Standard Formula Sheet

1. If $|r| < 1$, then $t + tr + tr^2 + \ldots + tr^n + \ldots$ converges to $S = t/(1-r)$.
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
3. $P(A \cap B) = P(A|B)P(B)$
4. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
5. $\mathrm{Var}(X) = \mathrm{E}[(X - \mathrm{E}[X])^2] = \mathrm{E}[X^2] - \mathrm{E}[X]^2$
6. binomial: $f_X(x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \ldots, n.\quad \mathrm{E}[X] = np;\quad \mathrm{Var}(X) = np(1-p).$
7. geometric: $f_X(x) = p(1-p)^{x-1},\ x = 1, 2, \ldots.\quad \mathrm{E}[X] = 1/p;\quad \mathrm{Var}(X) = (1-p)/p^2.$
8. negative binomial: $f_X(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r},\ x = r, r+1, r+2, \ldots.$
   $\mathrm{E}[X] = r/p;\quad \mathrm{Var}(X) = r(1-p)/p^2.$
9. hypergeometric: $f_X(x) = \binom{m}{x}\binom{n-m}{k-x}/\binom{n}{k},\ x = 0, 1, \ldots, k;\ m - (n-k) \le x \le m.$
   $\mathrm{E}[X] = k(m/n);\quad \mathrm{Var}(X) = k(m/n)\frac{(n-m)(n-k)}{n(n-1)}$
10. Poisson distribution: $f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!},\quad x = 0, 1, 2, \ldots,\quad \lambda > 0.\quad \mathrm{E}[X] = \mathrm{Var}(X) = \lambda.$
11. If $X \sim \text{uniform}(a, b)$, then $\mathrm{Var}(X) = (b-a)^2/12$.
12. triangular: For base $[a, b]$ and mode at m, where $a \le m \le b$,
    $$f_X(x) = \begin{cases} \frac{2(x-a)}{(b-a)(m-a)}, & a \le x \le m \\ \frac{2(b-x)}{(b-a)(b-m)}, & m < x \le b. \end{cases}$$
    $\mathrm{E}[X] = [a + b + m]/3;\ \mathrm{Var}(X) = [(b-a)^2 - (m-a)(b-m)]/18.$
13. normal: $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2},\quad -\infty < x < \infty.\ \mathrm{E}[X] = \mu;\ \mathrm{Var}(X) = \sigma^2.$
14. exponential: $f_X(x) = \lambda e^{-\lambda x},\quad 0 \le x < \infty,\quad \lambda > 0.\ F_X(x) = 1 - e^{-\lambda x}$ if $x > 0$.
    $\mathrm{E}[X] = 1/\lambda;\ \mathrm{Var}(X) = 1/\lambda^2.$
15. gamma function: $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx,\quad r > 0.$ If r is an integer, $\Gamma(r) = (r-1)!$
16. gamma/Erlang: For $\lambda > 0$, $f_X(x) = \frac{1}{\Gamma(r)} \lambda^r x^{r-1} e^{-\lambda x},\ x \ge 0.\ \mathrm{E}[X] = r/\lambda; \mathrm{Var}(X) = r/\lambda^2.$
17. $\mathrm{E}[X] = \mathrm{E}[\mathrm{E}[X|Y]]$
18. $\mathrm{Cov}(X, Y) = \mathrm{E}[(X - \mathrm{E}[X])(Y - \mathrm{E}[Y])] = \mathrm{E}[XY] - \mathrm{E}[X]\mathrm{E}[Y]$
19. $\mathrm{Cov}(aX + c, bY + d) = ab\mathrm{Cov}(X, Y)$
20. $\rho_{X,Y} = \mathrm{Cov}(X, Y)/[\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}]$
21. bivariate normal: For $\mu_X, \mu_Y, \sigma_X > 0, \sigma_Y > 0$, and $-1 < \rho < 1$,
    $$f_{X,Y}(x, y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - \frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right]},$$
    $-\infty < x < \infty, -\infty < y < \infty.$
22. multinomial: $f_{X_1, X_2, \ldots, X_k}(x_1, x_2, \ldots, x_k) = \frac{n!}{x_1! x_2! \cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$ for $x_1 + x_2 + \ldots + x_k = n$ and $p_1 + p_2 + \ldots + p_k = 1$.
23. $\mathrm{Var}\left(\sum_{i=1}^{p} c_i X_i\right) = \sum_{i=1}^{p} c_i^2 \mathrm{Var}(X_i) + 2 \sum_{1 \le i < j \le p} c_i c_j \mathrm{Cov}(X_i, X_j).$
24. $\mathrm{MSE}(\hat{\theta}, \theta) = \mathrm{E}[(\hat{\theta} - \theta)^2] = \mathrm{E}[(\hat{\theta} - \mathrm{E}[\hat{\theta}])^2] + (\mathrm{E}[\hat{\theta}] - \theta)^2.$
25. $L(\theta \mid x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f_X(\theta \mid x_i).$