Coherent optical imaging inside scattering media at substantial depths is key for biomedical applications, as well as various environmental and materials sensing situations, has remained elusive. We present a simple imaging method involving the correlation of speckle patterns as a function of object position that provides access to the object autocorrelation function and hence allows the formation of an image using phase retrieval. With multiple measured speckle patterns, our method uses the ensemble-averaged spatial correlations to enable the extraction of an image of the moving object. The approach is effective for thick and heavily scattering media. Subject to adequate signal level, the method is simplified by heavy scatter. Reconsstructions of both patch and aperture objects placed between heavily scattering acrylic slabs and between thick chicken breast slices demonstrate the feasibility of the approach.

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Coherent optical imaging in opaque and randomly scattering material offers the promise of high spatial resolution as well as spectroscopic information. When the scattering medium is thin, the memory effect [1], in which the speckle pattern translates with the incident light angle over a small range, has provided a basis for imaging using fluorescence [2]. Mapping of the field transmission matrix of the scattering medium and its inversion allow contrast-enhanced focusing [3] and pointwise imaging [4]. Wavefront shaping has allowed focusing inside scattering media using measurements with photacoustic [5] and ultrasound [6] guidestars. Despite such progress, these methods generally become less effective as the amount of scatter or the medium’s thickness increases, a consequence of the underlying principle and increasing experimental and computational complexities. We propose an approach in which heavy scatter is not prohibitive and can actually simplify the theory. Using a physical model combined with phase retrieval, we demonstrate the imaging of absorptive targets and intensity patterns hidden inside a thick scattering domain.

Speckle forms because of the constructive and destructive interference that occurs when coherent light interacts with a random medium. The polarized field statistics due to sufficient scatter with mean scatterer separation large relative to the wavelength, of relevance here, contract to zero-mean circular Gaussian [7, 8]. Previously, with a transmitted field defined by an aperture and a transmission speckle measurement as a function of aperture position, the imaging of a circular aperture scanned between two scattering slabs was demonstrated [9]. We now present a means to image an arbitrary object moving within a scattering medium and show that complex geometries can be imaged in heavy scatter using phase retrieval, further demonstrating the practical utility of the approach.

Figures 1(a) and (e) illustrate our approach. Speckle intensity images taken at different object positions in between two scattering slabs constitute the dataset. The spatial scan steps are large relative to the wavelength and short-range correlations are neglected. Cross-correlation coefficients between pairs of speckle patterns are computed as a function of object displacement. Changes in the speckle pattern are due to motion of the object. The averaged and interpolated correlation coefficients, such as in Fig. 1(e), contain deterministic information about the object, which is useful for object reconstruction.

In the temporal frequency domain, the complex scalar field (detected through a polarizer) at detector position rd is written as \( \Phi(d; r, \Delta r) = \Phi_{db}(r, \Delta r) + \Phi_{ds}(r, \Delta r) \), where \( \Phi_{db} \) is the background field without the object of interest and \( \Phi_{ds}(r, \Delta r) \) is the scattered field due to the object, with reference position \( r \) and displacement \( \Delta r \). The object at a reference position represented by a point location \( r \) and translated by a movement described by \( \Delta r \), we write the normalized average correlation of the scattered field at \( r_d \) as

\[
S^{(1)}_{db}(r, \Delta r) = \langle \Phi_d(r) \Phi_{ds}(r + \Delta r) \rangle \langle I_{ds}(r) \rangle^{-1/2} \langle I_{ds}(r + \Delta r) \rangle^{-1/2}
\]

\[
= \langle \int dr' O(r, r') \Phi(r, r') G(r_d, r') \int dr'' \times O(r + \Delta r, r'') \Phi(r + \Delta r, r'') G(r_d, r'') \rangle \langle I_{ds}(r) \rangle^{-1/2} \langle I_{ds}(r + \Delta r) \rangle^{-1/2},
\]

where \( \langle \cdot \rangle \) indicates averaging over scatter configuration for a point detector at \( r_d \). In Eq. (1), \( I_{ds} = |\Phi_{ds}|^2 \), \( O \) describes the object contrast (regulating the scattered field from the moving ob-
stationary. This allows us to use a pair of speckle images that correspond to given \( r \) and \( \Delta r \) to form an estimate of the scatterer reconfiguration average \( \langle \cdot \rangle \) for a point detector at \( r_0 \). Our approach uses multiple pairs of speckle patterns sharing a given offset. Thus, with the appropriate normalizations, we obtain an improved estimate of the average by averaging over the multiple reference positions represented by different \( r \). Our approach results in cross-correlation images such as that in Fig. 1(e) for a circular patch object. From Eq. (1), this additional averaging step results in

\[
\langle \tilde{g}_{ss}^{(1)}(r, \Delta r) \rangle = \tilde{g}_{ss}^{(1)}(\Delta r),
\]

where only dependence on the displacement of the object is retained. Consequently, we write all subsequent physical quantities with \( \langle \cdot \rangle \) as being dependent only on \( \Delta r \).

The average correlation of \( \tilde{I}_d \) over object scan position is

\[
\langle \tilde{I}_d(r) \tilde{I}_d(r + \Delta r) \rangle = \frac{|\langle \Phi_d^*(r) \Phi_d(r + \Delta r) \rangle|^2}{\langle \tilde{I}_d(r) \rangle^2},
\]

where we have assumed the fields measured are zero-mean circular Gaussian [8] and hence can apply a theorem to write the fourth-order field moment in terms of the second order moments [10]. The numerator of Eq. (3) can be expanded as

\[
|\langle \Phi_d^*(r) \Phi_d(r + \Delta r) \rangle|^2 = |\langle \Phi_{db}^* \Phi_{db} + \Phi_{ds}^* \Phi_{ds} \rangle |^2.
\]

Substituting Eq. (4) into Eq. (3) and with Eq. (1) and Eq. (2), we can write

\[
\langle \tilde{I}_d(r) \tilde{I}_d(r + \Delta r) \rangle = C_0(\Delta r) + \Re \left\{ C_1(\Delta r) \tilde{g}_{ss}^{(1)}(\Delta r) \right\} + C_2(\Delta r) \tilde{g}_{ss}^{(1)}(\Delta r)^2,
\]

where \( C_0(\Delta r), C_1(\Delta r), \) and \( C_2(\Delta r) \) are spatially-dependent coefficients obtained when grouping terms in the expansion by order of \( \tilde{g}_{ss}^{(1)} \). The cross-correlation terms involving the unknown \( \Phi_{db} \) and \( \Phi_{ds} \) are retained in the expansion of Eq. (4), and the mean intensities in Eq. (3) are incorporated into \( C_0 \) and \( C_1 \).

Referring to Fig. 1, macroscopic imaging requires that the scan distance be greater than the size of the object. From our experiments with various circular patches (3.5 - 7.5 mm diameter), with a sufficiently heavily scattering medium on either side, we find \( \langle \tilde{I}_d(r + \Delta r) \rangle \simeq \langle \tilde{I}_d(r) \rangle \). Therefore, \( C_0, C_1 \) and \( C_2 \) can be approximated as constants over \( \Delta r \). Therefore, in this heavy scatter regime, Eq. (5) can be approximated as

\[
\langle \tilde{I}_d(r) \tilde{I}_d(r + \Delta r) \rangle = C_0 + \Re \left\{ C_1 \tilde{g}_{ss}^{(1)}(\Delta r) \right\} + C_2 \tilde{g}_{ss}^{(1)}(\Delta r)^2.
\]

For an aperture-type object, where the aperture forms the source of detected intensity, \( \langle \tilde{I}_d(r + \Delta r) \rangle = \langle \Phi_{db}^* \Phi_{db} \rangle = 0 \), so \( C_0 = 0 \) and \( C_1 = 0 \). Also, by the definitions of the normalized intensity and normalized averaged scattered field correlation, \( C_2 = 1 \). We thus have

\[
\langle \tilde{I}_d(r) \tilde{I}_d(r + \Delta r) \rangle_{\text{ap}} = \tilde{g}_{ss}^{(1)}(\Delta r)^2.
\]

Our experimental data for circular absorptive patches of various sizes in a background bright field consistently indicated that \( \langle \tilde{I}_d(r) \tilde{I}_d(r + \Delta r) \rangle \) is quadratic in \( \tilde{g}_{ss}^{(1)}(\Delta r) \). Using the known object function in these cases suggests that \( C_1 \) in Eq. (6) is sufficiently small to allow the second term to be neglected, giving

\[
\langle \tilde{I}_d(r) \tilde{I}_d(r + \Delta r) \rangle_{\text{pa}} = C_0 + C_2 \tilde{g}_{ss}^{(1)}(\Delta r)^2.
\]
The minimum of the measured intensity correlation not only indicates the value of C_2 but also reveals the size of the object, allowing us to remove the data points outside the joint spatial support. From Eq. (1) and Eq. (2), given that |Δr| is large compared to the wavelength, assuming heavily scattering environment so that we have Gaussian field statistics and only the joint spatial support of the object and the translated object contributes to the average second-order field moment, we have

$$\tilde{g}_ss^{(1)}(\Delta r) = \int dr \tilde{O}^\ast(r) \tilde{O}(r + \Delta r),$$

where \(\tilde{O}\) is the object function from Eq. (1) normalized so that \(\int dr \tilde{O}^\ast(r) \tilde{O}(r) = 1\). From Eq. (7), and upon subtracting C_2 from Eq. (8) and rescaling, we can write

$$\langle I_i(r) I_d(r + \Delta r) \rangle = \left| \tilde{g}_ss^{(1)}(\Delta r) \right|^2 = \left| \mathcal{F}^{-1}\left\{ \left| \tilde{O}(k) \right|^2 \right\} \right|^2,$$

where in the second equality we use the Wiener-Khinchin theorem. With \(\tilde{g}_ss^{(1)}(\Delta r) = \mathcal{F}^{-1}\left\{ \left| \tilde{O}(k) \right|^2 \right\}\), we recognize that \(\tilde{g}_ss^{(1)}\) can be a complex quantity. For our experiments, the continuous nature of the intensity correlation data and the positive and symmetric properties of \(\left| \tilde{O}(k) \right|^2\) for a real object function indicate that \(\tilde{g}_ss^{(1)}\) is continuous, positive and symmetric. Hence, we can directly obtain \(\tilde{O}(k)\) and subsequently carry out iterative phase retrieval [11] to reconstruct the embedded object.

We designed three experiments with different objects and scattering materials that used the setup shown in Fig. 1(a). A 59-mW 850-nm laser diode with a linewidth less than 10 MHz was used for illumination. The beam diameter at the scattering material was approximately 0.4 mm. As shown in Figs. 1(b)-(d), we used two layers of scattering material separated by a small distance (about 5 cm) and a pair of stages to move the objects of interest in the transverse plane between the layers. An area of approximately 1.8 mm by 1.8 mm on the back of the second scattering layer was imaged by the camera through a polarizer (to obtain Gaussian field statistics) using magnifying optics. A PRIME sCMOS (for "LUX"), 500 ms integration time, and for the circular patch, 300 ms) and a CoolSNAP HQ CCD (for "π" with 3 s integration time) were used in the experiment. The intensity statistics of the speckle patterns used were measured to be negative exponential, indicating Gaussian field statistics.

Figure 2(a) shows a 7.5-mm-diameter circular absorbing patch object that was placed on a transparent plastic window (10 cm × 13 cm). This object was translated along the y-axis, referring to Fig. 1(a), between two 6-mm-thick acrylic scattering slabs. The scattering slabs (14 cm × 14 cm) were made of commercial acrylic with negligible optical absorption, embedded with TiO_2 scatterers having a mean diameter of 50 nm. The reduced scattering coefficient (\(μ_s' = 4 \text{ cm}^{-1}\)) of the scattering acrylic slabs is comparable to that of human breast tissue in vivo [12]. This is the scattering material shown in Figs. 1(b)-(d), and the very heavy scatter is evident. As Fig. 1(d) illustrates, it is not possible to see the small circular patch behind a 6 mm thick acrylic scattering slab. The background intensity and the object scatter are both significant in this situation. The object was moved in a 1D uniformly-spaced scan of 40 points over 13.16 mm. Averaged intensity correlations were computed using the 40 speckle patterns obtained at these object locations. We extracted C_2 in Eq. (8) from the minimum of the measured intensity correlation, along with the renormalizing constant C_2. In Fig. 2(b), we compare the speckle intensity correlation over object position with a prediction using Eq. (8). Note the excellent agreement. The measured correlation increases after the minimum because the speckle patterns become similar to the bright background when the object is displaced farthest away from the center. To reconstruct the circular patch object, we discarded measured data points for Δr > 7.5 mm, beyond the minimum of the measured intensity correlation and the joint spatial support, and took advantage of rotational symmetry, using the 1D data to form a 2D map. Inversion of Eq. (10) was achieved using iterative phase retrieval to form the object function [13]. Specifically, 2000 iterations of the hybrid input-output method with a varying spatial frequency filter implemented outside the support area was used to obtain Fig. 2(c) [11], which is true to the object size. The inversion procedure conforms to a well-defined phase retrieval problem where the spatial support can be determined by the global minimum of the decorrelation data. We used a loose support about 15 mm × 15 mm. The near-perfect prediction of the correlation suggests that imperfect smoothing at Δr = 7.5 mm and a local minimum may be responsible to the noise in phase reconstructing the symmetric object in Fig. 2(c).

We describe two experiments and demonstrate the reconstruction of small and complex apertures in Fig. 3. In the experiment to image the character "π", we used fresh chicken breast that was 2 mm thick on the source side and 7 mm on the detector side, held between two microscope slides. For the "LUX" aperture, 3-mm and 9-mm-thick acrylic scattering slabs (\(μ_s' = 4 \text{ cm}^{-1}\)) were used on the source and detector side, respectively. The two aperture objects were fabricated as reflective metal coatings on rectangular glass substrates, forming the apertures "LUX" and "π" shown in Figs. 3(a) and (d). The apertures were scanned over 20 diagonal lines having a uniform angular increment (π/10) within a circular region. Each line scan had a length at least 1.5 times the maximum spatial support of the object. Cross-correlation coefficients of speckle images that share the same displacement are averaged if the speckle images involved are acquired on the same line scan. This data was interpolated onto a square 2D Cartesian grid using the gridfit function in MATLAB™. The central regions of the interpolated correlations are shown in Figs. 3(b) and (e) for the "LUX" and "π" objects, respectively. Note that the correlations approach a minimum when Δr is about the object's size, consistent with the picture of the joint support of the object contributing to the measured correlations. This provides a convenient way to select the data set used in reconstruction, and we did so using a threshold (about 0.05 of the maximum). A Cartesian 2D-tapered cosine (Tukey) window (twice the object’s size, selected by aN/2, with a = 0.45 and N the number of samples along a given dimension), was applied to the data. The filtered data set was...
tinctly different from recent applications of the memory effect, the approach applies to thick and very heavily scattering medium regime that is representative of many applications, and the speckle decorrelation (in a 2D space in our case). Employing a windowing method that is based on the minima of the intensity correlation raw data before the Fourier transform suggests that the primary contributor to resolved intensity correlations, and our investigation of the reconstruction process, suggest the possibility to identify and even image circulating melanoma cells [16]. Movement could also be induced by an acoustic wave in tissue to produce controlled motion, and motion of tens of microns has been achieved [14]. Other possible applications include detecting and imaging fast-moving objects through clouds, fog, or other obscuring environments. More generally, spatial speckle correlations allow new sensing, imaging, and communication opportunities in scenarios in which traditional methods are not feasible.

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REFERENCES