TECHNICAL NOTE: REEXAMINATION OF ALL-OR-NONE INSPECTION POLICIES IN A SUPPLY CHAIN WITH ENDOGENOUS PRODUCT QUALITY

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Abstract. This note studies the optimal inspection policies in a supply chain, in which a manufacturer purchases components from a supplier but has no direct control of component quality. The manufacturer uses an inspection policy and a damage cost sharing contract to encourage the supplier to improve the component quality. We find that All-or-None inspection policies are optimal for the manufacturer if the supplier’s share of the damage cost is larger than a threshold; otherwise, the manufacturer should inspect a fraction of a batch.

1. Introduction and Literature Review

Inspection is a major quality control tool for quality assurance [8, 14]. With inspection, a manufacturer can identify defective parts and prevent them from further processing. The simplest inspection policy is an All-or-None inspection policy (see, e.g., Chapter 15 of [5]). For an All-or-None inspection policy, a manufacturer either inspects all units in a batch or none, depending on the tradeoffs among inspection, rework and damage costs. Traditional models focus on the manufacturer’s inspection decision problem by taking parts quality as exogenous. Under this assumption, an All-or-None inspection policy is optimal for a manufacturer if the following two conditions hold: (C1) the quality
of units in a batch is modeled as i.i.d. random variables and (C2) inspection cost is identical and summable across units of a batch (see more discussions in [12, 13]).

The assumption of exogenous parts quality does not seem very demanding if a manufacturer produces parts in house, since she can always fix parts quality to a desired level. But this assumption becomes very questionable if a manufacturer procures parts from an outside supplier, who often lacks an incentive to deliver a quality level required by the manufacturer. Under this scenario, incoming inspection not only serves as a tool for identifying defects but also as a mechanism for encouraging a supplier to improve parts quality. For instance, if a supplier bears the rework cost of defects, then inspection provides an incentive for the supplier to reduce the defect rate.

The quality coordination literature focuses on finding optimal economic contracts that provide an incentive for suppliers to deliver a required quality level, but explicitly or implicitly assumes All-or-None inspection policies (see, e.g., [1, 2, 6, 9]). Unlike this stream of literature, we focus on finding the optimal inspection policy that provides suppliers an incentive to improve parts quality. Under Conditions C1 and C2, we find that there exists a threshold such that an All-or-None inspection policy is optimal for the manufacturer if the supplier’s share of the damage cost is higher than the threshold. In contrast, the manufacturer should inspect a fraction of a batch if the supplier’s share of the damage cost is lower than the threshold. Hence, All-or-None inspection policies may be suboptimal in a supply chain even if Conditions C1 and C2 hold.

The note is organized as follows. We study the optimal inspection policy and the resulting supplier’s quality level in Section 2. We consider a quadratic quality overhead function in Section 3. Conclusions are summarized in Section 4.

2. The Model

We assume that a manufacturer orders $B$ units of a component from a supplier. Before production, the supplier invests in quality improvement, which determines the probability that a component is defective (denoted by $q$), with an overhead cost function $S(q)$. Notice that a low value of $q$ means
high component quality. For simplicity, we assume that the unit production cost is independent of the defective probability $q$ and the quality of the components within the batch is independent.

After receiving the order from the supplier, the manufacturer randomly selects $n$ units from the batch to inspect with a unit cost $c_I$ and let the remaining $B - n$ units pass through the inspection process. Since the quality of the components within the batch is independent, the inspection sample does not provide any additional information of the defect rate of the remaining $B - n$ units. Hence, we do not consider the alternative of rejecting the remainder of the batch even if the manufacturer finds many defects in the inspection sample.

The total inspection cost for the manufacturer is $c_I n$. We assume that the inspection process does not reject a good unit, but may miss a defect. Let $p$ be the probability that a defective unit is correctly identified. After the inspection, the manufacturer returns the identified defects to the supplier. Hence, the expected number of units that will be returned to the supplier is $npq$. The supplier repairs the returned defective units with a rework cost $c_R$ per unit and delivers them back to the manufacturer. For simplicity, we assume that every unit becomes good after the rework. Hence, the expected number of defects remaining in the batch after the inspection process is $(B - n)q + nq(1 - p) = (B - np)q$.

For a defective component that passes through inspection, there is a damage cost of $D$. The damage cost $D$ includes the scrap cost in further processing and the external failure cost of a defective component. To avoid triviality, we assume that $D > c_R$, which implies that the rework cost is cheaper than the damage cost. Furthermore, we assume that the supplier shares $100\alpha\%$ of the damage cost $D$, where $\alpha \in [0, \bar{\alpha}]$ and $\bar{\alpha}$ represents the maximal damage cost sharing proportion that the manufacturer can charge the supplier. The value of $\bar{\alpha}$ is mainly determined by legal and business factors (e.g., by a court and/or by the supplier’s negotiation power).

Therefore, the expected cost of the supplier is $C_S(n, q; \alpha) = c_R npq + \alpha D(B - np)q + S(q) - t$ and the expected cost of the manufacturer is $C_M(n, q; \alpha) = c_I n + (1 - \alpha)D(B - np)q + t$, where
t is a monetary transfer from the manufacturer to the supplier such that the supplier is willing to undertake production.

2.1. The Centralized System. We first consider a centralized supply chain, in which the manufacturer can fully control the quality of components. This gives the first-best solution for the whole supply chain. The joint cost in the centralized case is

$$C_T(n, q) = C_M(n, q; \alpha) + C_S(n, q; \alpha) = c_I n + D(B - np)q + c_R np q + S(q).$$

The manufacturer chooses an optimal inspection policy (i.e., an optimal number of inspections $n^*_c$) and an optimal defective probability ($q^*_c$) to minimize the total cost $C_T(n, q)$. Notice that the joint cost $C_T(n, q)$ is linear in $n$ for a fixed $q$. Deming [5] and Vander Wiel and Vardeman [12] proved that All-or-None inspection policies are optimal for a linear cost function with fixed quality. It is clear that endogenizing quality decisions does not affect their results; that is, All-or-None inspection policies are optimal for the centralized case.

2.2. The Decentralized System. Second, we consider a decentralized supply chain, in which the manufacturer cannot control the quality of components. Instead, she commits to an inspection policy (e.g., with a contract in [1, 2, 7]). In this section, we assume that the damage cost sharing proportion $\alpha$ is fixed. The endogeneity of $\alpha$ is studied in the next section.

Given the manufacturer's inspection policy and damage cost sharing contract, the supplier decides the quality of components. This problem is formulated as a Stackelberg leadership game with the manufacturer as a leader and the supplier as a follower [11]. Throughout this paper, we assume that the manufacturer knows the supplier's cost parameters (rework cost $c_R$ and quality overhead cost $S(q)$). First, we make the following assumptions of the quality overhead function $S(q)$:

**Assumption 1.** $S(q) = \int_q^1 f(x)dx$, where $q \in [0, 1]$ and $f(q) \geq 0$.

**Assumption 2.** $f(q)$ is decreasing and convex in $q \in [0, 1]$ and $f(1) = 0$.

**Assumption 3.** $f(q)$ is log-concave in $q \in [0, 1]$.

Assumption 1 means that there is no quality cost if all components are defective. Assumption 2 implies that the quality overhead cost function $S(q)$ is convex in $q$. Notice that the quality overhead
function \( S(q) = c_Q(1 - q)^\gamma \) and \( f(q) = c_Q\gamma(1 - q)^{\gamma-1} \), where \( \gamma \geq 2 \), satisfy Assumptions 1-3. By Assumptions 2 and 3, \( f(q) \) is bounded in \([0,1]\), which implies that perfect quality can be achieved with a finite overhead quality cost (i.e., \( S(0) < +\infty \)). A "six sigma" manufacturing process allows less than 3.4 defects per million products. In this case, perfect quality with a finite overhead quality cost is a good approximation. Notice that Baiman et al. [1] and Balachandran and Radhakrishnan [2] used a different assumption. They assumed that perfect quality is not achievable with limited cost.

First, we solve the supplier’s quality decision given the manufacturer’s inspection policy (i.e., the number of inspections \( n \)). The supplier minimizes her cost \( C_S(n, q; \alpha) = c_Rnpq + \alpha D(B - np)q + S(q) - t \) by choosing a quality level. By Assumptions 1 and 2, the supplier should choose \( q_d(n; \alpha) = f^{-1}((c_Rnp + \alpha D(B - np)) \land f(0)) \), where \( f^{-1} \) is the inverse function of \( f \) and \( x \land y = \min(x, y) \).

**Proposition 1.** (1) If \( \alpha < \frac{c_R}{B} \), then \( q_d(n; \alpha) \) is decreasing in \( n \); (2) If \( \alpha \geq \frac{c_R}{B} \), then \( q_d(n; \alpha) \) is increasing in \( n \).

Proposition 1 suggests that the manufacturer’s inspection policy provides an incentive for the supplier to improve quality when the supplier has little share of the damage cost. In contrast, when the supplier holds a large share of the damage cost, since inspection reduces the chance that a defective component causes damage, the supplier’s incentive for quality improvement fades as the manufacturer increases the number of inspections; that is, the manufacturer’s inspection discourages the supplier’s quality improvement effort when the latter holds a large share of the damage cost.

We denote the reservation cost of the supplier as \( \bar{C}_S \). In order to make the supplier undertake production, the manufacturer must offer the supplier an amount of money (\( t \)) such that the supplier’s cost is less than \( \bar{C}_S \). It is easy to see that the manufacturer should offer the supplier \( t_d(n; \alpha) = c_Rnpq_d(n; \alpha) + \alpha D(B - np)q_d(n; \alpha) + S(q_d(n; \alpha)) - \bar{C}_S \). Hence, the manufacturer’s total cost \( C_M(n; \alpha) = c_I n + D(B - np)q_d(n; \alpha) + c_Rnpq_d(n; \alpha) + S(q_d(n; \alpha)) - \bar{C}_S \). Without loss of generality, we assume that \( \bar{C}_S = 0 \). Let \( n_d^*(\alpha) = \arg \min_{n \in [0, B]} C_M(n; \alpha) \) and \( q_d^*(\alpha) = q_d(n_d^*(\alpha); \alpha) \).
Proposition 2. (1). If \( \alpha < \frac{c_R}{B} \), then \( C_M(n; \alpha) \) is convex in \( n \); (2). If \( \alpha \geq \frac{c_R}{B} \), then \( C_M(n; \alpha) \) is quasi-concave in \( n \).

Proposition 2 suggests that the shape of the manufacturer’s cost function critically depends on whether the supplier’s share of the damage cost is larger than the threshold \( \frac{c_R}{B} \). By Proposition 2, it is easy to calculate the optimal number of inspections \( n^*_d(\alpha) \).

Theorem 1. (1). If \( \alpha \geq \frac{c_R}{B} \), then the optimal inspection policy for the manufacturer is an All-or-None inspection policy (i.e., \( n^*_c \in \{0\} \cup \{B\} \)); (2). If \( \alpha < \frac{c_R}{B} \) and \( c_I \in I \), then the optimal inspection policy for the manufacturer is a fractional inspection policy (i.e., \( n^*_d(\alpha) \in (0, B) \)), where

\[
I = \begin{cases} 
(0, (D - c_R)p^{-1}(\alpha DB) - \frac{(1-\alpha)DB(c_R - \alpha D)p}{f(J^{-1}(\alpha DB))}) & \text{if } \alpha DB(1 - p) + c_R B p > f(0) > \alpha DB; \\
((D - c_R)p^{-1}(\alpha DB(1 - p) + c_R B p) - \frac{(1-\alpha)DB(1-p)(c_R - \alpha D)p}{f(J^{-1}(\alpha DB(1 - p) + c_R B p))}, (D - c_R)p^{-1}(\alpha DB) - \frac{(1-\alpha)DB(c_R - \alpha D)p}{f(J^{-1}(\alpha DB))}) & \text{if } f(0) \geq \alpha DB(1 - p) + c_R B p.
\end{cases}
\]

Theorem 1 suggests that an All-or-None inspection policy continues to be optimal if the supplier’s share of the damage cost is larger than the threshold \( \frac{c_R}{B} \), but a fractional inspection policy becomes optimal for the manufacturer if the supplier’s share of the damage cost is lower than the threshold and the manufacturer’s inspection cost is moderate. Notice that the key driver of the suboptimality of All-or-None inspection policies is the endogeneity of the supplier’s quality decision, which requires incoming inspection to serve as not only a tool for identifying defects but also an incentive for the supplier to improve parts quality. In contrast, Lim [7] studied an incomplete information game, in which the quality level of a supplier is prefixed but unknown to a manufacturer. She found that All-or-None inspection policies are optimal.

Proposition 3. (1). If \( n^*_d(\beta) = B \), where \( \beta \) is a number larger than \( \frac{c_R}{B} \), then \( n^*_d(\alpha) = B \) for \( \alpha \in [0, \frac{c_R}{B}] \); (2). \( n^*_d(\alpha) \) is decreasing in \( \alpha \), where \( \alpha \in [0, \frac{c_R}{B}] \); (3). If \( p = 1 \), then \( n^*_d(\alpha) \) is decreasing in \( \alpha \in [0, 1] \); (4). If \( p = 1 \), then \( q^*_d(\alpha) \) is decreasing in \( \alpha \in [\frac{c_R}{B}, 1] \).
Proposition 3 suggests that the manufacturer cuts the number of inspections as the supplier’s share of damage cost increases. This is because the damage sharing contract provides the supplier with an incentive to improve the component quality. Moreover, when the supplier pays for a large share of the damage cost, the component quality is improved. Deere & Co. reported that its warranty costs dropped by 17% after starting to share warranty costs with its suppliers. The drop of warranty costs indicates that the suppliers improved their parts quality [3].

2.3. The Optimal Damage Cost Sharing Contract. Third, we consider the optimal damage cost sharing contract to minimize the manufacturer’s total cost. We let the optimal damage cost sharing proportion \( \alpha^* = \arg \min_{\alpha \in [0,1]} C_M(n_d^*(\alpha); \alpha) \), the optimal sample size \( n_d^* = n_d^*(\alpha^*) \) and the optimal quality level \( q_d^* = q_d^*(\alpha^*) \), where \( \alpha \in [0,1] \).

**Theorem 2.** (1) \( C_M(n_d^*(\alpha); \alpha) \) is decreasing in \( \alpha \); (2) \( \alpha^* = \overline{\alpha} \); (3) If \( \overline{\alpha} = 1 \), then \( C_M(n_d^*(\alpha^*); \alpha^*) = C_T(n_c^*, q_c^*) \), \( n_d^* = n_c^* \) and \( q_d^* = q_c^* \).

Theorem 2 suggests that the manufacturer should recoup the damage cost from the supplier as much as she can. If the manufacturer is able to force the supplier to bear all cost of component failures alone (i.e., \( \overline{\alpha} = 1 \)), then she should choose the number of inspections \( n_c^* \), which induces the supplier choosing the first best quality level \( q_c^* \). This results in the manufacturer’s total cost \( C_M \) achieving the first-best outcome \( C_T \).

But, in reality, it is rare to see that suppliers bear all risk and cost of component failures alone. For instance, automobile suppliers paid $500 million of warranty costs to automakers in 2002, which is merely 5% of the $10 billion warranty bill of automakers [10]. The negotiation power of suppliers and legal constraints may prevent a manufacturer from recouping all damage costs, as pointed out in [2]. Also, the cause identification of product failures in practice is often costly, which may prevent the manufacturer from charging the supplier all damage cost of component failures (see more discussions in [4]). These constraints limit how much a manufacturer is able to charge suppliers for component
failures, which implies $\bar{\sigma}$ may be much less than one. If $\bar{\sigma} < \frac{c_R}{B}$, by Theorem 1, it is optimal for the manufacturer to adopt a fractional inspection policy.

Finally, we compare the total cost of the manufacturer in the decentralized supply chain with the cost in the centralized supply chain. We let $\rho(\alpha) = \frac{C_T(n^*_d,q^*_d)}{C_M(n^*_d(\alpha);\alpha)} (\leq 1)$. $\rho(\alpha)$ measures the supply chain efficiency. When $\rho(\alpha) = 100\%$, the first-best solution is achieved in the decentralized supply chain. Often, there is an efficiency loss due to decentralization (i.e., the manufacturer has no direct control of the component quality), which implies that $\rho < 100\%$. By Theorem 2, $\rho(\alpha)$ is increasing in $\alpha$ and $\rho(1) = 1$. Hence, the supply chain efficiency can be improved by increasing the supplier’s share of the damage cost, and the supply chain is fully coordinated when the manufacturer recoups all damage cost from the supplier, who bears all risk of component failures. Proposition 2B of [1] showed a similar contract to achieve full supply chain efficiency.

3. A Quadratic Quality Overhead Cost Function

We now study a quadratic quality overhead cost function, $S(q) = c_Q(1-q)^2$, where $c_Q > 0$. Then $f(q) = 2C_Q(1-q)$ and $q_d(n;\alpha) = \lceil 1 - \frac{c_Rnp+\alpha D(B-np)}{2c_Q} \rceil \lor 0$, where $x \lor y = \max(x,y)$. The manufacturer’s cost $C_M(n;\alpha) = c_I n + c_Q$ if $2c_Q - \alpha DB < (c_R - \alpha D)np$; otherwise, $C_M(n;\alpha) = c_I n + [DB - (D - c_R)np][1 - \frac{c_Rnp+\alpha D(B-np)}{2c_Q}] + [\frac{c_Rnp+\alpha D(B-np)}{4c_Q}]^2$ if $2c_Q - \alpha DB \geq (c_R - \alpha D)np$.

By Proposition 2, if $\alpha \geq \frac{c_R}{B}$, then $n^*_d(\alpha) = 0$ or $B$. If $\alpha < \frac{c_R}{B}$, $n^*_d(\alpha) = 0 \lor n_0$, where $n_0 = \frac{2c_Q - \alpha DB}{(c_R - \alpha D)p} \land \frac{-DBp(\alpha(2-\alpha)D-c_R)-2c_Qc_I+2pc_Q(D-c_R)}{(2D-c_R)(c_R - \alpha D)p^2} \land B$.

Now we see how the number of inspections, component quality level and supply chain efficiency change with the supplier’s share of the damage cost.

Example 1. Let $c_I = 1, c_R = 4, c_Q = 225, D = 8, B = 100$ and $p = 1$. Figure 1(a) shows that the number of inspections $n^*_d(\alpha)$ is decreasing in the supplier’s share of the damage cost ($\alpha$), which is consistent with Proposition 3. As shown in Figure 1(b), the decrease in the number of inspections by the manufacturer initially results in component quality deterioration when the supplier shares little damage cost of a component failure (i.e., $\alpha$ is small). But as $\alpha$ increases, the supplier shares more
of the damage cost of component failures, which provides an incentive to improve the component quality. Hence, the component defective probability \( q_d^*(\alpha) \) is decreasing in \( \alpha \) when \( \alpha \) is large. This is also consistent with Proposition 3. Finally, as shown in Figure 1(c), the supply chain efficiency \( \rho(\alpha) \) is increasing in \( \alpha \) and achieves 100% when the supplier pays more than 50% of the damage cost.

In Proposition 3, the monotonicity of the optimal number of inspections requires 100% inspection accuracy. This condition is necessary, as shown in Example 2.

**Example 2.** Let \( c_I = 0, c_R = 1, c_Q = 150, D = 6, B = 100 \) and \( p = 0.7 \). By Proposition 2, \( n_d^*(\alpha) = 0 \) or \( B \) for \( \alpha \in \left[ \frac{c_R D}{B}, 1 \right] \). Table 1 shows the manufacturer’s cost and supplier’s component quality level when the number of inspections is 0 or 100. When \( \alpha = 0.5 \), \( n_d^*(0.5) = 0 \) and \( q_d^*(0.5) = 0 \).

When \( \alpha = 1 \), \( n_d^*(1) = 100 \) and \( q_d^*(1) = 0.17 \). Hence, the optimal number of inspections is not decreasing in \( \alpha \) if the inspection accuracy is not 100%.

Proposition 1 suggests that, as the manufacturer increases inspections, the supplier reduces the component quality if her share of the damage cost is large (\( \alpha > \frac{c_R}{D} \)). Hence, an all-inspection policy results in lower component quality than does a none-inspection policy. Moreover, if the inspection accuracy is not high, then an all-inspection policy may generate more damage to the supply chain than does a none-inspection policy. Hence, the manufacturer may choose a none-inspection policy to force the supplier to improve the component quality since the latter holds a large share of the damage cost. However, the manufacturer may switch to the all-inspection policy, as the supplier’s share of
\[ \alpha = 0.5 \quad \text{and} \quad \alpha = 1 \]

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<th>( n = 0 )</th>
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<tbody>
<tr>
<td>( C_M(n; \alpha) )</td>
<td>150.0</td>
<td>159.3</td>
<td>150.0</td>
</tr>
<tr>
<td>( q_d(n; \alpha) )</td>
<td>0</td>
<td>0.47</td>
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Table 1. Total cost \( C_M(n; \alpha) \) and quality level \( q_d(n; \alpha) \).

the damage cost increases up to a point, at which the damage cost sharing contract solely provides a sufficient incentive for the supplier to improve the component quality.

4. Conclusions

We have studied the optimal inspection policy in a supply chain, in which a manufacturer purchases components from a supplier but has no direct control of component quality. We find that All-or-None inspection policies are optimal for the manufacturer if the supplier’s share of the damage cost is larger than a threshold; otherwise, the manufacturer should inspect a fraction of the batch. Hence, the optimality of All-or-None inspection policies in a supply chain with endogenous product quality critically depends on the supplier’s share of the damage cost.

Appendix: Proofs

Proof of Proposition 1. Notice that \( q_d(n; \alpha) = f^{-1}((\alpha DB + (c_R - \alpha D)np) \wedge f(0)) \) and \( \alpha DB + (c_R - \alpha D)np \) is increasing in \( n \) if \( \alpha < \frac{c_R}{B} \), but decreasing in \( n \) if \( \alpha \geq \frac{c_R}{B} \). Since \( f(q) \) is decreasing in \( q \), the claims follow. \( \square \)

Proof of Proposition 2. Recall that \( q_d(n; \alpha) = f^{-1}((\alpha D(B - np) + c_R np) \wedge f(0)) \). If \( \alpha D(B - np) + c_R np > f(0) \), \( C_M(n; \alpha) = c_I n + \int_0^1 f(x)dx \), which is linear in \( n \). Notice that if \( \alpha D(B - np) + c_R np < f(0) \), \( \frac{dC_M(n; \alpha)}{dn} = c_I - (D-c_R)pf^{-1}(\alpha D(B-np)+c_R np)+(1-\alpha)D(B-np)(c_Rp-\alpha Dp)/f'(f^{-1}(\alpha D(B-np)+c_R np)) \). Since \( f \) is a decreasing function of \( q \), by Assumption 2, \( 1/f'(q) \) is decreasing in \( q \in [0,1] \) and \( 1/f'(f^{-1}(x)) \) is increasing in \( x \).
By Assumption 3, \( f(q)/f'(q) \) is increasing in \( q \in [0,1] \) and \( x/f'(f^{-1}(x)) \) is decreasing in \( x \). Hence, 
\[
\frac{dC_M(n;\alpha)}{dn} \text{ is increasing in } \alpha D(B - np) + crnp.
\]

If \( \alpha < \frac{c_B}{p} \), since \( \alpha D(B - np) + crnp \) is increasing in \( n \) and \( f(q) \) is decreasing in \( q \), Claim 1 holds. If \( \alpha \geq \frac{c_B}{p} \), since \( \alpha D(B - np) + crnp \) is decreasing in \( n \) and \( \frac{dC_M(n;\alpha)}{dn} = 0 \) has at most one zero-point, Claim 2 holds. \( \Box \)

**Proof of Theorem 1.** For Claim 1, by Claim 2 of Proposition 2, \( n_d^*(\alpha) \in \{0\} \cup \{B\} \).

For Claim 2, if \( f(0) > \alpha DB \), then 
\[
\frac{dC_M(0;\alpha)}{dn} = c_I - (D - c_R)pf^{-1}(\alpha DB) + \frac{(1 - \alpha)DB(c_B - \alpha D)p}{f(B^{-1}(\alpha DB))} < 0.
\]
Hence, \( n_d^*(\alpha) \neq 0 \). If \( \alpha DB(1 - p) + c_R Bp > f(0) \), then \( \frac{dC_M(B;\alpha)}{dn} = c_I > 0 \); otherwise, \( \frac{dC_M(B;\alpha)}{dn} = c_I - (D - c_R)pf^{-1}(\alpha DB(1 - p) + c_R Bp) + \frac{(1 - \alpha)DB(1 - p)c_R Bp}{f(B^{-1}(\alpha DB(1 - p) + c_R Bp))} > 0 \). Hence, \( n_d^*(\alpha) \neq B \). \( \Box \)

**Proof of Proposition 3.** For Claim 1, \( 0 \geq C_M(B;\beta) - C_M(0;\beta) = c_I B + [DB(1 - p) + c_R Bp]f^{-1}((\beta DB(1 - p) + c_R Bp) \land f(0)] + \int_{f^{-1}(\beta DB(1 - p) + c_R Bp) \land f(0))} f(x)dx - DBf^{-1}(\beta DB \land f(0)) - \int_{f^{-1}(\beta DB \land f(0))} f(x)dx \). Since \( f(q) \) is decreasing in \( q \) and \( \beta > \frac{c_B}{B} \), \( \int_{f^{-1}(\beta DB \land f(0))} f(x)dx \leq DB[(f^{-1}((\beta DB(1 - p) + c_R Bp) \land f(0)) - f^{-1}(\beta DB \land f(0))] \). Hence, \( 0 \geq B[c_I - (D - c_R)pf^{-1}((\beta DB(1 - p) + c_R Bp) \land f(0))] \), which implies \( \beta DB(1 - p) + c_R Bp < f(0) \). Since \( f(q) \) is decreasing in \( q \), \( 0 \geq c_I - (D - c_R)pf^{-1}(\beta DB(1 - p) + c_R Bp) \geq c_I - (D - c_R)pf^{-1}(\alpha DB(1 - p) + c_R Bp) + (1 - \alpha)DB(1 - p)(c_R Bp - \alpha Dp)/f'(f^{-1}(\alpha DB(1 - p) + c_R Bp)) = \frac{dC_M(B;\alpha)}{dn} \), where \( \alpha < \frac{c_B}{B} \). By Claim 1 of Proposition 2, \( n_d^*(\alpha) = B \).

For Claim 2, recall that 
\[
\frac{dC_M(n;\alpha)}{dn} = c_I - (D - c_R)pf^{-1}(\alpha DB - np) + c_R np + (1 - \alpha)D(B - np)(c_R Bp - \alpha Dp)/f'(f^{-1}(\alpha DB - np) + c_R np) \] if \( \alpha DB - np + c_R np < f(0) \) and
\[
\frac{dC_M(n;\alpha)}{dn} = c_I \] if \( \alpha DB - np + c_R np > f(0) \). By Assumption 2, it is easy to see that \( \frac{dC_M(n;\alpha)}{dn} \) is increasing in \( \alpha \in [0, \frac{c_B}{B}] \). By Claim 1 of Proposition 2, \( n_d^*(\alpha) \) is decreasing in \( \alpha \in [0, \frac{c_B}{B}] \).

For Claim 3, since \( n_d^*(\alpha) = 0 \) or \( B \), where \( \alpha > \frac{c_B}{B} \), we only need to prove that \( n_d^*(\beta) = B \) implies \( n_d^*(\alpha) = B \) for \( \alpha \in [\frac{c_B}{B}, \beta) \). Notice that if \( p = 1 \), then \( C_M(B;\alpha) = c_I B + c_R B^{-1}(c_B B \land f(0)) + \int_{f^{-1}(c_B B \land f(0))} f(x)dx \), which is independent of \( \alpha \). If \( \alpha DB < f(0) \), then \( \frac{dC_M(0;\alpha)}{da} = (1 - \alpha)D^2 B f^{-1}([\alpha DB]) \) and \( \frac{dC_M(0;\alpha)}{dn} = 0 \) if \( \alpha DB > f(0) \). Since \( f(q) \) is decreasing in \( q \), \( C_M(0;\alpha) \) is
decreasing in $\alpha$. Moreover, since $n^*_d(\beta) = B$, $C_M(B; \alpha) = C_M(B; \beta) \leq C_M(0; \beta) < C_M(0; \alpha)$. Hence, $n^*_d(\alpha) = B$.

For Claim 4, let $\frac{cR}{B} \leq \alpha < \beta \leq 1$, $q^*_d(\alpha) = f^{-1}((cRn^*_d(\alpha) + \alpha D(B - n^*_d(\alpha))) \land f(0))$ and $q^*_d(\beta) = f^{-1}((cRn^*_d(\beta) + \beta D(B - n^*_d(\beta))) \land f(0))$. If $n^*_d(\alpha) = n^*_d(\beta)$, then $q^*_d(\alpha) \geq q^*_d(\beta)$, since $f(q)$ is decreasing in $q$. If $n^*_d(\alpha) = B$ and $n^*_d(\beta) = 0$, then $q^*_d(\alpha) \geq q^*_d(\beta)$, since $cR B < \beta DB$. Hence, Claim 4 holds. □

Proof of Theorem 2. Since $f(q)$ is decreasing in $q$, $\frac{\partial C_M(n; \alpha)}{\partial \alpha} = (1 - \alpha) D(B - np) \frac{\partial q_d(n; \alpha)}{\partial \alpha}$. Since $f(q)$ is decreasing in $q$, $q_d(n; \alpha)$ is decreasing in $\alpha$. Hence, $C_M(n; \alpha)$ is decreasing in $\alpha$ and Claim 1 holds. Claim 1 immediately implies Claim 2.

For Claim 3, notice that $C_M(n; \alpha) \geq C_T(n^*_c, q^*_c)$. Since $q_d(n^*_c; 1) = q^*_c$, $C_M(n^*_c, 1) = C_T(n^*_c, q^*_c)$. Hence, if $\bar{\sigma} = 1$, the manufacturer should choose the number of inspections $n^*_c$, which induces the supplier choosing the first best quality level $q^*_c$ for her own interest. □

References


