663, Homework III, (2 problems)

Problem 1

In the previous homework you condsidered two types of interaction of a spin 0 particle with fermions:

$$V = \int d^3x \frac{\lambda}{\sqrt{2}} \phi \bar{\psi} \psi \tag{0.1}$$

and

$$V = \int d^3x \frac{i\lambda}{\sqrt{2}} \phi \bar{\psi} \gamma^5 \psi \tag{0.2}$$

where the second one is appropriate for a pseudo-scalar. Assuming the second type of interaction and that the fermions have charge e, compute the decay probability of the pseudoscalar into two photons at lowest order in perturbation theory (triangle diagram). Using appropriate values for the pion (π^0) find the mean life of the pion under this type of decay, compare with the experimental result. For this purpose take $\frac{\lambda}{\sqrt{2}} = \frac{m_f}{f_\pi}$ with m_f the fermion mass and $f_\pi \simeq 93$MeV. Assume also that $m_\pi \ll m_f$. You can consider just one fermion or you can use the quark model with two fermions (u, d) with charges $\frac{2}{3}e, \frac{1}{3}e$ and opposite values of λ.

Note: the experimental decay rate is $\Gamma \simeq 7.7 \pm 0.6$ eV.

Problem 2

Consider the $U(1)$ Abelian Higgs model with Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + |D_\mu \phi|^2 - m^2 |\phi|^2 - \frac{\lambda}{6} |\phi|^4 \tag{0.3}$$

where ϕ ia a complex scalar and $D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$.

a) Compute the 1-loop effective potential in Landau gauge ($\partial^\mu A_\mu = 0$).

b) Take the limit of zero renormalized mass in the potential obtained in point a) and discuss if radiative corrections lead to symmetry breaking.

c) Compute the β-functions for e and λ and sketch the renormalization group flow in the (λ, e^2) plane.