Parametric Vs NonParametric Density Estimation

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Following closely Chris Bishops’ PRML book, Chapter 2
Density Estimation: How do you model a density $p(x)$ given a finite set of samples $x_1, x_2, \ldots, x_N$?

- This is an ill-posed problem. Any distribution $p(x)$ that is non-zero at the data $x_1, x_2, \ldots, x_N$ is a potential candidate.

- Choosing $p(x)$ is a model selection problem.

- We often work with parametric distributions (e.g. a Gaussian) where density estimation becomes a selection of the parameters (e.g. mean and variance in the case of the Gaussian).
Density Estimation: given a finite set $x_1, \ldots, x_N$ of observations, find distribution $p(x)$ of $x$

- **Frequentist’s Way:** chose specific parameter values by optimizing criterion (e.g., maximum likelihood)

- **Bayesian Way:** prior distribution over parameters, compute posterior distribution with Bayes’ rule

  - **Conjugate Priors:** of particular interests as they lead to a posterior distribution of the same functional form as the prior.
We are interested in Bayesian approaches to density estimation.

We need to introduce priors for the parameters and compute the posterior given the data.

Emphasis here is given to conjugate priors – they lead to posteriors that have the same form as the prior.

Many of the examples considered are of the so-called exponential family.
An alternative approach to parametric density estimation is *nonparametric density estimation* methods.

- Here the form of the distribution typically depends on the size of the data set.
- These models still contain *parameters*, but these control the model complexity (rather than the form of the distribution).
- Typical non-parametric methods include
  - histograms,
  - nearest-neighbors
  - and kernels.