Economics 690
Problem Set: Direct Simulation

(a) First, note that

$$\Pr(0 < x \leq 1) = \Pr(-1 \leq x \leq 0) = 1/2.$$

Thus,

$$p(x|0 < x \leq 1) = 2 - 2|x|, \;\; 0 < x \leq 1$$

or

$$p(x|0 < x \leq 1) = 2 - 2x, \;\; 0 < x \leq 1.$$

Similarly,

$$p(x| -1 \leq x \leq 0) = 2 - 2|x|, \;\; -1 \leq x \leq 0$$

or

$$p(x| -1 \leq x \leq 0) = 2 + 2x, \;\; -1 \leq x \leq 0.$$

(b) To implement the method of inversion, we must calculate the cdf's for each of these truncated densities. Let us first consider $p(x|0 < x \leq 1)$.

For $c \in (0, 1]$,

$$\Pr(x \leq c|0 < x \leq 1) = \int_0^c (2 - 2x)dx = 2c - c^2.$$

The method of inversion thus states that a draw from this truncated density can be obtained by solving for c in the equation

$$u = 2c - c^2,$$

where $u \sim U(0, 1)$. Applying the quadratic formula, we obtain:

$$c = 1 \pm \sqrt{1 - u}.$$

Since c must be contained in $(0, 1]$, we must take the "minus" solution (otherwise the value will exceed one) and thus $c = 1 - \sqrt{1 - u}$ provides a draw from the desired truncated density.

As for the other density, namely $p(x| -1 \leq x \leq 0)$, for $c \in [-1, 0]$

$$\Pr(x \leq c| -1 \leq x \leq 0) = \int_{-1}^c (2 + 2x)dx = 2c + c^2 + 1.$$

The method of inversion thus states that a draw from this truncated density can be obtained by solving for c in the equation

$$u = (c + 1)^2$$

(where $u \sim U(0, 1)$), or

$$c = -1 \pm \sqrt{u}.$$

Since c must be contained in $[-1, 0]$, we must take the "plus" solution, or set

$$c = -1 + \sqrt{u}$$

as a draw from the desired truncated density.

(c) MATLAB code which implements this estimator is provided on the course web site.