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Adaptive Robust Control (ARC) for an Altitude Control of a Quadrotor Type UAV Carrying an Unknown Payloads

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Abstract: This research deals with an altitude controller of a quadrotor type UAV with an unknown total mass of the structure. We assume that the uncertainty results from the flight mission in which the UAV carries unknown payloads. Since the quadrotor type UAV involves both translational and rotational motions due to its inherent dynamics, it is of importance to know accurate information on the vehicles the moment of inertia and the total mass in order to guarantee the UAVs attitude and position controls. An Adaptive Robust Control (ARC) is utilized to compensate for the parametric uncertainty. Then, Lyapunov based stability analysis shows that the proposed control design guarantees asymptotic tracking error for the UAVs altitude control. Numerical simulation results which are time-based are presented to illustrate the good tracking performance of the designed control law.

Keywords: Adaptive Robust Control, ARC, UAV, Quadrotor, Altitude control

1. INTRODUCTION

Numerous research efforts have been conducted on Unmanned Aerial Vehicles (UAVs) due to their wide use in many practical applications, e.g., research, surveillance and reconnaissance in specific regions. This family of UAVs can be classified into two categories by their shapes and flying methods: fixed-wing type aircraft and vertical take-off and landing (VTOL) type aircraft. Fixed-wing type aircrafts are generally deployed where extensive areas are to be covered in a short time because they can fly with considerable speeds. On the other hand, VTOL type aircrafts (single rotor helicopter, coaxial type helicopter and quad-rotor) are suitable for a limited area or application that requires a stationary or a slow flight because they have specific characteristics like hovering and vertical landing.

Focusing on the VTOL category of UAVs, a quad-rotor helicopter is representative of a VTOL type helicopter. By using four rotors which can rotate with individually controllable speeds, the mechanical structure of the quad-rotor aircraft becomes simpler than that of the single rotor aircraft or the coaxial type helicopter [1]. Such advantages result in making the quad-rotor type UAV a simple and light rotor actuator system. Furthermore, because of its high payload and relatively easily balanced center of mass, the quad-rotor helicopter is often used in missions which carry objects [2]. To achieve such a mission, however, a highly sophisticated attitude and position control is required. The position control of the quad-rotor helicopter is highly linked with its attitude. If rolling and pitching moments are produced, the helicopter may fly towards x or y direction, respectively. If yawing moment is produced, the head of the helicopter may turn at the same place.

Over the years, various researches on quad-rotor helicopter control have been conducted, including a backstepping and sliding mode control [3-7,9] and a PID control [7-10]. Bouadi $et\ al.$ presented stabilizing control laws synthesis by sliding mode based on the backstepping approach [3]. S. Bouabdallah $et\ al.$ used the integral backstepping technique for attitude, altitude and position control [6]. In this paper, the results of this technique showed a powerful and flexible control structure. Furthermore, he showed that OS4 quadrotor was able to perform autonomous hovering with altitude control and autonomous take-off and landing E. Altug $et\ al.$ implemented a proportional derivative (PD) controller to control attitude and altitude using dual cameras via feedback linearization [4]. There is another approach combining a PID controller, the backstepping and the sliding mode controller together [4,7,9]. The authors divided the quadrotor model into several subsystems. U. Ozguner $et\ al.$, for instance, divided the model into two subsystems: a fully-actuated subsystem and an under-actuated subsystem [9]. Then, he controlled them with a PID controller and a sliding mode controller, respectively. As a result, this combined controller stabilized the overall quadrotor system with the model errors, parametric uncertainties and other disturbances.

As the quad-rotor helicopter has both translational and rotational motions and operates, in three dimensional environments with the rotors, it is of importance for us to know accurate information on the vehicle's moment of inertia and the total mass. We may obtain them by experiments if they can be measured in advance and not changed. If the mission of the UAV is to carry an unknown load or object, it is hard to know the exact value of the its weight in advance. If we fail to obtain the exact value, the controller for UAV's attitude and position will be collapsed with inaccuracy parameters of the UAV because they are closely associated with the rotational motion and the translational motion. Thus, in this paper, we apply the ARC control law to the UAV in case of which we do not know the exact value of its mass. Due to the parameter estimation we will obtain the exact value of the UAV's mass and will use the value in an altitude controller.

The rest of this paper is arranged as follows. The dynamics of the quad-rotor type UAV will be introduced in Section 2 so that the adaptive robust control for the altitude control of the helicopter can be designed in Section 3. Then, Section 3 will discuss an adaptive robust control law, and Lyapunov based stability analysis shows that the designed control law guarantees a zero tracking error for the UAVs altitude control. In Section 4, to verify the performance of the control, the results of numerical simulations will be given by describing the operations of the designed controller. Finally, the conclusions and future scope of this research will be summarized in Section 5.

2. MODELING

Deriving mathematical modeling or differential equations is necessary for the control of the quad-rotor helicopter position and altitude. However, it is hard for the complicated structure of the quad-rotor type to express its motion with simple modeling. In addition, since the quad-rotor type aircraft includes highly nonlinear factors, we need to consider several assumptions in order to get a desired model [3].

- The body is rigid and symmetrical.
- The rotors are rigid, i.e. no blade flapping occurs.
- The difference of gravity by the spin of the earth is minor.
- The center of mass and body fixed frame origin coincide.

These assumptions can be formed because of slower speed and lower altitude of the quad-rotor aircraft whose body having 6 DOF (Degree of Freedom) is rigid as compared to a regular aircraft. Under these assumptions, it is possible to describe the fuselage dynamics, and the coordinate system can be divided into an earth frame {E} and a body frame {B} as shown in Fig. 1. Using the formalism of Newton-Euler, the dynamic equations are written in the following form [3]:

$$\begin{array}{rcl} \dot{\xi} & = & v \\ m\ddot{\xi} & = & F_f + F_d + F_g \\ \dot{R}_{EB} & = & R_{EB}S(\Omega) \\ J\Omega & = & -\Omega \wedge J\Omega + \Gamma_f - \Gamma_a - \Gamma_g. \end{array} \quad (1)$$

As a consequence, the complete dynamic model which governs the quad-rotor helicopter is as follows:

$$\begin{cases} \ddot{x} = \dfrac{(C_\phi S_\theta C_\psi + S_\phi S_\psi)U_1}{m} \\ \ddot{y} = \dfrac{(C_\phi S_\theta S_\psi + S_\phi C_\psi)U_1}{m} \\ \ddot{z} = \dfrac{(C_\phi C_\theta)U_1}{m} - g \\ \ddot{\phi} = \dfrac{\dot{\theta}\dot{\psi}(I_y - I_z) + dU_2 - K_{ax}\dot{\phi}^2 - J_r\dot{\theta}\Omega_r}{I_x} \\ \ddot{\theta} = \dfrac{\dot{\phi}\dot{\psi}(I_z - I_x) + dU_3 - K_{ay}\dot{\theta}^2 - J_r\dot{\phi}\Omega_r}{I_y} \\ \ddot{\psi} = \dfrac{\dot{\phi}\dot{\theta}(I_x - I_y) + U_4 - K_{az}\dot{\psi}^2}{I_z} \end{cases} \quad (2)$$

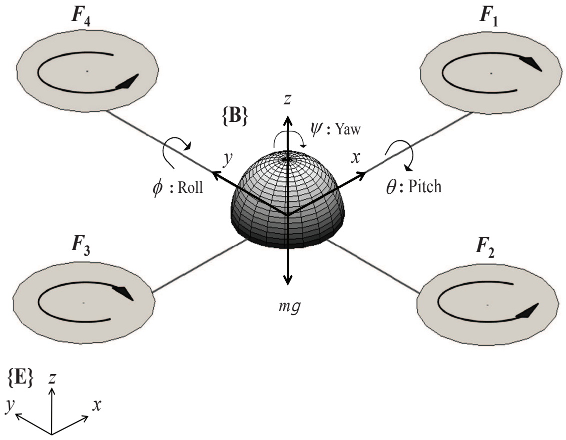

Fig. 1 The coordinate system with an earth frame {E} and a body frame {B}.

where $\Omega_r = (w_1 - w_2 + w_3 - w_4)$, and U_i (i=1, 2, 3, 4) are control inputs of the model as follows:

$$\begin{cases} U_1 = F_1 + F_2 + F_3 + F_4 \\ U_2 = F_3 - F_1 \\ U_3 = F_4 - F_2 \\ U_4 = F_1 + F_3 - F_2 - F_4. \end{cases} \quad (3)$$

3. ADAPTIVE ROBUST CONTROL (ARC)

The ARC control strategy is implemented for an altitude control of the quadrotor UAV, with the total mass which is unknown. This idea of using ARC control law was first proposed in [11], and applied in [12]. The controller for the altitude system is illustrated in Fig. 2.

In Fig. 2, u_a can be considered as the correct model compensation that is needed for the UAV system to track a time-varying trajectory along z-axis. With this approximate model compensation u_a, however, a perfect tracking may not be achieved. Thus, certain feedback action is needed to keep the output tracking error $e = z - z_d$. z_d is the desired altitude to be tracked by z. Since the UAVs dynamics along z-axis dynamics (2) is of second order, we can introduce a Proportional Derivative (PD) feedback control design p shown in Fig. 2 [13]. p is given by $p = \dot{e} + k_1 e = \dot{z} - \dot{z}_{eq}$, $\dot{z}_{eq} = \dot{z}_d - k_1 e$ where k_1 is a positive feedback gain. Since the transfer function from p to e, $G_p(s) = e(s)/p(s) = 1/(s + k_1)$ is stable, if we are able to regulate p well, it will guarantee that the output tracking error e will go to zero. Then, control input and the model uncertainty with p in the motion of the UAV along z axis (13) are as follows:

$$\begin{array}{rcl} m\dot{p} & = & m(\ddot{z} + g) - m(\ddot{z}_{eq} + g) \\ & = & \Delta U - m(\ddot{z}_{eq} + g) \end{array} \quad (4)$$

where U is control input of the model, Δ represents an input disturbance that results from the tilted body of the UAV, c.f, $C_\phi C_\theta$, m is the total mass of the UAV that is the model uncertainty, and $\ddot{z}_{eq} = \ddot{z}_d - k_1\dot{e}$. Let the unknown parameter and the known basis function be $\theta = m$ and

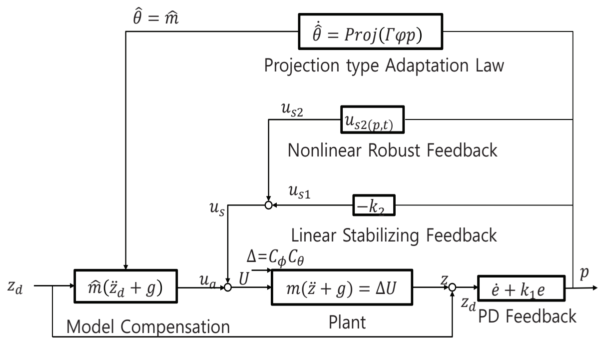

Fig. 2 The structure of the ARC control for the altitude control.

$\varphi = \ddot{z}_{eq} + g$, respectively. In [14], $\Delta(t)$ is assumed to be bounded a known function. The assumption is as follws:

Assumption. The unknown parameter lies in the known bounded region Ω_θ and unknown nonlinear function Δ is bounded by the known function $\delta(t)$

$$\begin{aligned} \theta &\in \Omega_\theta \triangleq \{\theta : \theta_{\min} < \theta < \theta_{\max}\} \\ \Delta &\in \Omega_\Delta \triangleq \{\Delta : |\Delta(t)| \leq \delta(t)d(t)\} \end{aligned} \tag{5}$$

where $\theta_{\min}$ and $\theta_{\max}$ are known as lower and upper bound of θ, and $d(t)$ is an time-varying disturbance.

Then, the ARC law that is shown in Fig. 2 can be obtained as follows:

$$\begin{aligned} U &= u_a + u_s,\ u_a = -\varphi\hat{\theta},\ \varphi = -(\ddot{z}_{eq} + g) \\ u_s &= u_{s1} + u_{s2},\ u_{s1} = -k_2 p \end{aligned} \tag{6}$$

in which u_{s1} represents a nominal stabilizing feedback.u_{s2} is the additional feedback that is needed to achieve a guaranteed robust performance when model uncertainty exists. Thus, it should satisfy the following robust performance conditions

$$\begin{aligned} &i. \quad pu_{s2} \leq 0 \\ &ii. \quad p\{u_{s2} - \varphi_d\tilde{\theta} + \Delta\} \leq \varepsilon \end{aligned} \tag{7}$$

where ε is a design parameter that can be arbitrarily small. Then, a projection type adaptation law is given by [13]

$$\dot{\hat{\theta}} = \Pr oj_{\hat{\theta}}(\Gamma\tau),\ \tau = \varphi p \tag{8}$$

where $\hat{\theta}$ is the estimate of θ, and $\Gamma > 0$ is a diagonal matrix. Let $\tilde{\theta}$ denote the estimation error (i.e., $\tilde{\theta} = \hat{\theta} - \theta$). If the desired trajectory satisfies the persistent exciting (PE) condition that is introduced in [13], the parameter estimates asymptotically converge to their true values (i.e., $\tilde{\theta} \to 0$ as $t \to \infty$).

Let Lyapunov theorem be considered and a positive definite (p.d.) function define, in order to show that the designed control law guarantees asymptotic tracking error for the UAV's altitude control

$$V_s = \frac{1}{2}p^2. \tag{9}$$

Then, it is bounded above by

$$V_s \leq \exp(-\lambda t)V_s(0) + \frac{\varepsilon}{\lambda}[1 - \exp(-\lambda t)] \tag{10}$$

where $\lambda = 2k_2/\theta_{\max}$. The time derivative of V_s is then,

$$\dot{V}_s = -k_2 p^2 + p\{u_{s2} - \varphi\tilde{\theta} + \Delta\}. \tag{11}$$

With ii of (7) and $\lambda = \min\{2_{k2}/\theta_{\max}\}$, we can have,

$$\dot{V}_s = -k_2 p^2 + \varepsilon \leq -\lambda V_s + \varepsilon \tag{12}$$

which leads to (10) and proves that all signals are bounded. Let another positive definite (p.d.) function define as

$$V_a = V_s + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}. \tag{13}$$

Then, the derivative of V_a satisfies

$$\begin{aligned} \dot{V}_a &= p\dot{p} + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \\ &= (-\varphi^T\tilde{\theta} - k_2 p)p + \tilde{\theta}^T\Gamma^{-1}\Gamma\varphi p \leq -k_2 p^2. \end{aligned} \tag{14}$$

Therefore, by Barbalats lemma [13], $p \to 0$ as $t \to \infty$, which implies the output tracking error asymptotically goes to zero.

4. SIMULATION

In this section, the results of a numerical simulation in terms of time history are presented in order to demonstrate the performance of the ARC control law utilized in this paper. The desired altitude is first set as $z_d(t)$=1 m with a disturbance $\Delta(t) = (1 - \cos\phi\cos\theta)(\sin(t^2))$ where ϕ and θ are assumed to be less than 5 degrees respectively, which comes from roll and pitch angles when the body of the UAV is fluctuated. To test the tracking performance of the controller with different unknown loads, then, the desired altitude is set as $z_d(t) = 0.5(1 - \cos(\pi(t))$ with the same disturbance. In this test,

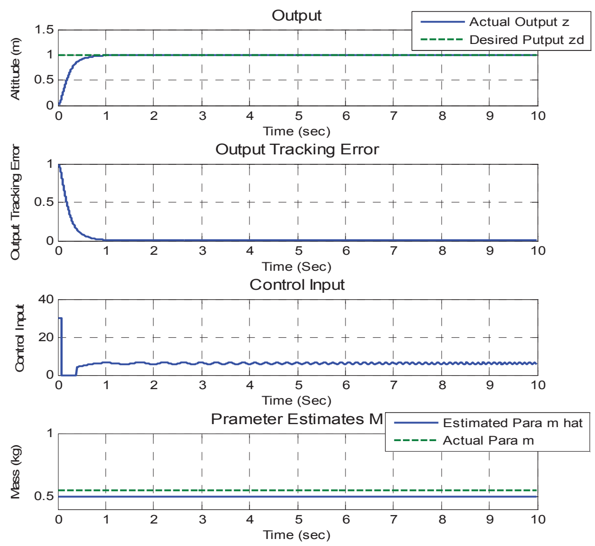

Fig. 3 ARC controller with a constant desired output and true parameter m = 0.550.

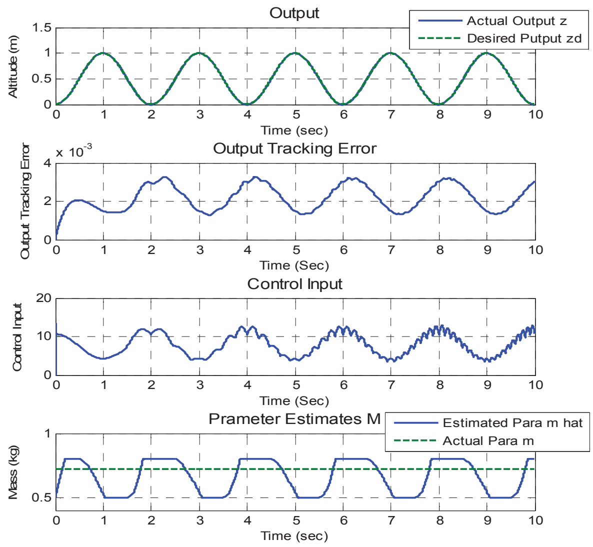

Fig. 4 PID controller with a sinusoidal desired output, true parameter $m = 0.720$, $\Gamma = 5000$.

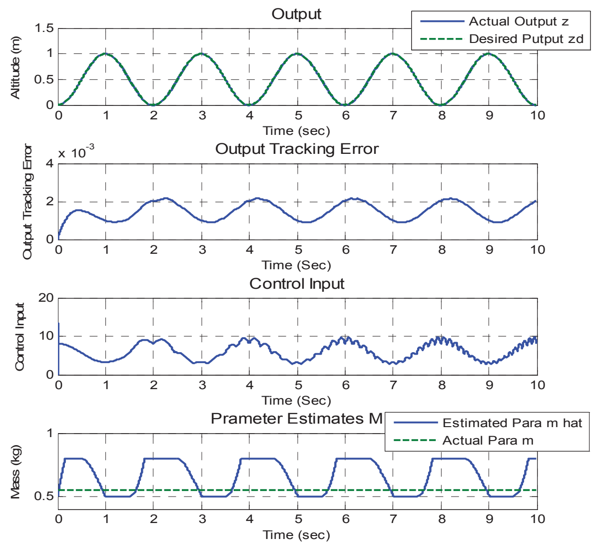

Fig. 6 PID controller with a sinusoidal desired output, true parameter $m = 0.550$, $\Gamma = 5000$.

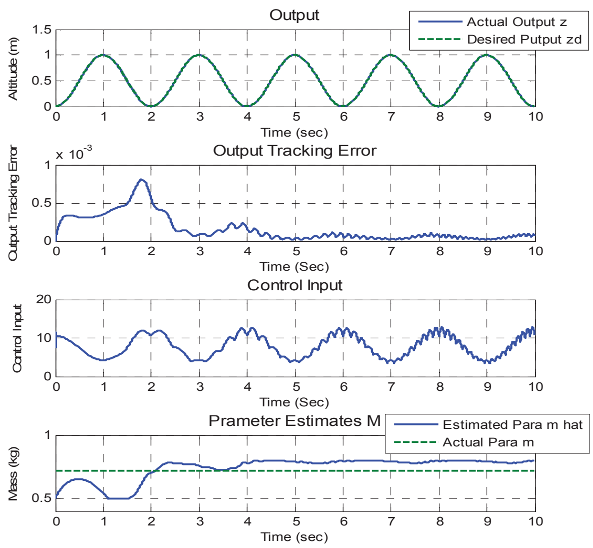

Fig. 5 ARC controller with a sinusoidal desired output, true parameter $m = 0.720$, $\Gamma = 5000$.

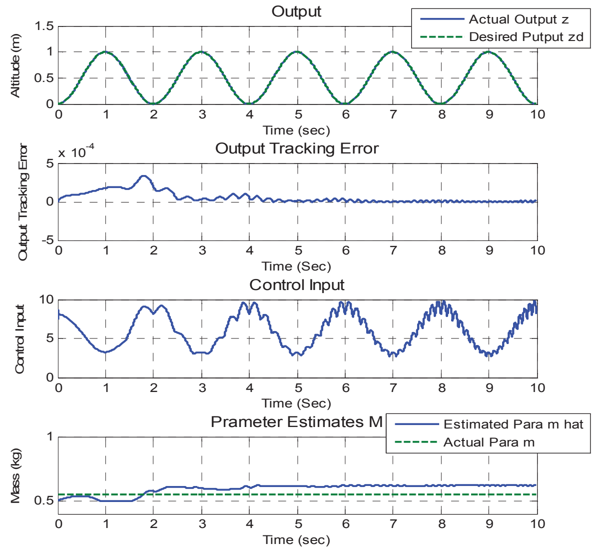

Fig. 7 ARC controller with a sinusoidal desired output, true parameter $m = 0.550$, $\Gamma = 5000$.

we first employ a classical PID controller and then, utilize the ARC control law for comparison. The gains of the PID controller are set as P = 0.04, I = 0.006, and D = 0.009, respectively. The bounds of the parameter variations are set as $\theta_{\min} = 0.500$ and $\theta_{\max} = 0.800$. The initial parameter estimates are set as 0.5000, for all cases. The adaptive rate, Γ, is 5000.

As a result, the responses and tracking errors of those simulations are shown in Fig. 3-7. As shown in the Fig. 3, when a desired output z_d was a constant, the UAV could track the desired output with almost zero tracking errors. With a constant z_d, however, the parameter estimate did not converge to its true value because the PE condition [13] was not satisfied with a constant desired output. Next, a desired output z_d is sinusoidal. When the PID controller was employed for an altitude control, the outputs could converge to the desired value with errors about $2 \times 10^{-3}m$, shown in Fig.4 and Fig. 6. When the ARC control law was employed, the outputs could converge to the desired value with the smaller tracking errors than those of the PID controller, shown in Fig.5 and Fig. 7. As the parameter estimates could nearly approach to their true values, which are 0.720 and 0.550, respectively, the tracking errors with the ARC control law could become almost zero. Since the desired output was of sinusoidal, the PE condition could be satisfied.

5. CONCLUSION

The final goal of our research is the success of the flight mission in that a quad-rotor helicopter carries unknown payloads. However, since the quad-rotor helicopter involves both translational and rotational motions, knowing of the accurate information on the vehicle, such as the moment of inertia and the total mass, is very significant, in order to guarantee the vehicles attitude and position controls.

Generally, if parameter uncertainty exists in a vehicles system in terms of a total mass of the structure, then the uncertainty is compensated by introducing an integral (I) controller in the PID controller. If the I gain has to increase highly to compensate the model uncertainty, however, it might make the stability of the system worse, making the system unstable overall.

Therefore, an adaptive robust controller was employed in case there is the parametric uncertainty in the quadrotor type UAV system. As shown in numerical simulation results, by utilizing the ARC law, the UAV could track the desired altitude in spite of the parameter uncertainty of the model. By comparing results of the PID controller with those of the ARC controller, tracking errors of the ARC controller could become smaller than those of the PID controller as parameter estimates nearly approached to their true values.

The parameter estimates could successfully converge to their true values when the desired outputs were differentiable. However, when the desired output was constant, the parameter estimate could not approach to its true value. In such a case, the quadrotor UAV system again involved the parametric uncertainty, even though the UAV could track the desired output.

Our future project will focus on the design of an attitude controller, by extending the ARC control strategy that is introduced in this paper.

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