Problem 1
Consider an $SU(N)$ gauge theory and show the validity of the Bianchi identity
\[ D_\mu F_{\nu \rho} + D_\nu F_{\rho \mu} + D_\rho F_{\mu \nu} = 0 \] (0.1)

Problem 2
Consider an $SU(N)$ gauge theory with a fermion in the fundamental. In the BRST approach one introduces extra fields $c, \bar{c}$ and $B$. We say, by definition, that $c$ has ghost number 1 and $\bar{c}$ has ghost number -1.

a) Use the gauge fixing function $\partial_\mu A_\mu$ (Lorentz gauge) and compute the dimensions of the different fields (including $B$).

b) Consider the Lagrangian density to be a BRST, Lorentz, and global $SU(N)$ invariant polynomial in the fields. Show that a BRST variation preserves the number $N_B + N_{\bar{c}}$, where $N_B$ is the number of fields $B$ in a given term, and the same for $N_{\bar{c}}$. Notice that the operator that counts this number can be written as
\[ N_B + N_{\bar{c}} = B \frac{\delta}{\delta B} + \bar{c} \frac{\delta}{\delta \bar{c}} \] (0.2)

c) Show that, if a function $L$ of the fields is BRST invariant then
\[ \delta_{BRST} \left[ \left( \bar{c} \frac{\delta}{\delta B} \right) L \right] = (N_B + N_{\bar{c}})L \] (0.3)

**Hint:** Use that the BRST variation can be written as $\delta_{BRST} = B \frac{\delta}{\delta B} + \tilde{\delta}$ where $\tilde{\delta}$ does not involve $B$ or $\bar{c}$.

d) Conclude that any BRST invariant Lagrangian of ghost number zero can be written as the usual Yang-Mills Lagrangian plus the BRST variation of another function $\Psi$

e) Using that the dimension of all the vertices in the Lagrangian is 4 or less, write the most general function $\Psi$ and the corresponding Lagrangian.