Economics 690
Problem Set # 1

(1) The derivations are provided on the lecture notes, as are the MATLAB graphs.

(2) Using Bayes theorem, the posterior is (up to proportionality):

$$
\begin{aligned}
p(\theta|y) \;\propto\;& \theta^{\underline{\alpha}+m-1}(1-\theta)^{T-m+\underline{\delta}-1}I(a<\theta<b) \\
=\;& \theta^{\overline{\alpha}-1}(1-\theta)^{\overline{\delta}-1}I(a<\theta<b),
\end{aligned}
$$

where $\overline{\alpha}$ and $\overline{\beta}$ retain the same definitions as in the lecture notes. The posterior above is in the same form as the prior and both are in the form of a truncated beta distribution. Therefore, the truncated beta prior is also a conjugate prior for the Bernoulli sampling model.

(3) The posterior density for θ in this case is (via Bayes theorem):

$$
\begin{aligned}
p(\theta|y) \;\propto\;& \theta^T \exp(-T\overline{y}\theta)\theta^{-1}\exp\left(-\frac{1}{2\sigma^2}(\ln\theta-\mu)^2\right) \\
=\;& \theta^{T-1}\exp\left(-\frac{1}{2\sigma^2}(\ln\theta-\mu)^2 - T\overline{y}\theta\right).
\end{aligned}
$$

Matlab code for carrying out the remainder of this exercise is provided on the course website.