Towards a Gravity Dual of Charmonium in the Strongly Coupled Plasma

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Outline

• Introduction/Motivation
• A New Model at $T=0$
• Finite Temperature
• Results
• Conclusion
How can one know that they have a QGP?

- One way is through $J/\psi$ suppression. [Masui and Satz, 1986]
- Charmonium states in the presence of a deconfined state can dissociated via color Debye screening.
- The observation of $J/\psi$ suppression could be a tell-tale experimental signal for the presence of a QGP.

- However, suppression is a complicated process, thus we will focus only on the dissociation of charmonium.
Field Theory

• Consider the charm current operator. \[ J^\mu = \bar{c}\gamma^\mu c \]
  – The 1^- state of this operator is J/ψ.

• Construct the current-current correlator and its spectral function.

• The correlator and the spectral function are well defined in field theory and can be calculated.

• At T=0, the spectral function (in the large Nc limit) consists of delta functions at the location of the mass states.

• At finite T, these spikes broaden with the strength of the peaks reducing.

• At some point, this broadening process makes the peak unrecognizable.

• Attribute the lack of a spectral peak to the state dissociating.
Lattice Results

T=0

Finite T

Spectral peak corresponding to J/ψ appears to be present up to 2.4 \( T_c \).

Potential Models

J/ψ spectral peak disappears at a much lower temperature, \( \sim 1.2-1.5 \, T_c \).

Holography and AdS/CFT Correspondence

- Relationship between a strongly coupled field theory and a weakly coupled theory of gravity.
  - N=4 SYM and 5D Anti-deSitter gravity.
- Operators of the field theory correspond to 5D fields in the bulk space.
  \[ J^\mu \rightarrow V^\mu \]
- Expectation values of the 4D operators can be found by calculating derivatives of the 5D action wrt the boundary value of a bulk field.
  \[ G_R(q) = \frac{\partial^2 S}{\partial V_0(q) \partial V_0(-q)} \bigg|_{z \rightarrow 0} \]
“Top-Down” perspective

• A string theory provides a background geometry upon which fluctuations are investigated.

• Flavor can be introduced to the gravity theory by introducing probe D7 branes.  

• Heavy meson systems have been investigated in the context of a D3/D7 construction where the branes are separated from one another by some gap, \( L \sim m_q \).

• Only one scale in the problem! 
  
  – All states in mass spectrum set by this scale.
    • Charmonium spectrum is set but 2 scales, \( m_q \) and \( \Lambda_{QCD} \)
  – Dissociation temperature also set by same scale.

Karch and Katz, 2002
Kruczenski et al. 2003
“Bottom-up” approach

• Construct a gravity theory with desirable properties.
• Such as the soft wall model of light vector mesons.
• Rescaled $\rho$
  – The standard soft wall model for light vector mesons has one parameter which is fit to the $\rho$ mass.
  – Instead the parameter is fixed to the $J/\psi$ mass
  – Reasonable procedure since both light and heavy vector mesons exhibit a Regge behavior in spectrum.
• $T_D$ is smaller, 1.2 $T_c$.
• Again there is only one scale in the problem.
  – Both the mass of $J/\psi$ and Regge behavior is fixed from the same parameter
  – However, separation energy between excited states is parametrically $\Lambda_{QCD}$ and not $m_q$

Fujita et al, 2009
To understand the dissociation of charmonium states, we will need to understand the finite T spectral functions (correlators).

But the finite T spectral function is related to the T=0 spectral function (correlator).

- So we should get this correct as well.

At T=0, the correlator can be written as a series of poles located at the mass states, with residues equal to the decay const.

\[
\Pi_V(-q^2) = \sum_n \frac{f_n^2}{q^2 - m_n^2 + i\epsilon}
\]

Therefore to reproduce the correlator (spectral function), both the masses and the decay consts of each state should be reproduced.
• Goal 1: Construct a new “bottom-up” model at T=0 for a U(1) field whose correlator approximates that of charmonium.

• Goal 2: Turn on temperature, and watch J/ψ disocciate.
  – Corollary: We will apply some new techniques in order to analyze the resulting spectral functions.
The model

Action

\[ S = -\frac{1}{4g_5^2} \int d^5 x \sqrt{g} e^{-\Phi} V_{MN} V^{MN} \]

\[ V_{MN} = \partial_M V_N - \partial_N V_M \]

Metric

\[ ds^2 = e^{2A(z)} \left[ \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right] \]

A(z) is a warping factor \quad \Phi(z) is a “dilaton” field

Both fields are initially unspecified and will be determined from charmonium considerations.

\[ B(z) = A(z) - \Phi(z) \]
\[ \partial_z \left[ e^{B(z)} \partial_z V \right] + q^2 e^{B(z)} V = 0 \]

Normalizable modes \[ V = \nu_n \quad V \big|_{z=0} = 0 \]

Discrete spectrum \[ q^2 = m_n^2 \]

Decay constants \[ \langle 0 | J_\mu(0) | n \rangle = f_n m_n \epsilon_n \quad f_n = \frac{1}{g_5 m_n} \nu'_n e^{B(z)} \big|_{z \to 0} \]

Schrödinger Equation \[ -d^2 \Psi / dz^2 + U(z) \Psi = q^2 \Psi \]

Schrödinger Potential \[ U(z) = \frac{B''(z)}{2} + \left( \frac{B'}{2} \right)^2 \]
• The properties of charmonium which we are interested in calculating, namely the masses and the decay constants depend on the Schrödinger potential.

• Therefore we will chose $U(z)$ such that the resulting masses and decay constants for $J/\psi$ and $\psi'$ are consistent with experiment.

• This is considered the inverse-scattering problem.

• The potential is uniquely determined from the spectrum and the derivative of the eigenfunctions (decay const.) for the Dirichlet problem.

Poeschel and Trubowitz, 1987
Determining the Schrödinger Potential

• Prerequisites
  – Want the potential to approach AdS potential near UV boundary.
    \[ U \sim \frac{3}{4z^2} \quad B(z) \sim -\log z \]
  – For Regge like behavior, want soft-wall near the IR.
    \[ U \sim z^2 \]

• Spectrum
  – Need a mass shift in the ground state which is independent of the excited state spacings.
    \[ U(z) = \frac{3}{4z^2} + a^4 z^2 + c^2 \]
    
    \[ \text{Shift} \]
    
    Standard soft wall
Decay constants

<table>
<thead>
<tr>
<th>Observable</th>
<th>Experiment (MeV)</th>
<th>$U_{(a)}$ (MeV)</th>
<th>$U_{(a,c)}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{J/\psi}$</td>
<td>3096</td>
<td>3096*</td>
<td>3096*</td>
</tr>
<tr>
<td>$m_{\psi'}$</td>
<td>3685</td>
<td>4378</td>
<td>3685*</td>
</tr>
<tr>
<td>$f_{J/\psi}$</td>
<td>416</td>
<td>348</td>
<td>145</td>
</tr>
<tr>
<td>$f_{\psi'}$</td>
<td>296</td>
<td>348</td>
<td>173</td>
</tr>
</tbody>
</table>

Decay constants are too small, we need to add a new feature to the potential to increase them.

$$f_n = \frac{1}{g_5 m_n} v''_n(0) = \frac{1}{g_5 m_n} \left( \sqrt{z} \psi_n \right)'' \bigg|_{z \to 0}$$

We need to include something localized near the UV boundary which makes the wavefunction steeper.
We will introduce a “Dip!”

A Dip is a region of finite width of stronger attraction.

We will consider the simplest Dip possible, namely a delta function.

\[ U(z) = \frac{3}{4z^2} \theta(z_d - z) + \left( (a^2 z)^2 + c^2 \right) \theta(z - z_d) - \alpha \delta(z - z_d) \]

4 parameters which are fitted to the masses and decay constants of $J/\psi$ and $\psi'$.

\[ a = 0.970 \text{ GeV}, \; c = 2.781 \text{ GeV}, \; \alpha = 1.876 \text{ GeV}, \; z_d^{-1} = 2.211 \text{ GeV} \]
Schrödinger Potential

Normalized Wavefunction
Does this model respect QCD sum rules?

\[ \pi^{(c)}(Q^2) \]

Graph showing the behavior of \( \pi^{(c)}(Q^2) \) as a function of \( Q^2 \) (in GeV\(^2\)). The graph compares different models and rules, indicating how the model aligns with QCD sum rules.
Finite Temperature

Metric with Black hole ansatz

$$ds^2 = e^{2A(z)} \left[ h dt^2 - dx^2 - h^{-1} dz^2 \right]$$

$$h(z_h) = 0 \quad T = \frac{1}{4\pi} |h'(z_h)|$$

We will consider the simplest form for $h(z)$.

$$h(z) = 1 - \left( \frac{z}{z_h} \right)^4$$

Ideally, $h(z)$ should be determined dynamically.

In turn, $B(z)$ should have some temperature dependence.

But we haven’t modeled any backgrounds dynamically.

And this allows us to focus on the effects of the new potential.
\[
\partial_z \left( h e^{B(z)} \partial_z V \right) + \omega^2 h^{-1} e^{B(z)} V = 0
\]

Look for solutions with the b.c.

\[
V(\omega, \epsilon) = 1
\]
\[
V(\omega, z) \xrightarrow{z \to z_h} C(\omega) \left(1 - z/z_h\right)^{-i\omega/(4\pi T)}
\]

Green’s function

\[
G_R(\omega) = -\frac{1}{g_5^2} h e^{B} V'(z, \omega) \bigg|_{z=\epsilon} = -\frac{1}{g_5^2} \left. \frac{V'(\epsilon, \omega)}{\epsilon} \right.
\]

Schrödinger Equation

\[
-d^2 \Psi/\, d\zeta^2 + U_T(\zeta) \Psi = \omega^2 \Psi
\]

Schrödinger Potential

\[
U_T(\zeta) = \left[ \frac{B''(z)}{2} + \left( \frac{B'}{2} \right)^2 + \frac{B'(z)h'(z)}{2h(z)} \right] h(z)^2
\]
\[
U_T(\zeta) = \frac{d^2 B/\, d\zeta^2}{2} + \left( \frac{dB/\, d\zeta}{2} \right)^2
\]
Results

Spectral function

\[ \bar{\rho}(\omega) \equiv \left. \frac{\rho/\omega^2}{(\rho/\omega^2)} \right|_{\omega \to \infty} = \frac{2g_5^2}{\pi} \frac{\rho}{\omega^2} \quad \bar{\rho}(\infty) = 1 \]
Quasinormal modes

States corresponding to the poles of the finite T Green’s function.

Satisfy the b.c.

\[ v_n(\epsilon) = 0 \quad v_n(z) \xrightarrow{z \rightarrow z_h} c_n \left( 1 - \frac{z}{z_h} \right)^{-i\omega/(4\pi T')} \]

Look like the normal modes at T=0.

Form a discrete spectrum. \( \omega_n \)

Analytic continuation of the mass at finite T.

For the spectral function,

\( \text{Re } \omega_n \) corresponds to the peak location,
\( \text{Im } \omega_n \) corresponds to the peak width.
Quasinormal modes

$\omega_1$

$\omega_2$

$T=100 \text{ MeV}$

$T=600 \text{ MeV}$

$T=300 \text{ MeV}$

$\text{Re}(\omega) \text{ (GeV)}$

$\text{Im}(\omega) \text{ (GeV)}$
Residues

The residues of the poles can also be calculated.

\[ r_n \equiv \lim_{\omega \to \omega_n} (\omega - \omega_n) G_R(\omega) \]

Can be expressed in terms of the quasinormal wavefunction.

\[ r_n = \frac{1}{g_5^2} \frac{(\nu'_n(\epsilon)/\epsilon)^2}{2\omega_n} \]

T=0 \quad r_n = \frac{f_n^2 m_n^2}{2\omega_n}

\[ \lim_{\delta \to 0} \left[ \int_{\epsilon}^{(1-\delta)z_n} \frac{dz}{\hbar} e^{B\nu_n^2} - i \frac{e^{B\nu_n^2}}{2\omega_n} \right] = 1 \]
Spectral peak deconstruction

\[ \bar{\rho}_n(\omega) \equiv \text{Im} \left( \frac{\bar{r}_n}{\omega - \omega_n} \right) \]

\[ \bar{r}_n = \frac{2g_5^2}{\pi} \frac{r_n}{\omega_n^2} \]
Comparison of the spectral peak widths

\[ \Gamma_n = -\text{Im} \omega_n \]
Comparison of the spectral peak heights

Criteria for melting: A state dissociated when its spectral peak becomes smaller than the spectral functions limiting value.

\[ H_n = \frac{\text{Re} \rho_n}{\text{Im} \omega_n} \]

Rescaled \( \rho \)

This Model

\[ \bar{\rho}(\infty) \]

Dissociation Temp:

<table>
<thead>
<tr>
<th></th>
<th>Rescaled ( \rho )</th>
<th>This Model</th>
<th>( T_c = 190 \text{ MeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dissociation Temp:</td>
<td>230 MeV = 1.2 ( T_c )</td>
<td>540 MeV = 2.8 ( T_c )</td>
<td></td>
</tr>
</tbody>
</table>
How can one explain the melting temperatures?

- In D3/D7 constructions, melting temperatures is fixed by heavy quark mass.
- In the bottom-up models, as the temperature increases the soft wall in the potential “melts.”
- The model with “dip” has weaker soft wall, melt widths grow initially faster than rescaled ρ model.
- However, for higher temperatures, the “dip” stabilizes the state, which prevents dissociation.
Potentials at finite temperature
Effects of delta function "dip" on width and height of spectral peaks.
Summary

• We’ve constructed a new gravity dual theory to describe J/ψ.
  – In the new model, charmonium dissociates ~540 MeV.
• We’ve applied new techniques to analysis the spectral function.
  – Analyzed spectral function by calculating quasinormal modes and residues of poles.
• Fertile area for future exploration
  – Fleshing out how such a system could arise dynamically.
    (Including considering if top-down constructions can be used to realize the features described here.)