662, Homework III, (1 problem, GRADED)

Problem 1

The Higgs boson can decay into two photons. One of the diagrams producing such transition is shown in the figure.

a) Compute the decay width as a function of the mass of the Higgs, the mass and charge of the fermion running in the loop and the Fermi decay constant $G_F$. Consider the diagram in the figure and also the one with $p_1, p_2$ interchanged. Assume $m_H < 2m_f$.

b) Compute the numerical value of the width when the fermion is a top quark (remember the color factor of 3) and compare with the decay width of Higgs into $b\bar{b}$.

Problem 1b

(Optional)

a) Discuss helicity properties of the fermion running in the loop and what does that imply for the amplitude as a function of the fermion mass.

b) Determine a low energy effective vertex that produces this transition and the implications of gauge invariance for the momentum dependence of the diagram.

c) Can you use a) and b) to argue that the diagram is finite?

d) Discuss the difference between the cases $m_H < 2m_f$ and $m_H > 2m_f$. Compare the decay width in the cases of bottom and top quark running in the loop.

Vertices

\[ V_1 = q\bar{\psi}\gamma^\mu\psi A_\mu \]  \hspace{1cm} (0.1)

\[ V_2 = g\bar{\psi}\psi \phi_H \]  \hspace{1cm} (0.2)
Figure 1: Diagram contributing to $H_0 \rightarrow \gamma + \gamma$. 
\[ g = -\frac{m_f}{v}, \quad \frac{1}{v^2} = \sqrt{2}G_F \] (0.3)

For the top quark: \( m_t = 173 \text{GeV}, \quad q = +\frac{2}{3}e. \)

Also: \( m_H = 125 \text{GeV} \) and \( G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}. \)

Finally:

\[
\Gamma_{i \to f} = \frac{(2\pi)^4}{2m_i} \prod_{f=1}^{N_f} \int \frac{d^3 p_f}{(2\pi)^3 2\omega_f} \delta^{(4)}(P_f - P_i)|\mathcal{M}_{fi}|^2
\] (0.4)