

## Significant Figures

The number of **significant figures** is the number of digits whose values are known with certainty.

The **numerical values of the experimental results** must be written according to specific rules. The number of significant figures that should be used in stating a result is inseparably connected with the accuracy with which the result is known.

- (1) The number of significant figures in the experimental uncertainty is limited to one or (when the experimental uncertainty is small, e.g.,  $\pm 0.15$ ) to two significant figures. You should not use more than two significant digits when stating the experimental uncertainty.
- (2) Now the best estimate (usually the average value) and its uncertainty (experimental error) must always have the same number of digits after the decimal point, even if the uncertainty does not contain the same number of significant figures as the best estimate. If the uncertainty has more number of places after the decimal as compared to the best estimate, adding it to (or subtracting it from) the best estimate will leave the best estimate with more number of decimal places than your apparatus is capable of measuring. For example,  $2.4 \pm 0.16$  implies that the result lies in the range  $2.24 - 2.56$ . But the apparatus can measure only up to a precision of one place after the decimal. Hence the correct way to express the answer is  $2.4 \pm 0.2$ .

If the uncertainty has less number of places after the decimal than the best estimate (could be because of the restriction of one or two significant figures), then your best estimate is rounded off appropriately. This is because even if your apparatus has a higher resolution (i.e., it can display more places after the decimal), it does not have that accuracy (as proved by the error having less number of places after the decimal). Thus  $2.456 \pm 0.12$  should be written as  $2.46 \pm 0.12$ .

- (3) The value of the experimental uncertainty and rules (1) and (2) determine the number of significant digits in the best estimate (usually the average value) of the measured value.
- (4) The number "zero" presents problems in significant figures. Zeros before the first non-zero number are not significant. Zeros after numbers after the decimal place are significant. Zeros before the decimal place may or may not be significant.

Each of the following numbers has three significant figures:

2.34    234    -0.234    0.00234     $-2.34 \times 10^5$      $2.34 \times 10^{-4}$

2.30   -0.230    $2.30 \times 10^{-7}$    0.000230   2.03    $2.30 \times 10^4$

Each of the following numbers has two significant figures:

$2.3 \times 10^2$    2.3   -0.23   0.0023    $-2.3 \times 10^5$     $2.3 \times 10^{-4}$

- (5) Writing an integer number (e.g., 350 m) presents a problem, because it does not clearly defines the number of significant figures (two significant figures or three?). Use the scientific notation, such as  $3.50 \times 10^2$  m, to indicate that three significant figures are known.
- 6) Both measurements and experimental uncertainties must have the same units (always the case unless specified as relative error that is always dimensionless and usually written as a percentage).

Please, compare the following examples:

Correct:	Wrong:
$0.2 \pm 0.3$	$0.20 \pm 0.321$
$0 \pm 2$	$0.05 \pm 2.5$
$3.14 \pm 0.01$	$3.14 \pm 0.002$
$3.142 \pm 0.002$	$3.14213 \pm 0.002$
$3.14 \pm 0.02$	$3.142133 \pm 0.023523$
$(2.34 \pm 0.15) \times 10^{-4}$	$(2.34 \pm 0.152) \times 10^{-4}$
$7.2 \times 10^2$ (two digits)	720 ( ? )
$7.20 \times 10^2$ (three digits)	720 ( ? )

- (7) Do not mix notations - that is, do not write a measured value in scientific notation and its error as a decimal or vice versa.
- (8) When multiplying or dividing measurement figures, the final answer may not have more significant figures than the *least* number of significant figures in the figures being multiplied or divided. This simply means that an answer cannot be more accurate than the least accurate measurement entering calculation, and that you cannot improve the accuracy of a measurement by doing a calculation (even if you have a 10-digit, scientific calculator). To minimize rounding errors, you may use more significant figures during intermediate calculations and adjust the significant figures only for the final results.

Examples:  $3.5 * 22.3 = 78$  *not*  $78.05$ ,       $6.2 / 833 = 0.0074$  *not*  $0.007442977$   
Adding or subtracting:  $42.4 - 41.62 = 0.8$ ,     $4256 - 24.7 = 4231$ ,     $33.8 + 15.63 = 49.4$