Problem 1
Consider a real scalar field with a $\phi^4$ interaction in terms of the bare parameters and the renormalized ones, namely

$$S = \int d^d x \left[ \frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 - \frac{1}{2} m_0^2 \phi_0^2 - \frac{\lambda_0}{4!} \phi_0^4 \right]$$ (0.1)

$$= \int d^d x \left[ 1 + \delta Z^2 \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} (m^2 + \delta m) \phi^2 - \mu^\epsilon \frac{\lambda + \delta \lambda}{4!} \phi^4 \right]$$ (0.2)

a) Compute the counter–terms $\delta Z$, $\delta m$, $\delta \lambda$ to one loop in perturbation theory.

b) Compute the two loop diagrams assuming zero external momenta and get the counter–terms at two loops.

c) Rewrite the relation between bare and renormalized parameters as

$$\lambda_0 = \mu^\epsilon \left( 1 + \frac{a_1(\lambda)}{\epsilon} + \frac{a_2(\lambda)}{\epsilon^2} + \ldots \right)$$ (0.3)

$$m_0^2 = m^2 \left( 1 + \frac{b_1(\lambda)}{\epsilon} + \frac{b_2(\lambda)}{\epsilon^2} + \ldots \right)$$ (0.4)

$$\phi_0 = \left( 1 + \frac{c_1(\lambda)}{\epsilon} + \frac{c_2(\lambda)}{\epsilon^2} + \ldots \right)$$ (0.5)

namely compute the coefficients $a_j, b_j, c_j$ to second order in perturbation theory.