Lecture 39 – Buckling of columns

Reading assignment: Ch.18 lecturebook

PURDUE UNIVERSITY
Mechanical Engineering
Instructor: Prof. Marcial Gonzalez

Last modified: 9/24/22 3:44:40 PM

Thin wall pressure vessels (Lecture 30)

Cylindrical body with hemispherical end caps

$$\sigma_a = \frac{pr}{2t}$$ axial stress in the cylinder

$$\sigma_h = \frac{pr}{t}$$ Hoop stress in the cylinder

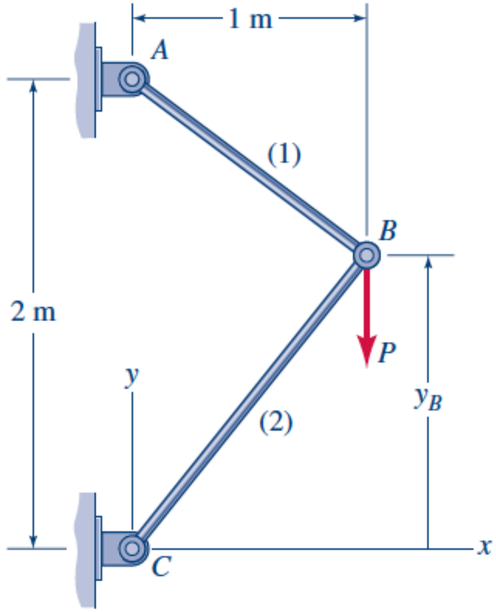
$$\sigma_s = \frac{pr}{2t}$$ normal stress in the sphere

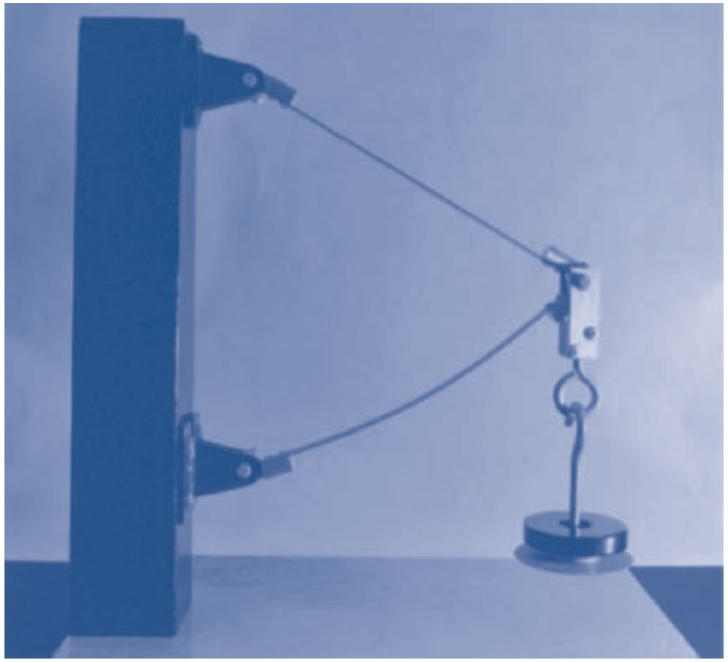
Collapse due to external pressure (no need for vacuum, many times a differential pressure is enough!)

Design of deformable bodies (Lecture 4)

Material properties from stress-strain diagrams:

- **Buckling** under compression





$$\sigma_{\text{buckling}} = \frac{P_{\text{buckling}}}{A}$$

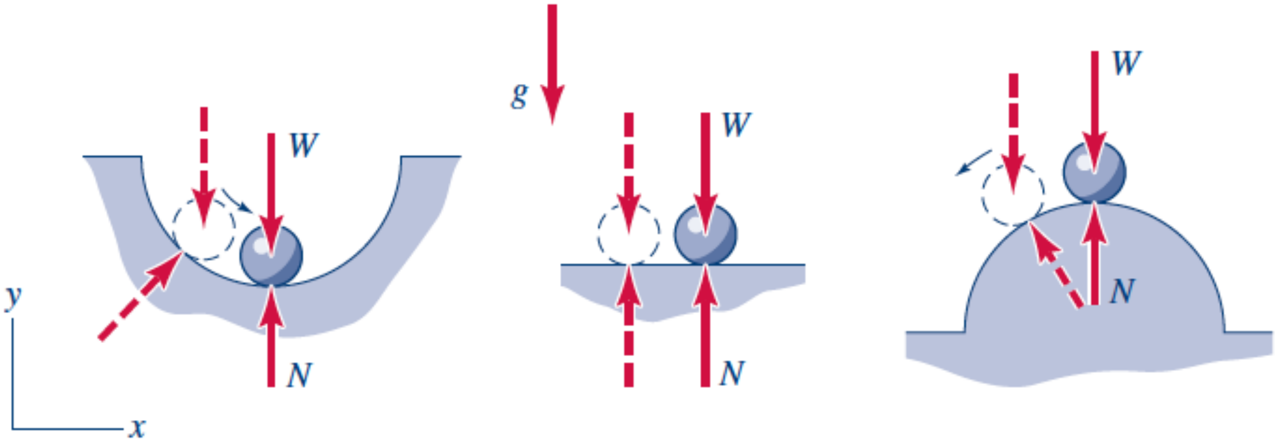
$$\sigma_{\text{allow}} = \sigma_{\text{buckling}} / FS$$

Q: Does the buckling load $P_{buckling}$ depend on the geometry of the bar?

A: Yes!

Buckling of columns

Stability of equilibrium:

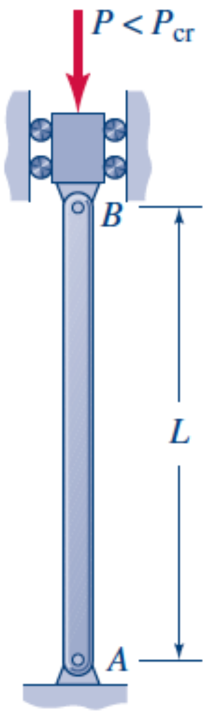


Stable equilibrium Neutral equilibrium Unstable equilibrium

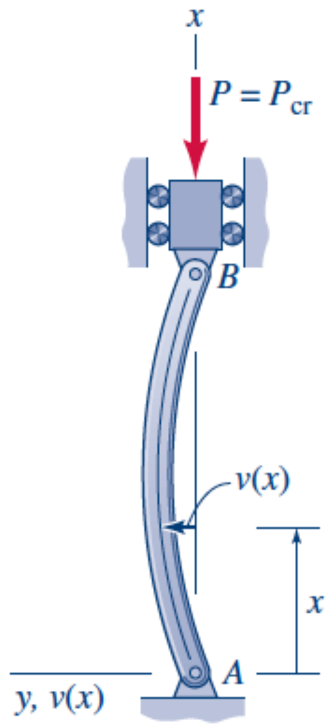
- Note:
  + *Work-Energy Principle* … implies equilibrium
  + *Castigliano's Second Theorem* … implies stable equilibrium
  + *Principle of minimum potential energy* … implies stable equilibrium

Buckling of columns

Pin-ended column:



Before buckling
(**stable configuration**)



Buckled configuration
(**neutral configuration**)

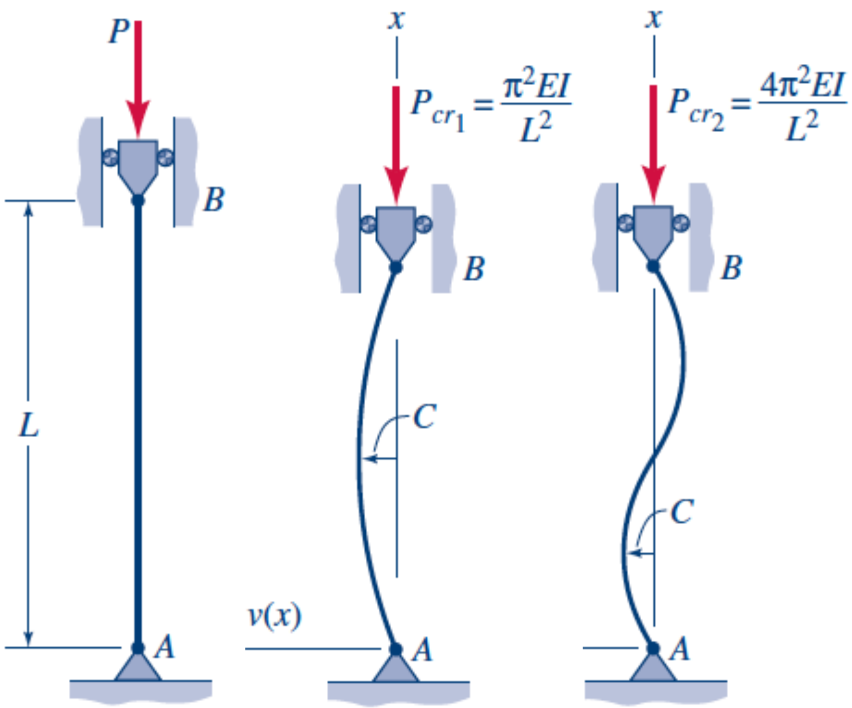
Critical load: $$P_{\text{cr}} = \frac{\pi^2 EI}{L^2}$$

Euler buckling load

Q1: Is this the only critical load?
A1: No!

Buckling of columns

Pin-ended column:



Critical load: $$P_{\mathrm{cr}} = \frac{\pi^2 EI}{L^2}$$

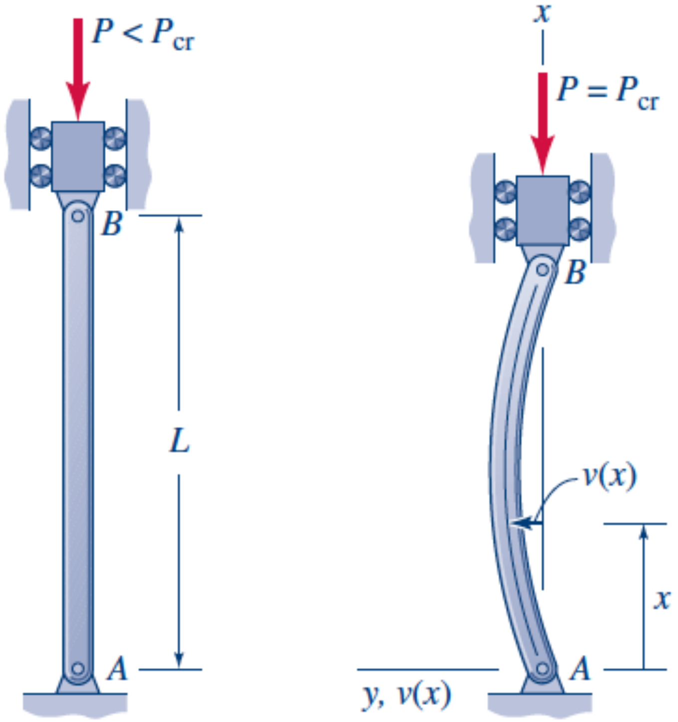
Euler buckling load

Q1: Is this the only critical load?
A1: No!
If we increase the load even further, there will be another neutral configuration (and thus another critical load) that will induce a second-buckling mode!

+ first buckling mode (Euler buckling load),
+ second buckling mode,
+ third buckling mode, …

Buckling of columns

Pin-ended column:



Before buckling
(**stable configuration**)

Buckled configuration
(**neutral configuration**)

Critical load: $$P_{\rm cr} = \frac{\pi^2 EI}{L^2}$$

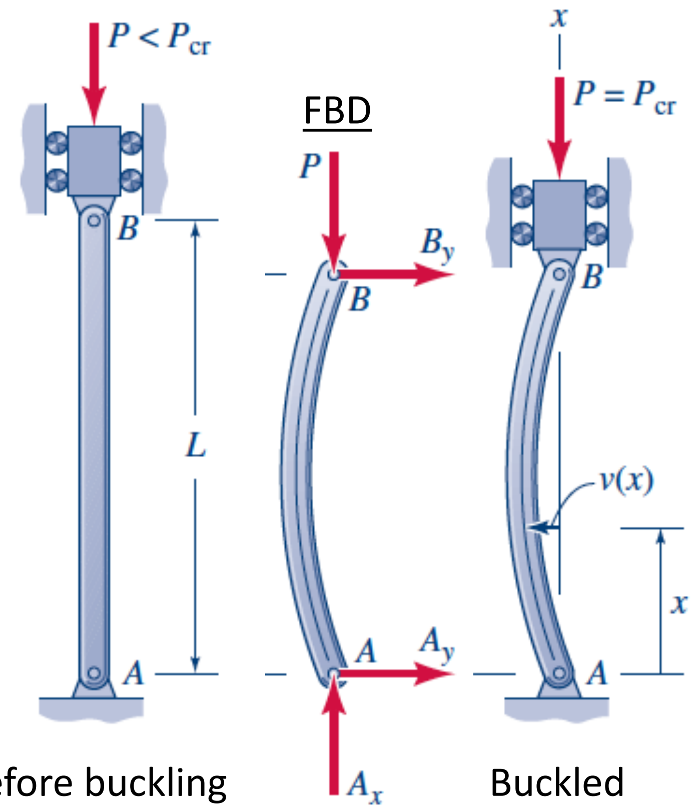
Euler buckling load

Q1: Is this the only critical load?
A1: No!
If we increase the load even further, there will be another neutral configuration (and thus another critical load) that will induce a second-buckling mode!

Q2: Are boundary conditions important?
A2: Yes!

Buckling of columns

Pin-ended column:



Before buckling (**stable configuration**)

Buckled configuration

Equilibrium (in the deformed configuration)

$$M(x) = -Pv(x)$$

Euler-Bernoulli beam theory:

$$EIv''(x) = M(x) = -Pv(x)$$

Boundary conditions:

$$v(0) = 0 \qquad v(L) = 0$$

Solution(s):

$$v(x) = C_1 \sin(\lambda x) + C_2 \cos(\lambda x)$$

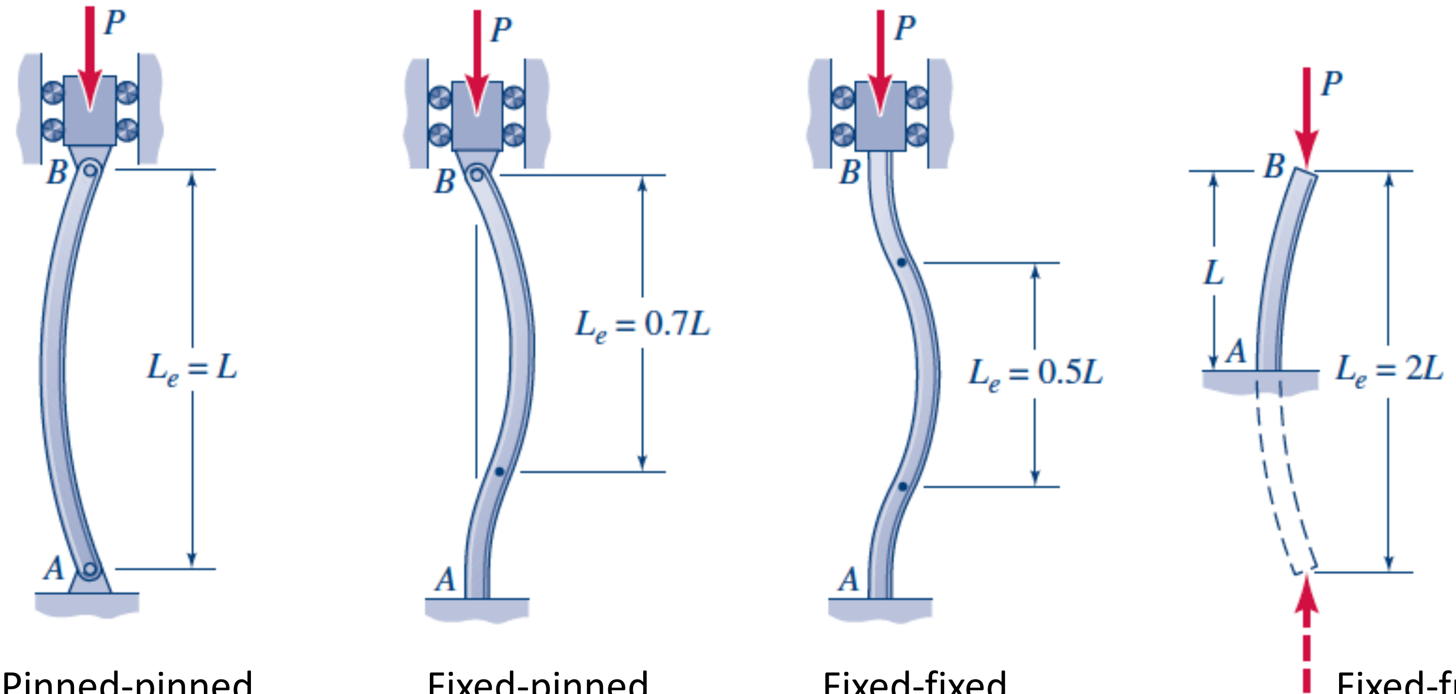
with $\lambda^2 = \dfrac{P}{EI}$

from B.C. $C_2 = 0$

$$C_1 \sin(\lambda L) = 0 \rightarrow \lambda_n = \frac{n\pi}{L}$$

Buckling of columns

Effect of end conditions:



Pinned-pinned Fixed-pinned Fixed-fixed Fixed-free

Critical load: $P_{cr} = \dfrac{\pi^2 EI}{L_e^2} = \dfrac{\pi^2 EI}{(KL)^2}$

K : effective length factor

Buckling of columns

Problem 85: Effect of end conditions …

Problem 86: Effect of cross-section size (e.g., of D) ...

Problem 87: … asymmetric cross section (e.g., rectangular)

Problem 88: Effect of length (e.g., of L) ...

Buckling of columns

Any questions?