Non-extremality and the IR limit of large N QCD

Mohammed Mia

Department of Physics, Columbia University,
New York, USA.

March 2 2012, Great Lakes Strings Conference
Purdue University, IN, USA.
Based On

- ‘Phase transitions in holographic QCD’, Mohammed Mia, Fang Chen and Miklos Gyulassy, To Appear.
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’.
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’. Anisotropic expansion of the matter is modeled with hydrodynamics- with minimal viscosity.
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’. Anisotropic expansion of the matter is modeled with hydrodynamics- with minimal viscosity.

The empirical evidence suggests QCD matter created at RHIC is strongly coupled.
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’. Anisotropic expansion of the matter is modeled with hydrodynamics- with minimal viscosity.

The empirical evidence suggests QCD matter created at RHIC is strongly coupled.

Elliptic flow at LHC is also consistent with strongly coupled quark matter.
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’. Anisotropic expansion of the matter is modeled with hydrodynamics- with minimal viscosity.

The empirical evidence suggests QCD matter created at RHIC is strongly coupled.

Elliptic flow at LHC is also consistent with strongly coupled quark matter.

Estimates of plasma temperature suggests $T \geq T_c$ but not asymptotically high.
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’. Anisotropic expansion of the matter is modeled with hydrodynamics- with minimal viscosity.

The empirical evidence suggests QCD matter created at RHIC is strongly coupled.

Elliptic flow at LHC is also consistent with strongly coupled quark matter.

Estimates of plasma temperature suggests $T \geq T_c$ but not asymptotically high.

Analysis of the plasma requires understanding of strongly coupled QCD. The conventional approach is to use lattice QCD- but it’s not the only option.
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’. Anisotropic expansion of the matter is modeled with hydrodynamics- with minimal viscosity.

The empirical evidence suggests QCD matter created at RHIC is strongly coupled.

Elliptic flow at LHC is also consistent with strongly coupled quark matter.

Estimates of plasma temperature suggests $T \geq T_c$ but not asymptotically high.

Analysis of the plasma requires understanding of strongly coupled QCD. The conventional approach is to use lattice QCD- but it’s not the only option.

Hilbert space of certain quantum field theories is contained in the Hilbert space of gravity.
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’. Anisotropic expansion of the matter is modeled with hydrodynamics- with minimal viscosity.

The empirical evidence suggests QCD matter created at RHIC is strongly coupled.

Elliptic flow at LHC is also consistent with strongly coupled quark matter.

Estimates of plasma temperature suggests $T \geq T_c$ but not asymptotically high.

Analysis of the plasma requires understanding of strongly coupled QCD. The conventional approach is to use lattice QCD- but it’s not the only option.

Hilbert space of certain quantum field theories is contained in the Hilbert space of gravity. Conformal Field Theory $\leftrightarrow$ Anti De Sitter Space [Maldacena, ’97]
The plasma created in ultra relativistic heavy ion collisions at RHIC is characterized as the most ‘perfect fluid’. Anisotropic expansion of the matter is modeled with hydrodynamics- with minimal viscosity.

The empirical evidence suggests QCD matter created at RHIC is strongly coupled.

Elliptic flow at LHC is also consistent with strongly coupled quark matter.

Estimates of plasma temperature suggests $T \geq T_c$ but not asymptotically high.

Analysis of the plasma requires understanding of strongly coupled QCD. The conventional approach is to use lattice QCD- but it’s not the only option.

Hilbert space of certain quantum field theories is contained in the Hilbert space of gravity. Conformal Field Theory $\leftrightarrow$ Anti De Sitter Space [Maldacena, ’97]

However QCD is non-conformal in the temperature regime explored by the heavy ion collisions.
Large conformal anomaly! Is there a gravity description for a QCD like theory?
Large conformal anomaly! Is there a gravity description for a QCD like theory? Yes!

Non Conformal Field Theory $\iff$ Conifold geometries with fluxes

[Klebanov-Tseytlin, 2000, Klebanov-Strassler,’01]
Large conformal anomaly! Is there a gravity description for a QCD like theory? Yes!

Non Conformal Field Theory $\iff$ Conifold geometries with fluxes [Klebanov-Tseytlin, 2000, Klebanov-Strassler,’01]

The gauge theory degrees of freedom $N_{\text{eff}}$ diverges in the UV.
Large conformal anomaly! Is there a gravity description for a QCD like theory? Yes!

Non Conformal Field Theory $\iff$ Conifold geometries with fluxes [Klebanov-Tseytlin, 2000, Klebanov-Strassler,'01]

The gauge theory degrees of freedom $N_{\text{eff}}$ diverges in the UV. We modify UV of KS model and introduce fundamental matter by embedding seven branes.
Large conformal anomaly! Is there a gravity description for a QCD like theory? Yes!

Non Conformal Field Theory \iff Conifold geometries with fluxes [Klebanov-Tseytlin, 2000, Klebanov-Strassler,’01]

The gauge theory degrees of freedom $N_{\text{eff}}$ diverges in the UV. We modify UV of KS model and introduce fundamental matter by embedding seven branes.

We find new non-extremal solutions of modified KS model derived directly from 10d type IIB action with fluxes.
Large conformal anomaly! Is there a gravity description for a QCD like theory? Yes!

Non Conformal Field Theory ⇐⇒ Conifold geometries with fluxes
[Klebanov-Tseytlin, 2000, Klebanov-Strassler,'01]

The gauge theory degrees of freedom $N_{\text{eff}}$ diverges in the UV. We modify UV of KS model and introduce fundamental matter by embedding seven branes.

We find new non-extremal solutions of modified KS model derived directly from 10d type IIB action with fluxes.

Using this black hole geometry, we explore the thermodynamics of the gauge theory and find qualitative consistency with thermal QCD.
Non-Conformal Field Theory: The Brane Setup

D3 Branes

D5 Branes

S_3

S_2
The gauge group is $SU(N + M) \times SU(N)$, logarithmic running of gauge coupling but $N_{\text{eff}}$ diverges in the UV and the theory has Landau Poles.
Non-Conformal Field Theory: The Brane Setup

S\textsuperscript{3} Branes

D\textsuperscript{3} Branes

D5 Branes

S\textsuperscript{2} Branes

Anti 5 Branes

7 Branes

Mohammed

Holographic Thermal QCD
Now we have $SU(N + M) \times SU(N)$ gauge group in the IR, $SU(N + M) \times SU(N + M)$ in the UV and $SU(N_f) \times SU(N_f)$ flavor symmetry with fundamental matter and logarithmic running of gauge coupling. $N_{\text{eff}}$ large but finite in the far UV and gauge group cascades to $SU(\bar{M})$ in the far IR.
Now we have $SU(N + M) \times SU(N)$ gauge group in the IR, $SU(N + M) \times SU(N + M)$ in the UV and $SU(N_f) \times SU(N_f)$ flavor symmetry with fundamental matter and logarithmic running of gauge coupling. $N_{\text{eff}}$ large but finite in the far UV and gauge group cascades to $SU(\bar{M})$ in the far IR.

How to analyze the strongly coupled gauge theory?
Dual Geometry

For large $M$, 'tHooft coupling $\Lambda_1 = g_1 (k + 1) M$, $\Lambda_2 = g_2 k M \gg 1$, and the gauge theory can be described by type IIB supergravity

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ R - \frac{\partial_a \tau \partial^a \tau}{2|\text{Im}\tau|^2} - \frac{G_3 \cdot \bar{G}_3}{12 \text{Im}\tau} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right]$$

$$+ \frac{1}{8i\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\text{Im}\tau} + S_{\text{loc}}$$
Dual Geometry

For large $M$, 'tHooft coupling $\Lambda_1 = g_1(k + 1)M$, $\Lambda_2 = g_2 kM \gg 1$, and the gauge theory can be described by type IIB supergravity

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ R - \frac{\partial_\alpha \partial^\alpha}{2|\text{Im}\tau|^2} - \frac{G_3 \cdot \bar{G}_3}{12\text{Im}\tau} - \frac{\bar{F}_5^2}{4 \cdot 5!} \right]$$

$$+ \frac{1}{8i\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\text{Im}\tau} + S_{\text{loc}}$$

The metric takes the form

$$ds^2 = \frac{1}{\sqrt{h}} \left[ -g_1(r)dt^2 + dx^2 + dy^2 + dz^2 \right] + \sqrt{h} \left[ g_2(r)^{-1}dr^2 + dM_5^2 \right]$$

$$\equiv -e^{2A+2B} dt^2 + e^{2A} \delta_{ij}dx^i dx^j + e^{-2A-2B} \bar{g}_{mn}dx^m dx^n$$
For large } M \text{, 'tHooft coupling } \Lambda_1 = g_1(k + 1)M, \Lambda_2 = g_2kM \gg 1\text{, and the gauge theory can be described by type IIB supergravity}

\[ S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ R - \frac{\partial a \tau \partial a \tau}{2|\text{Im}\, \tau|^2} - \frac{G_3 \cdot \bar{G}_3}{12\text{Im}\, \tau} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right] + \frac{1}{8i\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\text{Im}\, \tau} + S_{\text{loc}} \]

The metric takes the form

\[ ds^2 = \frac{1}{\sqrt{h}} \left[ -g_1(r)dt^2 + dx^2 + dy^2 + dz^2 \right] + \sqrt{h} \left[ g_2(r)^{-1}dr^2 + dM_5^2 \right] \]

\[ \equiv -e^{2A+2B}dt^2 + e^{2A}\delta_{ij}dx^i dx^j + e^{-2A-2B}\bar{g}_{mn}dx^m dx^n \]

Extremal limit is achieved when } e^{2B} = 1\text{. Non-extremality is the limit } e^{2B} \equiv e^{2B(x^m)} \text{ with a regular horizon } x^m_H \text{ such that } e^{2B(x^m_H)} = 0, e^{-2A(x^m_H)} \neq 0 \}
For large $M$, 'tHooft coupling $\Lambda_1 = g_1 (k + 1) M$, $\Lambda_2 = g_2 k M \gg 1$, and the gauge theory can be described by type IIB supergravity

$$S_{\text{IIB}} = \frac{1}{2 \kappa_{10}^2} \int d^{10} x \sqrt{-g} \left[ R - \frac{\partial a \tau \partial a \tau}{2 |\text{Im} \tau|^2} - \frac{G_3 \cdot \tilde{G}_3}{12 \text{Im} \tau} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right]$$

$$+ \frac{1}{8i \kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \tilde{G}_3}{\text{Im} \tau} + S_{\text{loc}}$$

The metric takes the form

$$ds^2 = \frac{1}{\sqrt{h}} \left[ -g_1(r) dt^2 + dx^2 + dy^2 + dz^2 \right] + \sqrt{h} \left[ g_2(r)^{-1} dr^2 + dM_5^2 \right]$$

$$\equiv -e^{2A+2B} dt^2 + e^{2A} \delta_{ij} dx^i dx^j + e^{-2A-2B} \tilde{g}_{mn} dx^m dx^n$$

Extremal limit is achieved when $e^{2B} = 1$. Non-extremality is the limit $e^{2B} = e^{2B(x^m)}$ with a regular horizon $x_H^m$ such that $e^{2B(x_H^m)} = 0$, $e^{-2A(x_H^m)} \neq 0$

Warped four dimensional Minkowski space and warped six dimensional cone with five dimensional compact space $S_5$ as it's base.
With fluxes and warp factors $A, B$ only depending on cone coordinates $x^m$, we have the Einstein equations

$$R_{\mu\nu} = -g_{\mu\nu} \left( \frac{G_3 \cdot \bar{G}_3}{48 \text{Im}\tau} + \frac{\bar{F}_5^2}{8 \cdot 5!} \right) + \frac{\bar{F}_{\mu abcd} \bar{F}_{\nu abcd}}{4 \cdot 4!} + \kappa_{10}^2 \left( T_{\mu\nu}^{\text{loc}} - \frac{1}{8} g_{\mu\nu} T^{\text{loc}} \right)$$

$$R_{mn} = -g_{mn} \left( \frac{G_3 \cdot \bar{G}_3}{48 \text{Im}\tau} + \frac{\bar{F}_5^2}{8 \cdot 5!} \right) + \frac{\bar{F}_{m abcd} \bar{F}_{n abcd}}{4 \cdot 4!} + \frac{G_{m b c} \bar{G}_{n b c}}{4 \text{Im}\tau} + \frac{\partial_m \tau \partial_n \bar{\tau}}{2 \text{Im}\tau^2}$$

$$+ \kappa_{10}^2 \left( T_{mn}^{\text{loc}} - \frac{1}{8} g_{mn} T^{\text{loc}} \right)$$

(1)
With fluxes and warp factors $A, B$ only depending on cone coordinates $x^m$, we have the Einstein equations

\[
R_{\mu\nu} = -g_{\mu\nu} \left[ \frac{G_3 \cdot \bar{G}_3}{48 \text{Im}\tau} + \frac{\tilde{F}_5^2}{8 \cdot 5!} \right] + \frac{\tilde{F}_{\mu abcd} \tilde{F}_{\nu}^{abcd}}{4 \cdot 4!} + \kappa_{10}^2 \left( T_{\mu\nu}^{\text{loc}} - \frac{1}{8} g_{\mu\nu} T^{\text{loc}} \right)
\]

\[
R_{mn} = -g_{mn} \left[ \frac{G_3 \cdot \bar{G}_3}{48 \text{Im}\tau} + \frac{\tilde{F}_5^2}{8 \cdot 5!} \right] + \frac{\tilde{F}_{m abcd} \tilde{F}_{n}^{abcd}}{4 \cdot 4!} + \frac{G_{m}^{bc} \bar{G}_{nbc}}{4 \text{Im}\tau} + \frac{\partial_m \tau \partial_n \tau}{2 |\text{Im}\tau|^2}
\]

\[
+ \kappa_{10}^2 \left( T_{mn}^{\text{loc}} - \frac{1}{8} g_{mn} T^{\text{loc}} \right)
\]

We consider closed three form fluxes $H_3, F_3$ (sourced by $M$ number of five branes) and self dual five form flux

\[
\tilde{F}_5 = (1 + \star) d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3
\]

Minimizing the action also gives the Bianchi identity for the five-form flux, namely

\[
d\tilde{F}_5 = H_3 \wedge F_3 + 2\kappa_{10}^2 T_3 \rho_3
\]
With fluxes and warp factors $A, B$ only depending on cone coordinates $x^m$, we have the Einstein equations

\[
R_{\mu\nu} = -g_{\mu\nu} \left[ \frac{G_3 \cdot \tilde{G}_3}{48 \Im \tau} + \frac{\tilde{F}_5^2}{8 \cdot 5!} \right] + \frac{\tilde{F}_{\mu abcd} \tilde{F}_{\nu abcd}}{4 \cdot 4!} + \kappa_{10}^2 \left( T_{\mu\nu}^{\text{loc}} - \frac{1}{8} g_{\mu\nu} T^{\text{loc}} \right)
\]

\[
R_{mn} = -g_{mn} \left[ \frac{G_3 \cdot \tilde{G}_3}{48 \Im \tau} + \frac{\tilde{F}_5^2}{8 \cdot 5!} \right] + \frac{\tilde{F}_{m abcd} \tilde{F}_{n abcd}}{4 \cdot 4!} + \frac{G_{m}^{bc} \tilde{G}_{nbc}}{4 \Im \tau} + \frac{\partial_m T \partial_n T}{2 |\Im \tau|^2} + \kappa_{10}^2 \left( T_{mn}^{\text{loc}} - \frac{1}{8} g_{mn} T^{\text{loc}} \right)
\]

(1)

We consider closed three form fluxes $H_3, F_3$ (sourced by $M$ number of five branes) and self dual five form flux

\[
\tilde{F}_5 = (1 + \star) d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3
\]

(2)

Minimizing the action also gives the Bianchi identity for the five-form flux, namely

\[
d\tilde{F}_5 = H_3 \wedge F_3 + 2\kappa_{10}^2 T_3 \rho_3
\]

Solve the Einstein equations and Bianchi identity to obtain $A, B$ and $\tilde{g}_{mn}$.
We have *four* equations and unknown functions $A$, $B$ and $\tilde{g}_{mn}$. We use the following ansatz

\[
\tilde{g}_{mn} dx^m dx^n = dr^2 + r^2 e^{2B} \left[ \frac{1}{9} (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 
+ \frac{1}{6} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{6} (1 + F)(1 + G) \left( \frac{d\theta_2^2}{1 + G} + \sin^2 \theta_2 d\phi_2^2 \right) \right]
\]
We have four equations and unknown functions $A$, $B$ and $\tilde{g}_{mn}$. We use the following ansatz

$$
\tilde{g}_{mn}dx^m dx^n = dr^2 + r^2 e^{2B} \left[ \frac{1}{9} (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 
+ \frac{1}{6} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) \right]
+ \frac{1}{6} (1 + F)(1 + G) \left( \frac{d\theta_2^2}{1 + G} + \sin^2 \theta_2 d\phi_2^2 \right)
$$

In the limit $M = 0$ and $\partial \tau \sim O(N_f) = 0$, an exact solution exist

$$
e^{-4A} = \alpha^{-1} = L^4/r^4, \quad e^{2B} = 1 - r_h^4/r^4, \quad F = G = 0
$$

the well known non-extremal limit of Klebanov-Witten geometry - $AdS_5 \times T^{1,1}$ with a black hole with $L^4 = g_s N \alpha'^2$
We have four equations and unknown functions $A$, $B$ and $\tilde{g}_{mn}$. We use the following ansatz

$$\tilde{g}_{mn}dx^m dx^n = dr^2 + r^2 e^{2B}\left[\frac{1}{9}(d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 + \frac{1}{6}(d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{6}(1 + F)(1 + G)\left(\frac{d\theta_2^2}{1 + G} + \sin^2 \theta_2 d\phi_2^2\right)\right]$$

In the limit $M = 0$ and $\partial \tau \sim O(N_f) = 0$, an exact solution exist

$$e^{-4A} = \alpha^{-1} = L^4/r^4, \quad e^{2B} = 1 - r_h^4/r^4, \quad F = G = 0$$

the well known non-extremal limit of Klebanov-Witten geometry - $AdS_5 \times T^{1,1}$ with a black hole with $L^4 = g_s N \alpha'/2$

When $N_f, M \neq 0$, but $B = 0$, we have the extremal Klebanov-Tseytlin geometry with Ouyang D7 embedding. Again exact solution exists with

$$e^{-4A} = \alpha^{-1} = h^0 = \frac{L^4}{r^4}\left\{1 + \frac{3g_s M^2}{2\pi N} \log r \left[1 + \frac{3g_s N_f}{2\pi} \left(\log r + \frac{1}{2}\right)\right]\right\} + \left(\frac{3g_s^2 M^2 N_f}{8\pi^2 N}\right) \log r \log \left(\sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2}\right)$$
Dual Geometry

Hence in the non-extremal limit, $F, G \sim O(M, r_h)$. With $g_s M^2 / N \ll 1$, ignoring $O(g_s M^2 / N) O(F, G)$ and considering up to linear order in $F, G$ we get the Bianchi identity

$$
\begin{align*}
\partial_r \partial_r h^1 + \frac{1}{g} \partial_{\theta_i} \left( \bar{g}_{0;\theta_i} \partial_{\theta_i} h^1 \right) + \frac{r_h^4}{g} \partial_{\theta_i} \left( \bar{g}_{0;\theta_i} \partial_{\theta_i} h^0 \right) &\equiv 5 r^4 \partial_r h^1 \\
&= 4 L^4 \partial_r (F + G / 2) \\
\end{align*}
$$

where $h^1 = e^{-4A} - h^0$. 

Mohammed Holographic Thermal QCD
Hence in the non-extremal limit, $F, G \sim O(M, r_h)$. With $g_s M^2 / N \ll 1$, ignoring $O(g_s M^2 / N) O(F, G)$ and considering upto linear order in $F, G$ we get the Bianchi identity

$$
\left[ \partial_r \partial_r h^1 + \frac{1}{g} \partial_{\theta_i} \left( \bar{g}_0^{\theta_i \theta_i} \partial_{\theta_i} h^1 \right) + \frac{r_s^4 / r_h^4}{g} \partial_{\theta_i} \left( \bar{g}_0^{\theta_i \theta_i} \partial_{\theta_i} h^0 \right) \right] r^5 + 5 r^4 \partial_r h^1
$$

$$= 4 L^4 \partial_r (F + G / 2)$$

where $h^1 = e^{-4A} - h^0$.

Note in the limit $N_f, M = 0, h^1 = 0$ which means $h^1 \sim O(M, N_f)$. Using this above gives

$$F, G \sim O(M / N) + O(g_s^2 M^2 N_f / N)$$

which give $F, G \ll 1$ for $N \gg M$. This also justifies ignoring $O(g_s M^2 / N) O(F, G)$ and the linear approximation.
Hence in the non-extremal limit, $F, G \sim O(M, r_h)$. With $g_s M^2 / N \ll 1$, ignoring $O(g_s M^2 / N)O(F, G)$ and considering upto linear order in $F, G$ we get the Bianchi identity

$$
\left[ \partial_r \partial_r h^1 + \frac{1}{g} \partial_{\theta_i} \left( \bar{g}_{0;\theta_i} \partial_{\theta_i} h^1 \right) + \frac{r_h^4 / r^4}{g} \partial_{\theta_i} \left( \bar{g}_{0;\theta_i} \partial_{\theta_i} h^0 \right) \right] r^5 + 5 r^4 \partial_r h^1
$$

$$
= 4 L^4 \partial_r (F + G/2)
$$

where $h^1 = e^{-4A} - h^0$.

Note in the limit $N_f, M = 0, h^1 = 0$ which means $h^1 \sim O(M, N_f)$. Using this above gives

$$
F, G \sim O(M/N) + O(g_s^2 M^2 N_f / N)
$$

which give $F, G \ll 1$ for $N \gg M$. This also justifies ignoring $O(g_s M^2 / N)O(F, G)$ and the linear approximation.

Writing $e^{2B} = 1 - \frac{r_h^4}{r^4} + G$, a similar analysis gives

$$
G \sim O(M/N) + O(g_s^2 M^2 N_f / N)
$$

Also writing $h^1 = \frac{L^4}{r^4} A^1$ gives $A^1 \sim O(M/N)$.
Thus in the limit $N \gg M$ and $g_s M^2 / N \ll 1$, we can consider only linear terms in $F$, $G$, $G$ and $A^1$. However, there are only three non-trivial equations up to linear order and hence we can set $G = 0$. In fact, $G_3$ do not enter explicitly in the equations for $F$, $G$ and $A^1$ and we find an exact solution to all equations with

$$h^1 = \frac{L^4}{r^4} \left( A_0 + A_1 \log r + A_2 \log^2 r \right)$$

$$e^{2B} \equiv g = 1 - \frac{\bar{r}^4}{r^4} + G \equiv 1 - \frac{\bar{r}^4}{r^4} + g_0 + g_1 \log r + g_2 \log^2 r$$

$$F = F_0 + F_1 \log r + F_2 \log^2 r$$
Thus in the limit $N \gg M$ and $g_s M^2/N \ll 1$, we can consider only linear terms in $F$, $G$, $G$ and $A^1$. However, there are only three non-trivial equations up to linear order and hence we can set $G = 0$. In fact, $G_3$ do not enter explicitly in the equations for $F$, $G$ and $A^1$ and we find an exact solution to all equations with

$$h^1 = \frac{L^4}{r^4} \left( A_0 + A_1 \log r + A_2 \log^2 r \right)$$

$$e^{2B} \equiv g = 1 - \frac{r_h^4}{r^4} + G \equiv 1 - \frac{r_h^4}{r^4} + g_0 + g_1 \log r + g_2 \log^2 r$$

$$F = F_0 + F_1 \log r + F_2 \log^2 r$$

The *background* warp factor $h^0$ couples directly to $G_3$ which is non-ISD. Our solution for the field strength is

$$\alpha = e^{4A} + O(F^2), \quad F_3, H_3 \sim M(1 + O(r_h, F))$$

where the non-ISD part $G_3$ is of $O(F, r_h)$. In the limit $F = r_h = 0$, we recover ISD $G_3$. 

Mohammed

Holographic Thermal QCD
In the limit $N_f = 0$ but $M \neq 0$, the equations drastically simplify and we get

$$A_0(r) = \sum_{k=1}^{\infty} \overline{a}_k^0 \left( \frac{r_h}{r} \right)^k, \quad F_0(r) = \sum_{k=1}^{\infty} \overline{f}_k^0 \left( \frac{r_h}{r} \right)^k, \quad g_0(r) = \sum_{k=1}^{\infty} \overline{\zeta}_k^0 \left( \frac{r_h}{r} \right)^k$$

with $A_i = F_i = g_i = 0$, $i = 2, 3$. These forms also satisfy the boundary conditions

$$A_0(\infty) = 0, \quad A_0'(\infty) = 0, \quad g_0(\infty) = 0, \quad g_0'(\infty) = 0, \quad F_0(\infty) = 0, \quad F_0'(\infty) = 0$$
In the limit $N_f = 0$ but $M \neq 0$, the equations drastically simplify and we get

$$A_0(r) = \sum_{k=1}^{\infty} \tilde{a}_k^0 \left( \frac{r_h}{r} \right)^k, \quad F_0(r) = \sum_{k=1}^{\infty} \tilde{r}_k^0 \left( \frac{r_h}{r} \right)^k, \quad g_0(r) = \sum_{k=1}^{\infty} \tilde{\zeta}_k^0 \left( \frac{r_h}{r} \right)^k$$

with $A_i = F_i = g_i = 0, i = 2, 3$. These forms also satisfy the boundary conditions

$$A_0(\infty) = 0, \quad A_0'(\infty) = 0, \quad g_0(\infty) = 0, \quad g_0'(\infty) = 0, \quad F_0(\infty) = 0, \quad F_0'(\infty) = 0$$
In the limit $N_f = 0$ but $M \neq 0$, the equations drastically simplify and we get

$$A_0(r) = \sum_{k=1}^{\infty} \bar{a}_k^0 \left( \frac{r_h}{r} \right)^k, \quad F_0(r) = \sum_{k=1}^{\infty} \bar{f}_k^0 \left( \frac{r_h}{r} \right)^k, \quad g_0(r) = \sum_{k=1}^{\infty} \bar{\zeta}_k^0 \left( \frac{r_h}{r} \right)^k$$

with $A_i = F_i = g_i = 0$, $i = 2, 3$. These forms also satisfy the boundary conditions

$$A_0(\infty) = 0, \quad A_0'(\infty) = 0, \quad g_0(\infty) = 0, \quad g_0'(\infty) = 0, \quad F_0(\infty) = 0, \quad F_0'(\infty) = 0$$
In the limit $N_f = 0$ but $M \neq 0$, the equations drastically simplify and we get

$$A_0(r) = \sum_{k=1}^{\infty} \bar{a}_k^0 \left( \frac{r_h}{r} \right)^k, \quad F_0(r) = \sum_{k=1}^{\infty} \bar{f}_k^0 \left( \frac{r_h}{r} \right)^k, \quad g_0(r) = \sum_{k=1}^{\infty} \bar{\zeta}_k^0 \left( \frac{r_h}{r} \right)^k$$

with $A_i = F_i = g_i = 0, i = 2, 3$. These forms also satisfy the boundary conditions

$$A_0(\infty) = 0, \quad A_0'(\infty) = 0, \quad g_0(\infty) = 0, \quad g_0'(\infty) = 0, \quad F_0(\infty) = 0, \quad F_0'(\infty) = 0$$
The exact solution is the non-extremal limit of Klebanov-Tseytlin model with a regular resolved cone with resolution function $F$ describing squashing between the two $S^2$'s.
The exact solution is the non-extremal limit of Klebanov-Tseytlin model with a regular resolved cone with resolution function $F$ describing squashing between the two $S^2$'s.

It can easily be generalized for the deformed resolved cone by using the deformed cone metric for $\tilde{g}_{mn}$ with radial coordinate $b \leq r \leq \infty$. 
The exact solution is the non-extremal limit of Klebanov-Tseytlin model with a regular resolved cone with resolution function $F$ describing squashing between the two $S^2$'s.

It can easily be generalized for the deformed resolved cone by using the deformed cone metric for $\tilde{g}_{mn}$ with radial coordinate $b \leq r \leq \infty$. However, for large radial distance, deformed cone becomes regular cone and we will consider $r \geq r_h \gg b$ which means the horizon ‘cloaks’ the deformed cone and the non-extremal geometry sees only a regular cone.
The exact solution is the non-extremal limit of Klebanov-Tseytlin model with a regular resolved cone with resolution function $F$ describing squashing between the two $S^2$'s.

It can easily be generalized for the deformed resolved cone by using the deformed cone metric for $\tilde{g}_{mn}$ with radial coordinate $b \leq r \leq \infty$. However, for large radial distance, deformed cone becomes regular cone and we will consider $r \geq r_h \gg b$ which means the horizon ‘cloaks’ the deformed cone and the non-extremal geometry sees only a regular cone.

Thus our analysis gives the non-extremal limit of Klebanov-Strassler model and $r_h \gg b$ means this geometry is dual to the high temperature deconfined phase of the gauge theory.
The exact solution is the non-extremal limit of Klebanov-Tseytlin model with a regular resolved cone with resolution function $F$ describing squashing between the two $S^2$’s. It can easily be generalized for the deformed resolved cone by using the deformed cone metric for $\tilde{g}_{mn}$ with radial coordinate $b \leq r \leq \infty$. However, for large radial distance, deformed cone becomes regular cone and we will consider $r \geq r_h \gg b$ which means the horizon ‘cloaks’ the deformed cone and the non-extremal geometry sees only a regular cone. Thus our analysis gives the non-extremal limit of Klebanov-Strassler model and $r_h \gg b$ means this geometry is dual to the high temperature deconfined phase of the gauge theory. On the other hand the extremal geometry with $B = 0, N_f, M \neq 0$ and a deformed cone as the internal space describes the confining phase of the gauge theory. The geometry near $r \sim b$ describes confinement in $SU(M)$ gauge theory at low temperature.
The exact solution is the non-extremal limit of Klebanov-Tseytlin model with a regular resolved cone with resolution function $F$ describing squashing between the two $S^2$’s.

It can easily be generalized for the deformed resolved cone by using the deformed cone metric for $\tilde{g}_{mn}$ with radial coordinate $b \leq r \leq \infty$. However, for large radial distance, deformed cone becomes regular cone and we will consider $r \geq r_h \gg b$ which means the horizon ‘cloaks’ the deformed cone and the non-extremal geometry sees only a regular cone.

Thus our analysis gives the non-extremal limit of Klebanov-Strassler model and $r_h \gg b$ means this geometry is dual to the high temperature deconfined phase of the gauge theory.

On the other hand the extremal geometry with $B = 0, N_f, M \neq 0$ and a deformed cone as the internal space describes the confining phase of the gauge theory. The geometry near $r \sim b$ describes confinement in $SU(M)$ gauge theory at low temperature.
The analysis can easily be generalized for KS type geometries with modifications at asymptotically large $r$.

Three form fluxes $\sim \tilde{M}(r)/r^A$ with $\tilde{M}(r) = M\left(1 - \frac{\exp[\alpha(r-r_0-b)]}{1+\exp[\alpha(r-r_0-b)]}\right)$. Near $r \sim r_0$ we have fluxes sourced by anti five branes.

$$\lim_{r \to b} M(r) \to M$$ and $$\lim_{r \to \infty} \tilde{M}(r) \to 0.$$ Hence $B_2 \sim M \ln(r)$ for $r \ll r_0$ and $B_2 \sim 0$ for $r \gg r_0$.

7 branes source $\tau$, we can arrange them in such a way that for $r \gg r_0$, $\tau \sim 1/r^n$. On the other hand for $r < r_0$, $\tau \sim \ln(r)$.

We expect the squashing function $F$ and warp factors $e^A$, $e^B$ to again be described by our ansatz with inverse power law behavior.
The Story so far

UV

SU(N+M) × SU(N+M)
g_i \sim 1/\Lambda^n
Asymptotic AdS geometry

Flow from UV to IR

Scale \Lambda

SU(N+M) × SU(N)
g_i \sim \log \Lambda
Klebanov–Strassler type geometry

Flow from UV to IR

IR

SU(\tilde{M})
g_i \sim \log \Lambda
Klebanov–Strassler type geometry
Entropy From Dual Gravity

- Entropy of the gauge theory given by entropy of black hole. The black hole entropy is obtained from Wald’s formula using our solution for the metric.
Entropy of the gauge theory given by entropy of black hole. The black hole entropy is obtained from Walds formula using our solution for the metric. In the limit $M = N_f = 0$, $s / T^3$ is flat [Gubser et al ’98], whereas for our non-AdS geometry a rapid change of entropy is observed.
We computed free energy of the thermal gauge theory from the ten dimensional type IIB supergravity action, $I_{\text{gravity}} = I_{\text{gauge}} = \beta F$.

We obtain pressure $p = -\frac{\partial F}{\partial V} = -f$ and then using the black hole entropy, we compute internal energy $e = f + Ts$. 
We computed free energy of the thermal gauge theory from the ten dimensional type IIB supergravity action, \( I_{\text{gravity}} = I_{\text{gauge}} = \beta F \).

We obtain pressure \( p = -\frac{\partial F}{\partial V} = -f \) and then using the black hole entropy, we compute internal energy \( e = f + Ts \).
We want to analyze the confinement mechanism for the fundamental matter arising from string theory. While lattice QCD gives linear confinement of quarks at low temperatures, do our quark strings confine?

We will study $Q\bar{Q}$ free energy as a function of inter quark separation and temperature. Free energy can be obtained from the Wilson loop

$$<W_C> \sim \exp(-F(d,T)/T)$$
Holography gives

\[ \langle W_C \rangle \sim \exp(-S_{NG}) \]
Lattice VS Dual Gravity

\[
F_1(r,T)/\sigma^{1/2}
\]

\[
T/T_c = 0.75 \\
T/T_c = 0.82 \\
T/T_c = 0.91 \\
T/T_c = 0.97 \\
T/T_c = 1.01 \\
T/T_c = 1.03 \\
T/T_c = 1.07 \\
T/T_c = 1.12 \\
T/T_c = 1.16 \\
T/T_c = 1.24
\]

\[
F_QQ
\]

[Olaf Kaczmarek et al]

Mohammed  Holographic Thermal QCD
Summary

- We have constructed non-extremal generalizations of warped resolved deformed conifold geometry.
- It is possible to construct the dual gravity of gauge theory which has logarithmic running of coupling in IR but behaves almost conformal in the UV.
- IR of the field theory we analyzed mimics large N QCD.
- Entropy of black hole is qualitatively similar to that of strongly coupled QCD while the conformal anomaly of the dual gauge theory is in agreement with QCD.
- The dual geometry realizes linear confinement of both heavy and light quarks.
- We expect the IR of the gauge theory to be thermodynamically equivalent to strongly coupled large N QCD.
Future Directions

- What are the phases and order of phase transitions?
- What about chemical potential and critical point?
- We are now studying various phases of nuclear matter at strong coupling by varying chemical potential and temperature coming from ten dimensional black hole geometries in string theory. We expect Hawking-Page phase transition ...

- For a trivial D7 embedding the chemical potential scales as
  \[ \mu \sim T(1 + \log(r_h)) \].

- Using this non-extremal geometries we are also studying trailing and falling strings giving rise to heavy and light quark energy loss [Andrej Ficnar, Miklos Gyulassy]... Stay tuned!