COLLECTIVE FIELDS IN $Sp(N)$ MODELS & HOLOGRAPHY

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$\boxed{\text{LARGE N AND HOLOGRAPHY}}$

THE KEY FEATURE OF LARGE-N LIMITS IS THE FACTORIZATION OF INVARIANT OPERATORS

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + O\left(\frac{1}{N}\right) \quad \text{VECTOR}$$

$$= \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + O\left(\frac{1}{N^2}\right) \quad \text{MATRIX}$$

THIS IMPLIES THAT WRITTEN IN TERMS OF SUITABLE INVARIANT FIELDS THE LARGE-N LIMIT IS A CLASSICAL LIMIT OF SUCH A COLLECTIVE FIELD THEORY — MASTER FIELD (Witten)

IN MOST INTERESTING SITUATIONS WE DO NOT KNOW HOW TO DERIVE THIS THEORY

—

IN FACT FOR THEORIES OF MATRICES (LIKE GAUGE THEORIES) THIS SHOULD BE SOME KIND OF STRING FIELD THEORY OF WILSON LOOPS
('t Hooft)

TYPICALLY WE DONT KNOW WHAT IS THIS STRING THEORY

THERE ARE A FEW CASES WHERE WE CAN DERIVE THIS THEORY SYSTEMATICALLY

A PARTICULARLY INTERESTING EXAMPLE IS MATRIX QUANTUM MECHANICS OF A SINGLE HERMITIAN MATRIX M_{ij}

$$H = -\frac{\partial^2}{\partial M_{ij} \partial M_{ij}} + V(M)$$

WHERE $V(M)$ IS INVARIANT UNDER $U(N)$

IN THIS CASE ALL THE INVARIANTS CAN BE PACKED INTO A SINGLE SCALAR FIELD IN 1+1 DIMENSIONS

$$\rho(x,t) \equiv \frac{1}{N} \mathrm{Tr}\, \delta(M - x\mathbb{I})$$

THE HAMILTONIAN WHICH DESCRIBES THE EVOLUTION OF SINGLET STATES, $H[\rho, \Pi_\rho]$ CAN BE DERIVED

COLLECTIVE FIELD THEORY

THIS IS WELL KNOWN TO DESCRIBE STRING THEORY IN 1+1 DIMENSIONS

A NEW HOLOGRAPHIC DIRECTION ARISES FROM THE SPACE OF EIGENVALUES.

(S.R.D & A.Jevicki, 1990)

THIS HAS TO BE INTERPRETED AS SPACE DIMENSION.

A MORE FAMILIAR EXAMPLE IS OF COURSE THE AdS/CFT CORRESPONDENCE

IN THIS CASE THE HIGHER DIMENSIONAL GRAVITATIONAL DESCRIPTION WAS FIGURED OUT DURING AN INCREDIBLE JOURNEY THROUGH D-BRANES AND BLACK HOLES (Maldacena; GKP; Witten)

AS EXPECTED THE DUALS ARE STRING THEORIES

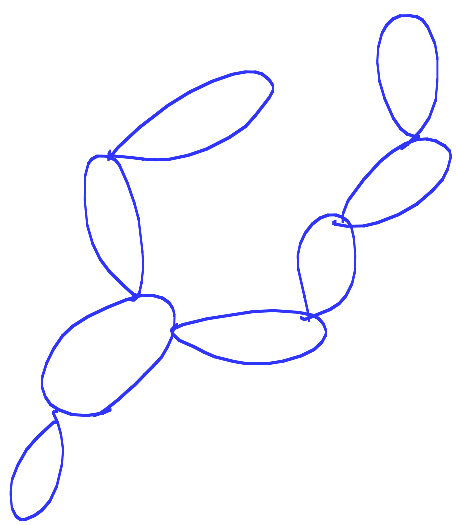
AdS/CFT WITH GAUGE THEORIES ARE STRONG-WEAK DUALITIES – WHICH IS WHY IT IS USEFUL

HOWEVER, FOR THE SAME REASON, IT IS PROBABLY IMPOSSIBLE TO DERIVE THIS CORRESPONDENCE – EXCEPT IN SPECIAL LIMITS

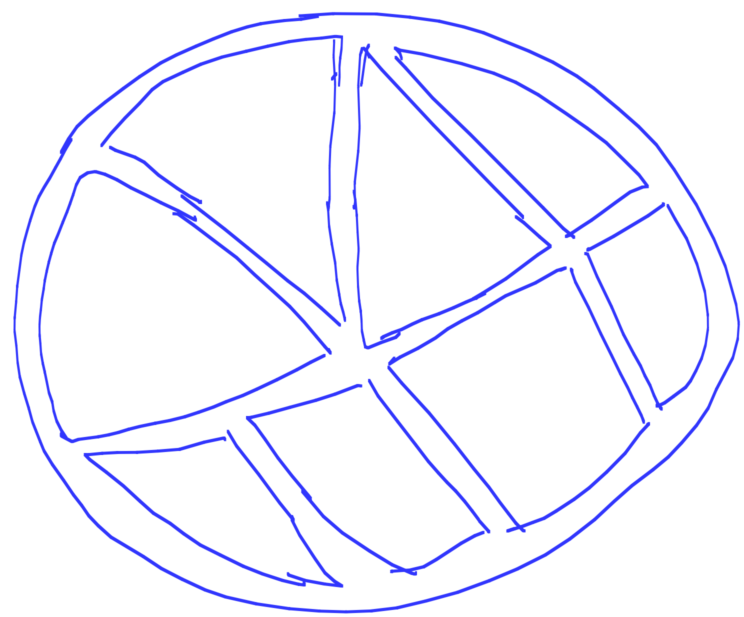
$\Rightarrow$ USEFUL TO FIND OTHER EXACT EXAMPLES

$\boxed{\text{VECTOR MODELS}}$

- LARGE N MODELS WITH FIELDS IN FUNDAMENTAL REPRESENTATION ARE NOT EXPECTED TO BE DUAL TO STRING THEORIES



VECTOR



MATRIX

- WE KNOW WHAT STRING THEORIES ARE LIKE

– BUT WHAT ARE THESE POLYMER-LIKE THEORIES WHICH DESCRIBE VECTOR MODELS ?

TEN YEARS AGO Klebanov & Polyakov PROVIDED A RATHER SURPRISING ANSWER TO THIS QUESTION FOR $O(N)$ MODELS

CONJECTURE : SINGLET SECTOR OF $O(N)$ VECTOR MODEL IN $2+1$ DIMENSIONS AT CONFORMAL POINTS IS DUAL TO VASILIEV HIGHER SPIN GAUGE THEORY IN AdS_4

THE TWO FIXED POINTS CORRESPOND TO TWO WAYS OF QUANTIZING THE CONFORMAL SCALAR IN THE MULTIPLET

THERE HAS BEEN A REVIVAL OF INTEREST IN THIS DUALITY RECENTLY

(Giombi & Yin,
Douglas & Razamat,
Shenker & Yin)

AND RELATED DUALITIES INVOLVING CFT_2

(Gaberdiel & Gopakumar)

$$\boxed{\text{COLLECTIVE FIELDS \& HOLOGRAPHY}}$$

(S.R.D & A. JEVICKI (2003))

THE HAMILTONIAN IS (SCHRÖDINGER PICTURE)

$$H = \int d^{d-1}x \, \frac{1}{2} \vec{\Pi}(x) \vec{\Pi}(x)$$

$$[\Pi_i(\vec{x}), \phi^j(\vec{x}')] = -i \, \delta_i^j \, \delta(\vec{x} - \vec{x}')$$

$$\Pi_i(\vec{x}) \rightarrow -i \frac{\delta}{\delta \phi^i(x)}$$

WE WANT TO WRITE DOWN A HAMILTONIAN WHICH CAPTURES THE PHYSICS OF THE SINGLET SECTOR OF THE THEORY

THE PROBLEM IS ENTIRELY ANALOGOUS TO THE PROBLEM OF A PARTICLE IN D DIMENSIONS

$$H = -\frac{\partial^2}{\partial x^i \partial x^i} + V(r) \qquad r = \sqrt{\vec{x}^2}$$

WHAT IS THE EFFECTIVE HAMILTONIAN IN THE SINGLET SECTOR?

THE KEY IS TO FIGURE OUT THE JACOBIAN

$$\prod_i dx_i = dr\, J(r)\, d\Omega$$

NOW DEFINE A RADIAL WAVEFUNCTION

$$\chi(r) = J^{1/2}(r)\, \Psi(r)$$

$$i \frac{\partial \chi}{\partial t} = H_r(r, \partial_r)\, \chi$$

H_r CAN BE OBTAINED BY DEMANDING HERMITICITY

(Jevicki & Sakita)

THE ANSWER IS

$$J(r) = r^{D-1} \qquad H_r = -\frac{d^2}{dr^2} + \frac{(D-1)(D-3)}{4r^2}$$

FOR A O(N) VECTOR MODEL IN $d-1$ SPACE DIMENSIONS
THE RADIAL VARIABLE IS A BILOCAL FIELD

$$\sigma(\vec{x}, \vec{y}) \equiv \vec{\phi}(x) \cdot \vec{\phi}(y)$$

THE ORIGINAL PROBLEM IS

$$i \frac{\partial \Psi[\vec{\phi}(x), t]}{\partial t} = \left[-\frac{\partial^2}{\partial \phi^i(x) \partial \phi^i(x)} + V \right] \Psi[\vec{\phi}(x), t]$$

WE WANT THE SINGLET SECTOR HAMILTONIAN

$$\Psi[\phi] = J^{1/2} \chi[\sigma]$$

$$i \partial_t \chi = H_c[\sigma, \pi_\sigma] \chi$$

FOR THE FREE THEORY

$$\boxed{H = \frac{2}{N} \operatorname{Tr}\left(\Pi_\sigma \sigma \Pi_\sigma\right) + \frac{N}{8} \operatorname{Tr} \sigma^{-1} + N \int dx \left(-\partial_x^2 \sigma(x,y)\right)\Big|_{x=y} + O(1/N)}$$

WHERE $\sigma(x,y)$ IS REGARDED AS A MATRIX

THE ACTION WHICH FOLLOWS FROM THIS HAS AN OVERALL FACTOR OF N

THUS THE LEADING ORDER IN $1/N$ EXPANSION IS EVALUATED AT THE SADDLE POINT

SINCE THE SADDLE POINT SOLUTION SHOULD BE TRANSLATIONALLY INVARIANT — GO TO MOMENTUM SPACE

THE SADDLE POINT SOLUTION FOR THE FREE THEORY IS

$$\left[\sigma_0(k_1,k_2)\right]^2 = \frac{N^2}{4\vec{k}_1^2}\,\delta(\vec{k}_1-\vec{k}_2).$$

WHICH SIGN DO WE CHOOSE

THIS HAS TO BE DONE TO ENSURE THAT

$$\sigma_0(k_1,k_2) = \langle \phi^i(k,t)\,\phi^i(k_2,t)\rangle = \frac{N}{2\sqrt{k_1^2}}\,\delta(k_1-k_2)$$

THUS WE MUST CHOOSE THE + SIGN

TO FIND THE SPECTRUM WE NEED TO EXPAND AROUND THE SADDLE POINT

$$\sigma = \sigma_0 + \frac{1}{\sqrt{N}}\eta \qquad \Pi_\sigma = \sqrt{N}\, P_\sigma$$

NOW EXPAND

$$\mathrm{Tr}\,\sigma^{-1} = \mathrm{Tr}\,\sigma_0^{-1} + \sum_{n=1}^{\infty} \frac{(-1)^n}{N^{n/2}} \mathrm{Tr}\left(\sigma_0 (\eta\, \sigma_0)^n\right)$$

LEADING TO AN INTERACTING THEORY

$$\boxed{\begin{array}{ll} \text{Quadratic:} & H_2 = 2\,\mathrm{Tr}(P_\sigma \sigma_0 P_\sigma) + \frac{1}{8}(\sigma_0 \eta\, \sigma_0 \eta\, \sigma_0) \\ \text{Cubic:} & H_3 = \frac{2}{\sqrt{N}} \mathrm{Tr}(P_\sigma \eta P_\sigma) - \frac{1}{8\sqrt{N}} \mathrm{Tr}(\sigma_0 \eta\, \sigma_0 \eta\, \sigma_0 \eta\, \sigma_0) \end{array}}$$

ADDITION OF A POTENTIAL IS TRIVIAL

$$V(\phi \cdot \phi) \to V(\sigma)$$

THIS SHIFTS THE SADDLE POINT

USING THIS HAMILTONIAN WE COULD SHOW THAT THE CORRELATORS ARE CORRECTLY REPRODUCED

IN PARTICULAR A CALCULATION OF THE 2 POINT FUNCTION OF η OF THE LINEAR O(N) SIGMA MODEL WE SHOWED THAT THE IR LIMIT LEADS TO THE WILSON-FISHER FIXED POINT

THE SUBLEADING $1/N$ TERMS WHICH I DID NOT WRITE DOWN PLAY A CRUCIAL ROLE IN LOOP CALCULATIONS

THE PROPOSAL :

$$\boxed{\text{THE BILOCAL FIELD IS IN FACT A COLLECTION OF ALL OF VASILIEV'S HIGHER SPIN FIELDS IN } d+1 \text{ DIMENSIONS}}$$

FOR THIS TO WORK, THE FIELD CONTENT SHOULD BE CORRECT. TO SEE THIS WRITE

$$\vec{u} = \frac{1}{2}(\vec{x}+\vec{y}) \qquad \vec{v} = (\vec{x}-\vec{y}) \equiv (\rho, \Omega_{d-2})$$

$$\sigma(\vec{x},\vec{y}) = \sum \sigma_{\ell,\{m\}}(\vec{u},\rho)\, Y_{\ell\{m\}}(\Omega_{d-2})$$

$\sigma_{\ell,\{m\}}(\vec{u},\rho)$: SPIN ℓ FIELD IN d SPACE DIM.

$\vec{x} \leftrightarrow \vec{y}$ SYMMETRY $\Rightarrow$ ℓ EVEN

CONSIDER NOW THE EXPANSION FOR 2+1 DIM.

$$\sigma(\vec{x},\vec{y}) = \sum_{\substack{l=0\\ \pm 2\\ \pm 4}} \sigma_l(\vec{x},\rho)\, e^{il\theta}$$

OUR INITIAL CONJECTURE WAS THAT ρ IS THE POINCARE RADIAL COORDINATE IN AdS_4

IT TURNS OUT THAT THIS NOT QUITE CORRECT

SOME TIME AGO de Mello Koch, Jevicki, Jin & Rodrigues SHOWED HOW TO CORRECTLY IDENTIFY THE USUAL POINCARE COORDINATE z IN AdS – THIS INVOLVES A NONLOCAL (BUT LINEAR) FIELD REDEFINITION IN LIGHT FRONT QUANTIZATION

MODES OF THIS FIELD IDENTIFIED WITH VASILIEV'S FIELDS IN LIGHT CONE GAUGE

SP(N) HOLOGRAPHY (?)

Anninos, Hartman & Strominger :
SINGLET SECTOR OF Sp(N) VECTOR MODEL IS DUAL TO HIGHER SPIN THEORY IN DE SITTER

IF THIS CLAIM IS CORRECT, THE HOLOGRAPHIC DIRECTION WHICH ARISES OUT OF LARGE-N DEGREES OF FREEDOM HAS TO BE INTERPRETED AS TIME

WE WILL NOW DERIVE A DUAL THEORY TO Sp(N) USING COLLECTIVE FIELD THEORY IDEAS

INDICATE WHY A dS RATHER THAN AdS CAN POSSIBLY ARISE

Sp(N) VECTOR MODEL HAS 2N GRASSMANN ODD SCALAR FIELDS (A THEORY OF GHOSTS !)

$$\chi_a^i(\vec{x},t) \qquad i = 1,2,\dots N. \quad a = 1,2$$

THE FREE ACTION IS (IN MOMENTUM SPACE)

$$\boxed{S = i\int dt\, d^d\vec{x} \left[\dot{\chi}_1^i \dot{\chi}_2^i - \partial_{\vec{x}}\chi_1^i \partial_{\vec{x}}\chi_2^i \right]}$$

THIS LEADS TO A HAMILTONIAN

$$\boxed{H = i\int d^dx \left[P_2^i P_1^i + \partial_{\vec{x}}\chi_1^i \partial_{\vec{x}}\chi_2^i \right]}$$

$$(\chi_a^i)^\dagger = \chi_a^i \qquad (P_a^i)^\dagger = -P_a^i$$

$$\boxed{\{ \chi_a^i(\vec{x}), P_b^j(\vec{y}) \} = -i\,\delta^{ij}\delta_{ab}\,\delta(\vec{x}-\vec{y})}$$

(Henneaux & Teitelboim)

A DIRECT CONSEQUENCE OF THIS IS

$$\langle T\, \chi_2^i(\vec{k},t)\, \chi_1^j(\vec{k}',t')\rangle = \frac{\delta^{ij}}{2|k|}\, e^{-i|k||t-t'|}\, \delta(\vec{k}-\vec{k}')$$

THE RESULT IS EXACTLY THE SAME AS THE BASIC PROPAGATOR OF THE $O(N)$ THEORY

$$\langle \phi^i(k,t)\, \phi^j(k',t')\rangle$$

SCHRODINGER PICTURE

$$\chi_a^i(\vec{x}) \longrightarrow \chi_a^i(\vec{x}) \qquad \text{(GRASSMAN)}$$

$$P_a^i(\vec{x}) \longrightarrow -i\,\frac{\delta}{\delta \chi_a^i(\vec{x})}$$

$$H = \int d^dx \left[-i\,\frac{\delta}{\delta \chi_2^i}\,\frac{\delta}{\delta \chi_1^i} + i\,\partial_x \chi_1^i\, \partial_x \chi_2^i \right]$$

AND CHANGE VARIABLES

WE WILL STUDY THE SINGLET SECTOR OF THIS MODEL BY CHANGING VARIABLES TO SUITABLE COLLECTIVE FIELDS

$$\rho(\vec{x},\vec{y}) \equiv i\,\chi_1^i(\vec{x})\,\chi_2^i(\vec{y})$$

AS BEFORE, THE MAIN JOB IS TO FIND THE JACOBIAN

$$\int \prod_i d\chi_1^i\, d\chi_2^i\; F[\rho] = \int d\rho(\vec{x},\vec{y})\; J[\rho]\; F[\rho]$$

CONSIDER FOR EXAMPLE 0+1 DIMENSIONS

$$J[\rho] = \int \prod_i d\chi_1^i\, d\chi_2^i\; \delta[\rho - i\chi_1^i \chi_2^i] = \rho^{-N}$$

COMPARE WITH BOSONIC CASE $J \sim \rho^{N/2}$

FOR FIELD THEORY A DIRECT CALCULATION IS NOT POSSIBLE – BUT THE JACOBIAN IS AGAIN OBTAINED BY REQUIRING HERMITICITY

THIS LEADS TO A CRUCIAL MINUS SIGN IN $\log J$ – AS EXPECTED FOR FERMIONS – THE SIGN BEING OPPOSITE TO THAT OBTAINED IN THE O(N) MODEL

REMARKABLY THE COLLECTIVE HAMILTONIAN $H[\pi_\rho, \rho]$ IS EXACTLY THE SAME AS THE O(N) MODEL !

$$\boxed{H = \frac{2}{N}\,\mathrm{Tr}\left(\Pi_\rho\, \rho\, \Pi_\rho\right) + \frac{N}{8}\,\mathrm{Tr}\,\rho^{-1} - \frac{N}{2}\int dx \left[\partial_x^2 \rho(x,y)\right]_{x=y}}$$

THIS MEANS THAT THE SADDLE POINT EQUATION IS ALSO THE SAME

$$\left[\rho_0(k_1,k_2)\right]^2 = \frac{N^2}{4k_1^2}\,\delta(k_1-k_2)$$

WHICH SIGN DO WE CHOOSE NOW ?

WE KNOW

$$\rho_0(k_1,k_2) = i\left\langle \chi_1^i(k_1)\,\chi_2^i(k_2)\right\rangle = -\frac{N}{2\sqrt{k_1^2}}\,\delta(k_1-k_2)$$

THIS IS BECAUSE WE SHOWED

$$i\left\langle \chi_2^i(k_1)\,\chi_1^i(k_2)\right\rangle = \frac{N}{2\sqrt{k_1^2}}\,\delta(k_1-k_2)$$

THEREFORE WE MUST CHOOSE <u>NEGATIVE</u> SIGN

IN SOME SENSE COLLECTIVE FIELD THEORY IS LIKE A BACKGROUND INDEPENDENT THEORY

DIFFERENT SOLUTIONS CORRESPOND TO DIFFERENT MICROSCOPIC HAMILTONIANS

(THIS KIND OF THING WAS ALREADY CLEAR FOR THE 2d STRING THEORY)

THIS HAS AN INTERESTING CONSEQUENCE

THE HAMILTONIAN FOR FLUCTUATIONS AROUND THE SADLLE POINT INVOLVES ρ_0

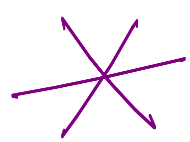
$$H_2 = 2\,\mathrm{Tr}(\Pi_\rho \rho_0 \Pi_\rho) + \frac{1}{8}\mathrm{Tr}(\rho_0 \eta \rho_0 \eta \rho_0)$$

$$H_3 = \frac{2}{\sqrt{N}}\mathrm{Tr}(\Pi_\rho \eta \Pi_\rho) - \frac{1}{8\sqrt{N}}\mathrm{Tr}(\rho_0 \eta \rho_0 \eta \rho_0 \eta \rho_0)$$

. . . . .

COMPARED TO THE CORRESPONDING EXPRESSIONS FOR $O(N)$

PROPAGATOR GETS A MINUS SIGN



n POINT VERTEX GETS $(-1)^{n+1}$

THIS IMPLIES THAT TO LEADING ORDER IN LARGE N

ANY m-POINT FUNCTION OF THE COLLECTIVE FIELD

$$\boxed{\begin{aligned}&\langle \rho(x_1 y_1 t_1)\, \rho(x_2 y_2, t_2) \cdots \rho(x_m y_m t_m) \rangle_{Sp(N)} \\ &= - \langle \sigma(x_1, y_1 t_1) \cdots \sigma(x_m y_m t_m) \rangle_{SO(N)}\end{aligned}}$$

dS SPACE

QUADRATIC ACTION OF HIGHER SPIN FIELDS WHICH WOULD FOLLOW FROM THE Sp(N) COLLECTIVE THEORY WOULD HAVE A NEGATIVE SIGN COMPARED TO O(N).

WE KNOW THAT THE LATTER DESCRIBES HIGHER SPIN FIELDS IN AdS SPACE

CONSIDER e.g. THE SCALAR ACTION

$$S_{Sp(N)} = -\int dt\,dz\,d^2x \left[\frac{1}{2z^2}\left\{ (\partial_t\phi)^2 - (\partial_x\phi)^2 - (\partial_z\phi)^2 \right\} + \frac{1}{z^4}\phi^2 \right]$$

$\sqrt{g}\, g^{AB}$ $\qquad$ $\frac{1}{6}\sqrt{g}\, R\phi^2$

$$ds^2 = \frac{1}{z^2}\left[-dt^2 + d\vec{x}^2 + dz^2 \right]$$

THE OVERALL MINUS SIGN IS TROUBLING

HOWEVER, FIRST EUCLIDEANIZE THE ORIGINAL $Sp(N)$ MODEL. SINCE t IS THE TIME IN THE $Sp(N)$ MODEL

$$S^{(E)}_{Sp(N)} = -\int dt\,dz\,d^2x \left[\frac{1}{2z^2}\left(\dot{\phi}^2 + (\partial_x\phi)^2 + (\partial_z\phi)^2\right) - \frac{1}{z^4}\phi^2 \right]$$

THIS CRIES OUT FOR A CONTINUATION $z \to iz$ LEADING TO

$$\tilde{S}_{Sp(N)} = \int dz\,dt\,d^2x \left[\frac{1}{2z^2}\left\{ (\partial_z\phi)^2 - (\partial_t\phi)^2 - (\partial_x\phi)^2 \right\} - \frac{1}{z^4}\phi^2 \right]$$

THE CONFORMAL COUPLING TERM DOES NOT CHANGE SIGN

$\Rightarrow$ THIS IS ACTION OF CONFORMALLY COUPLED SCALAR IN DE SITTER WHOSE TIME IS z

- THIS WORKS FOR THE QUADRATIC ACTION ONLY FOR THE EVEN SPIN FIELDS

MIGHT AS WELL – LIKE ITS $O(N)$ COUNTERPART, OUR COLLECTIVE FIELD THEORY ONLY CONTAINS EVEN SPIN FIELDS

THUS REWRITING THE $Sp(N)$ MODEL IN TERMS OF COLLECTIVE FIELDS LEADS TO A NATURAL INTERPRETATION WITH A DOUBLE ANALYTIC CONTINUATION

THIS MAKES THE $Sp(N)$ EUCLIDEAN AND THE HOLOGRAPHIC DIRECTION TIMELIKE

VERY SIMILAR TO THE WAY LIOUVILLE MODE APPEARS AS TIME FOR SUPERCRITICAL STRINGS

WE ARE NOW LOOKING AT

- INTERACTION TERMS
- dS/CFT VS AdS/CFT ISSUES
- A DERIVATION WHICH AVOIDS THE INTERMEDIATE STEP OF GOING TO LIGHT CONE

THE COLLECTIVE HAMILTONIAN H_c IS DERIVED USING A TRICK (Jevicki & Sakita)

CONSIDER THE PROBLEM OF QUANTUM MECHANICS IN D DIMENSIONS.
ACTING OF WAVE FUNCTIONS $\chi(r)$

$$-\frac{1}{2}\frac{\partial^2}{\partial x_i \partial x_i}\chi = \frac{1}{2\sqrt{J}}\Big[(\omega C - \Omega C^2 - \Omega \partial_r C)\chi + (\omega - 2\Omega C + \partial_r \Omega)\partial_r \chi - \partial_r(\Omega \partial_r)\chi\Big]$$

WHERE

$$\omega = -\partial_i \partial_i r \qquad \Omega = \partial_i r \, \partial_i r$$

$$C = -\frac{1}{2}\partial_r \log J$$

THIS WILL LEAD TO A HERMITIAN HAMILTONIAN WITH TRIVIAL MEASURE $\int dr$ PROVIDED

$$\omega - 2\Omega C + \partial_r \Omega = 0 \quad \Rightarrow \quad \boxed{J = r^{D-1}}$$

$$\eta(x_1^-, x_1 ; x_2^-, x_2) \rightarrow \eta(p_1^+, p_1 ; p_2^+, p_2)$$

IF WE DEFINE A FIELD

$$\Phi(p^+, p ; p^z, \theta)$$

$$\equiv \int dp_1^+ dp_1 \, dp_2^+ dp_2 \; \delta(p_1^+ + p_2^+ - p^+) \, \delta(p_1 + p_2 - p)$$
$$\delta\left(p^z - p_1\sqrt{p_2^+/p_1^+} - p_2\sqrt{p_1^+/p_2^+}\right)$$
$$\delta\left(\theta - 2\tan^{-1}\left(\sqrt{p_2^+/p_1^+}\right)\right)$$
$$\eta(p_1^+, p_1 ; p_2^+, p_2)$$

AND THEN FOURIER TRANSFORM IN θ

DENOTE THE CONJUGATES (x^-, x, z)

$$\{x^-, p^+\} = \{x, p^x\} = \{z, p^z\} = 1$$

THEN

- (x^-, x, z) ARE PRECISELY THE POINCARE COORDINATES

$$ds^2 = \frac{1}{z^2}\left[2dx^+dx^- + dx^2 + dz^2\right]$$

- THE FOURIER COMPONENTS ARE PRECISELY VASILIEV'S HIGHER SPIN FIELDS IN LIGHT CONE GAUGE

e.g.

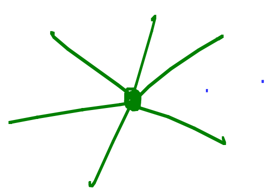
THE SCALAR IN VASILIEV THEORY

$$\varphi(x^-, x, z) = z\int d\theta \; \Phi(x^-, x, z, \theta)$$

THIS IMPLIES THAT TO LEADING ORDER IN LARGE N ANY m-POINT FUNCTION OF THE COLLECTIVE FIELD

$$\boxed{\left\langle \rho(x_1,y_1,t_1)\cdots\cdots\rho(x_m\,y_m\,t_m)\right\rangle_{Sp(N)} = -\left\langle \sigma(x_1\,y_1\,t_1),\cdots\sigma(x_m\,y_m\,t_m)\right\rangle_{SO(N)}}$$

THE LEADING ORDER ANSWER IS THE SUM OVER ALL TREE DIAGRAMS – THE ABOVE RULES SHOW THAT ALL TREE DIAGRAMS HAVE A MINUS SIGN



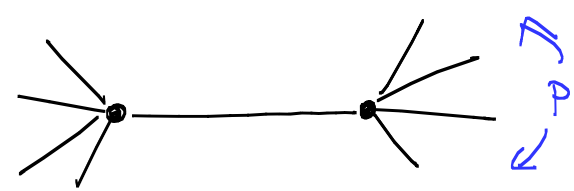
m POINT

$(-1)^m(-1)^{m+1}$

m LINES

VERTEX

$(m-p)$



p

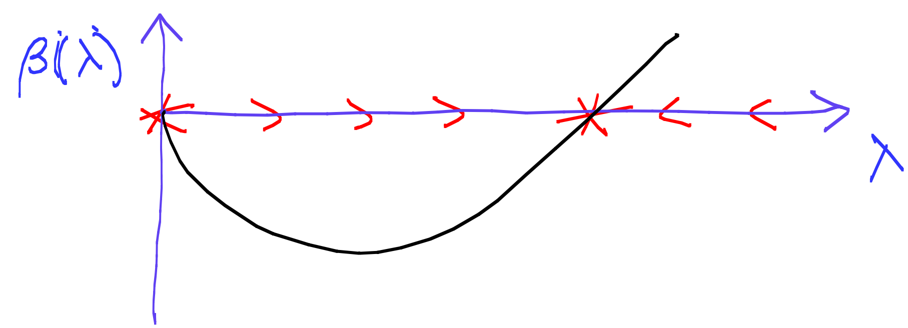
(-1) $\quad(-1)^{p+1}(-1)^{p-1}$

$= (-1)$

CONSIDER THE LINEAR VERSION

$$\mathcal{L} = -\frac{1}{2}(\partial\vec{\phi})^2 + \lambda\left(\vec{\phi}^2 - \frac{m^2}{2\lambda}\right)^2$$

IN 2+1 DIMENSIONS THIS MODEL HAS A NONTRIVIAL IR FIXED POINT



THE TWO CFT-S CORRESPOND TO TWO DIFFERENT QUANTIZATIONS OF VASILIEV THEORY IN AdS_4