Problem 1

In the previous homework you considered two types of interaction of a spin 0 particle with fermions:

\[ V = \int d^3x \frac{\lambda}{\sqrt{2}} \phi \bar{\psi} \psi \]  
\[ \text{(0.1)} \]

\[ V = \int d^3x \frac{i\lambda}{\sqrt{2}} \phi \bar{\psi} \gamma^5 \psi \]  
\[ \text{(0.2)} \]

where the second one is appropriate for a pseudo-scalar. Assuming the second type of interaction and that the fermions have charge \( e \), compute the decay probability of the pseudoscalar into two photons at lowest order in perturbation theory (triangle diagram). Using appropriate values for the pion (\( \pi^0 \)) find the mean life of the pion under this type of decay, compare with the experimental result. For this purpose take \( \frac{\lambda}{\sqrt{2}} = \frac{m_f}{f_\pi} \) with \( m_f \) the fermion mass and \( f_\pi \approx 93 \text{MeV} \). Assume also that \( m_\pi \ll m_f \). You can consider just one fermion or you can use the quark model with two fermions (\( u, d \)) with charges \( \frac{2}{3}e, \frac{1}{3}e \) and opposite values of \( \lambda \).

Note: the experimental decay rate is \( \Gamma \approx 7.7 \pm 0.6 \text{eV} \).

Problem 2

Consider the \( U(1) \) Abelian Higgs model with Lagrangian

\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu}^2 + |D_\mu \phi|^2 - m^2 |\phi|^2 - \frac{\lambda}{6} |\phi|^4 \]  
\[ \text{(0.3)} \]

where \( \phi \) is a complex scalar and \( D_\mu \phi = (\partial_\mu + ie A_\mu) \phi \).

a) Compute the 1-loop effective potential in Landau gauge (\( \partial^\mu A_\mu = 0 \)).

b) Take the limit of zero renormalized mass in the potential obtained in point a) and discuss if radiative corrections lead to symmetry breaking.

c) Compute the \( \beta \)-functions for \( e \) and \( \lambda \) and sketch the renormalization group flow in the \( (\lambda, e^2) \) plane.

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