Problem 1

Let

\[ H = \int d^3p f(p) \ln f(p) \]  
\[ (0.1) \]

where \( f(p,t) \) is arbitrary except for the conditions

\[ N = \int d^3p f(p) \]  
\[ (0.2) \]

\[ E = \frac{1}{N} \int d^3p \frac{p^2}{2m} f(p) \]  
\[ (0.3) \]

for some given values \( N, E \). Find the function \( f(p) \) that minimizes \( H \). How do you interpret the result?

Problem 2

A room of volume \( V = 10m^3 \) is under standard conditions of pressure and temperature (atmospheric pressure and \( T = 300^\circ K \))

1. Estimate the probability that, at an instant of time a \( 1cm^3 \) of volume anywhere in the room is totally devoid of air due to a statistical (or thermal) fluctuation.

2. The same for a volume \( 1 \, \text{A}^3 \).

Problem 3

Compute the density fluctuations in the grand canonical ensemble in a similar way as we did the fluctuations of energy in the canonical one. Write your answer using the isothermal compressibility

\[ \kappa_T = -\frac{1}{v} \frac{\partial v}{\partial P} \]  
\[ (0.4) \]

where \( v = \frac{V}{N} \) and \( P \) is the pressure. Argue that the fluctuations are small (compared to what?)
Problem 4

Consider the classical ideal gas in the grand canonical ensemble. Compute the grand partition function $\Xi$ and derive the thermodynamic quantities including the equation of state, specific heat, entropy, chemical potential, and energy. Compare with the results from the canonical ensemble.