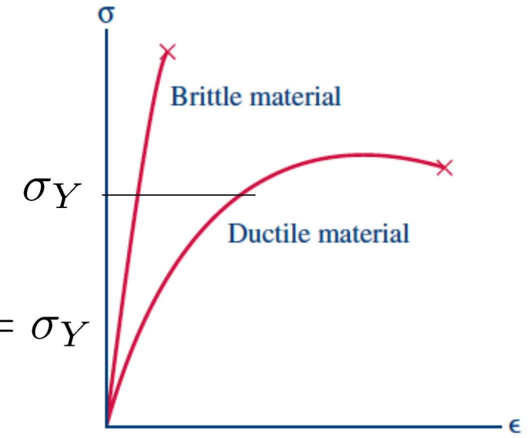
$$\sigma_M = \frac{1}{\sqrt{2}}[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_x-\sigma_z)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2)]^{1/2} = \sigma_Y$$

↑ Mises equivalent stress

Failure theories – Ductile materials

Failure criteria for ductile materials

- Maximum-distortion-energy theory



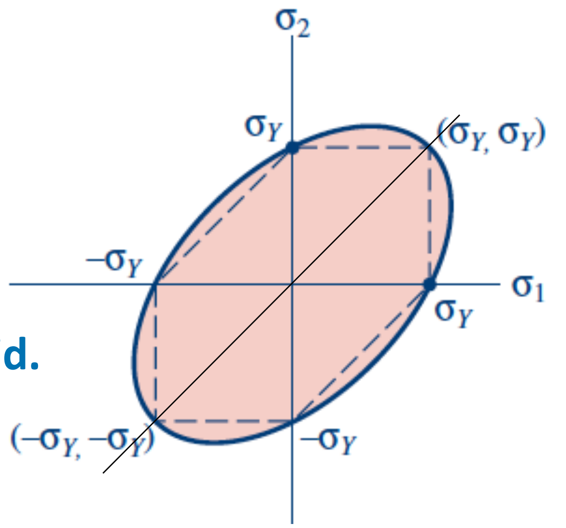
Failure criterion (for a triaxial state of stress)

$$\sigma_M = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2]^{1/2} = \sigma_Y$$

For plane stress, using $\sigma_3 = 0$ in this analysis!)

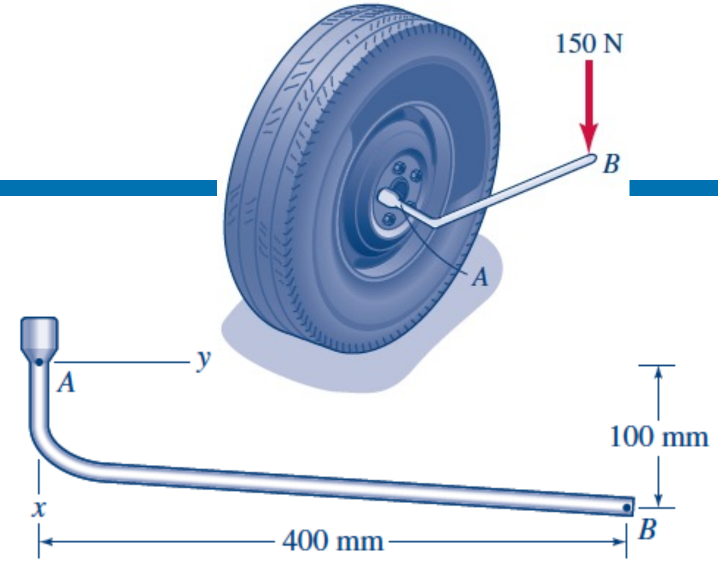
$$\sigma_M = [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2} = \sigma_Y$$ failure criterion

$$\sigma_M = [\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2]^{1/2} = \sigma_Y$$



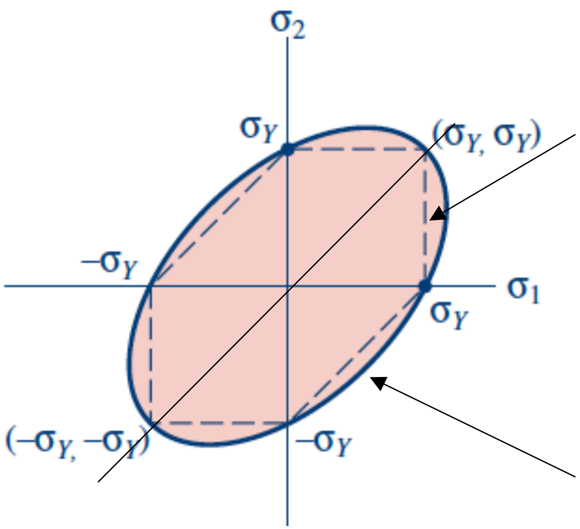
The failure envelope is an ellipsoid.
+ inside: no failure
+ outside: failure

Failure theories



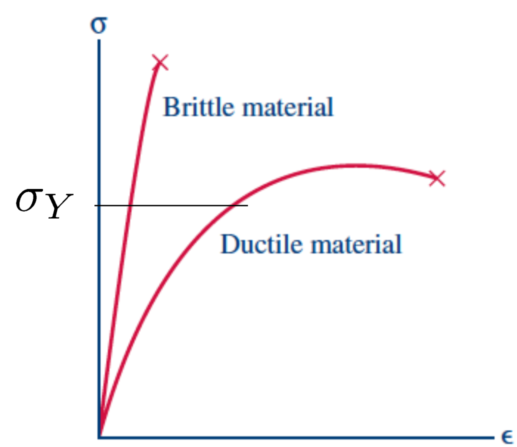
Problem 82 (from last class):

Determine the principal stresses and the maximum shear stress at point A (i.e., the point on top of the wrench handle). The diameter of the circular cross section is 12.5 mm. **The material is ductile.**
The yield stress is 300 MPa.



Maximum shear Stress theory

Maximum Distortion Energy theory



Failure theories – Brittle materials

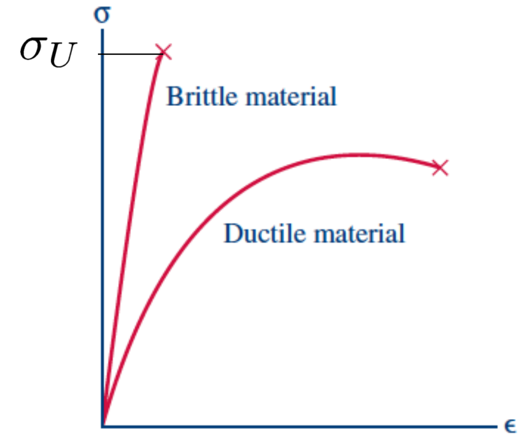
Failure criteria for brittle materials

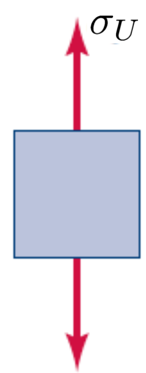
- Maximum-normal-stress theory

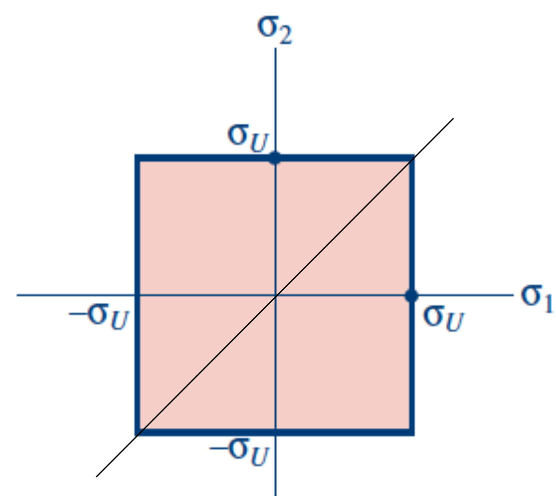
$$\max|\sigma_i| = \sigma_U \quad \text{failure criterion}$$

Notice that this theory assumes that failure under tension and compression occur under the same ultimate stress (not true for most materials).

For plane stress, using $\sigma_3 = 0$ in this analysis!)



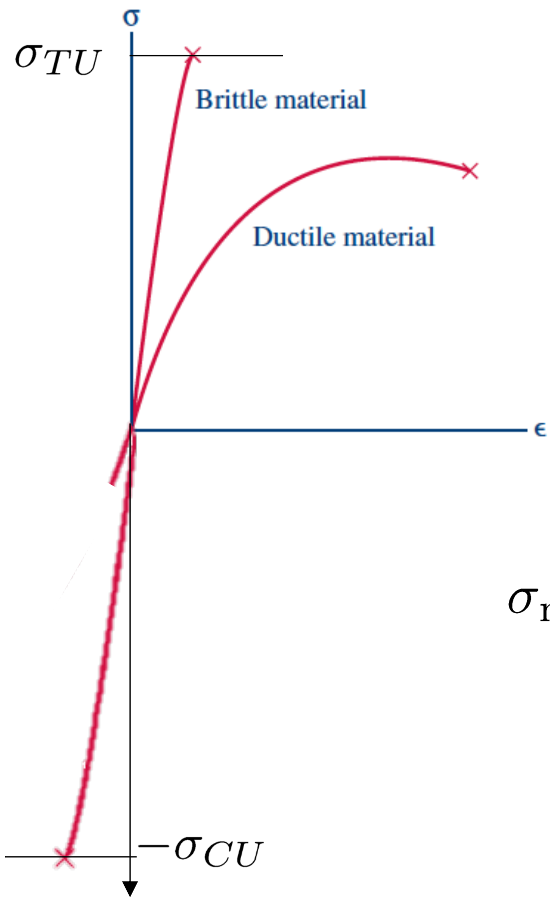




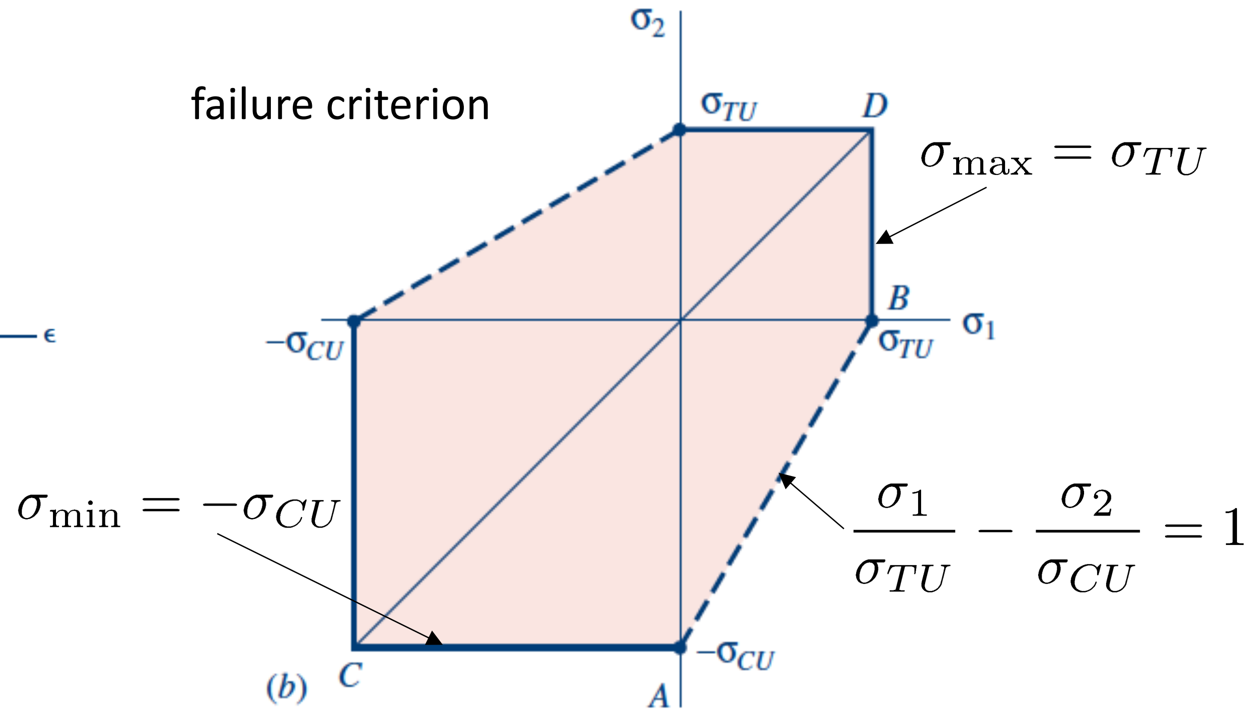
Failure theories – Brittle materials

Failure criteria for brittle materials

- Mohr's failure criterion (it incorporates $\sigma_{TU} < \sigma_{CU}$)



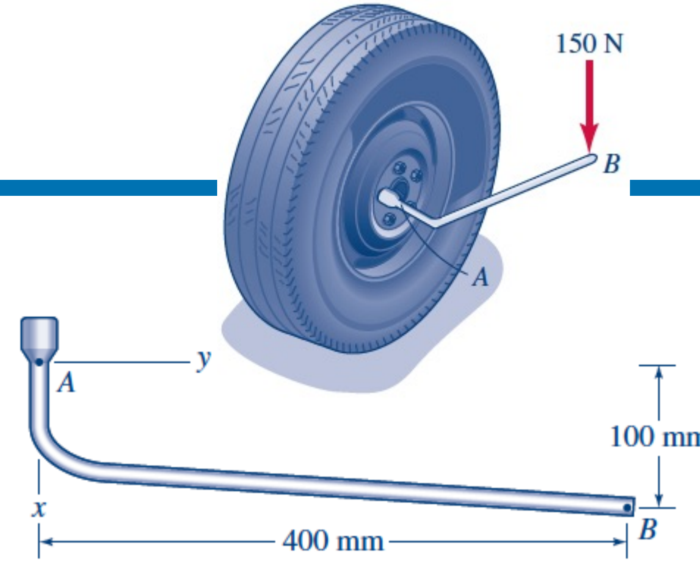
failure criterion

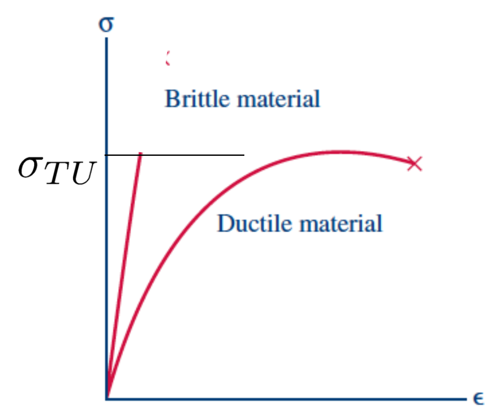


Failure theories

Problem 83:

Determine the principal stresses and the maximum shear stress at point A (i.e., the point on top of the wrench handle). The diameter of the circular cross section is 12.5 mm. **The material is brittle.**
The ultimate tensile stress is 300 MPa.

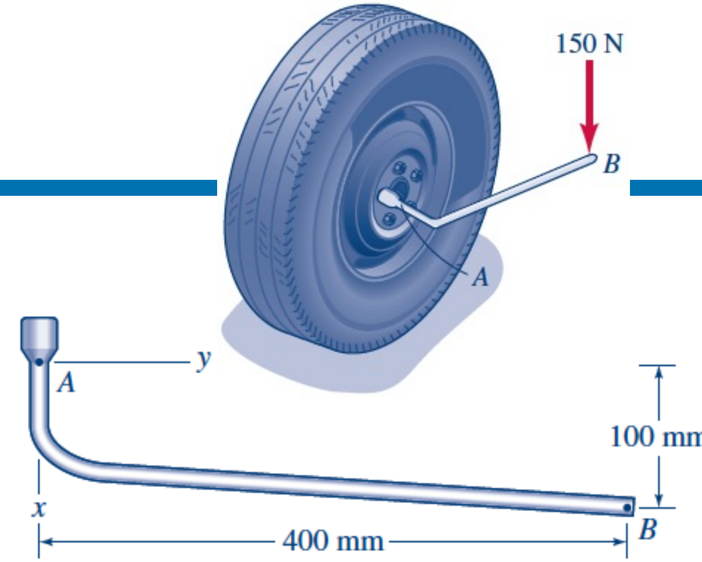


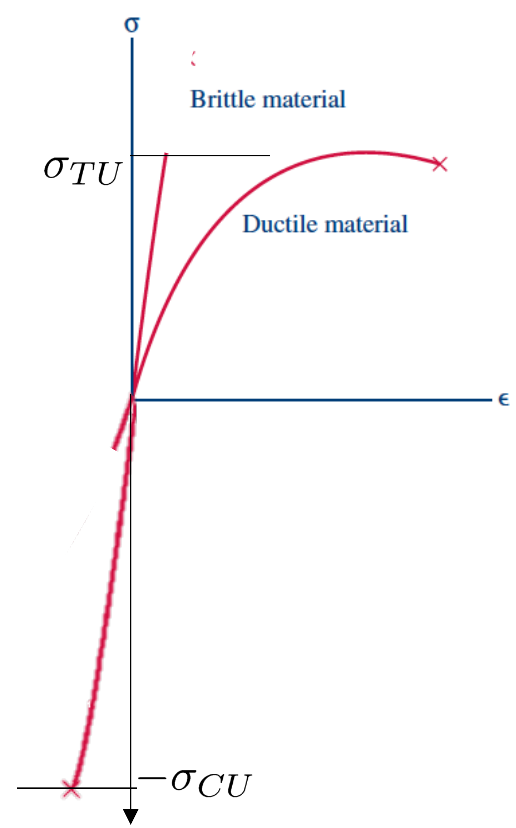


Failure theories

Problem 84:

Determine the principal stresses and the maximum shear stress at point A (i.e., the point on top of the wrench handle). The diameter of the circular cross section is 12.5 mm. **The material is brittle.**
The ultimate tensile stress is 300 MPa and
the ultimate compressive stress is 500 MPa.





Failure theories

Any questions?