Financing Innovative Activity and the Endogeneity of Patenting*

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Abstract

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Keywords: financing of innovation, endogeneity of patenting decision, debt, incomplete contracts.

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Abstract

It is common for lenders to require innovative firms to report all innovations and take actions, such as patenting, to protect their value. However, many future innovations are difficult to verify, making enforcement of such requirements difficult. This creates a tension for firms between protecting their innovations through patenting and creating assets that can be liquidated by a lender when not fully repaid. We show that the endogeneity of the patenting decision introduces an upper bound on the long-term payments that can be credibly promised to a lender, affecting the feasibility of obtaining financing and the terms of financial contracts.

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1 Introduction

For many innovative firms, intellectual capital is often their most important asset, with patents being increasingly used as collateral for loans (Mann, 2018a; Ma and Wang, 2019). Even firms that have yet to innovate pledge future patents as collateral through blanket liens (Mann, 1997). Thus, it is not surprising that lenders worry about protecting the value of the current and future intellectual capital of their borrowers. Indeed, anecdotal evidence shows that lenders require borrowers to disclose any new innovations and to take actions, such as patenting, to protect their value.[1] Such clauses notwithstanding, many innovations that are yet to materialize are likely difficult to verify, making the enforcement of these clauses difficult, if not impossible.

This unenforceability makes the patenting decision endogenous, and in a way that interacts with the contractual features of a firm's financing choices. One important aspect of patenting is that it creates a verifiable asset that a lender may liquidate, or threaten to liquidate, if not fully repaid. Anecdotal evidence and discussions with industry practitioners indicate that a firm facing the possibility of defaulting may not find it optimal to patent an innovation if the benefit obtained from patent protection does not outweigh the cost associated with increasing a lender's *ex-post* bargaining power.[2] Court cases also provide evidence that not securing intellectual property by registering collateral on patents prior to bankruptcy weakens or nullifies lenders' security interests.[3] Despite the importance of these

[1] See, for instance, the loan agreement between SI-Bone Inc. and Biopharma Credit Investments (October 13, 2017), available at: https://www.sec.gov/Archives/edgar/data/1459839/000119312518278797/d452987dex1020.htm

[2] We thank Trevor J. Belden, as well as an anonymous banker, for useful suggestions, including examples where lenders are worried that firms close to bankruptcy may have an incentive to delay patenting (or registering IP) to increase their bargaining power in bankruptcy negotiations with lenders. Another example was that lenders may extend additional loans to firms in order for them to litigate infringement of intellectual property by third parties, fearing that the firm may otherwise not want to dedicate resources to protecting IP that has a chance of transferring to the lender anyway.

[3] While a lender may have a blanket lien on all IP, unpatented innovations and/or unregistered intellectual property make the security of the collateral uncertain for the lender. Patent security interests are advised to be recorded under UCC and with the USPTO. A bankruptcy case involving Mitsui Manufacturers' Bank and Transportation Design and Technology, Inc (https://advance.lexis.com/api/document?collection=cases&id=urn:contentItem:3S4V-NPN0-0039-K2T6-00000-00&context=1516831) illustrates this issue well (see Haemmerli (1996) for a detailed discussion). In this instance, the lack of patents at the time

potential issues to lenders, as well as the substantial evidence that debt financing plays an important role in the funding of innovative firms (see, e.g. Amore et al., 2013; Chava et al., 2013; Cornaggia et al., 2015; Chava et al., 2017; Mann, 2018a), there is little or no research on how the endogeneity of the patenting decision interacts with a firm's financing decisions. We show that the endogeneity of patenting introduces a friction in contracting, especially for innovative firms whose existing assets are insufficient to serve as collateral against their needs for funding. While in equilibrium firms patent their innovations when doing so is efficient, this friction nevertheless affects the feasibility of obtaining financing, the terms of financial contracts, and has implications for policies related to patent protection and creditor rights.

In our model, a firm requires financing to invest in a two-period project which may produce an innovation at the end of the first period. The firm's existing assets and cash flows, however, are not sufficient to support the needed financing. In addition, cash flows and the innovation are not verifiable, so that contracts that directly depend on them are not enforceable in courts (as in Grossman and Hart, 1986; Hart and Moore, 1990). At the end of the first period, the firm learns whether the project produced an innovation and how much cash was generated and decides whether and how much to repay the lender, as well as whether to patent the innovation. If not fully repaid, the lender has the right to liquidate the project early. If the project is not liquidated in the first period, second period cash flows are realized. The contract can be renegotiated in each period, which ensures that the patenting decision will be efficient. Renegotiation of the contract effectively makes the terms of financing state-dependent and may introduce writedowns ("haircuts"), rollovers, or early repayment when it is optimal to do so.

A key feature of our framework is that if an innovation takes place, the firm must then decide whether to patent the innovation. Patenting not only protects intellectual capital but also converts it into a verifiable asset on which claims can be registered and secured, and which is then subject to repossession by an investor who has obtained control rights over

of bankruptcy by an innovating firm weakened the lender's claim on the firm's intellectual property in bankruptcy, including patents that were granted post-filing.

it. The firm's patenting decision hinges on the tradeoff between the value associated with protecting the innovation versus creating an asset that could be liquidated by the lender in the second period. As a result, the endogeneity of the patenting decision imposes an upper limit on the long-term repayment that the firm can credibly offer, which shapes the terms of financial contracts in equilibrium, limiting the feasibility of financing and leading to inefficiencies.

The upper limit on the long-term repayment depends, among other factors, on the difference between the value of the innovation with and without patenting. Of course, this difference depends on many considerations[4] and has an impact on not only the firm's patenting decisions but also its ability to obtain financing. While a firm with no need for external financing will patent its innovation any time this difference is positive, a financially constrained firm will not be able to obtain financing unless this difference is sufficiently large since a lender will infer, correctly, that the firm will subsequently choose not to patent its innovation. Moreover, even if the upper limit on repayments is not a binding constraint ex-ante, it may still limit what payments can be deferred when the firm cannot meet its first period obligation in full. This reduces the lender's expected payoff, which must then be compensated through an increase in the total promised payment. Hence, the need for greater promised payments leads to more projects being denied financing or, if financing is feasible, leads to a higher probability of liquidation of existing assets and/or the patent.

We find that when patenting is endogenous, innovative projects with a higher probability of success (low risk) will be financed even when their expected values are relatively low. This introduces a bias towards financing of low risk but also low payoff projects at the expense of projects that may yield higher returns but are riskier, even though investors have no particular preference for reducing risk. The effect of this bias is similar to credit rationing (Stiglitz and Weiss, 1981; Besanko and Thakor, 1987), although the mechanism

[4]For example, for some innovations the value when unpatented may be larger than when patented, giving an incentive to maintain trade secrets instead of patenting (see Saidi and Zaldokas, 2021; Friedman et al., 1991, for a review). We examine cases where patenting is optimal for firms in the absence of financing frictions to highlight the effects stemming from the endogeneity of patenting.

is very different; here, the constraint on long term payments limits the degree to which a lender can be compensated for risk.

Firm and industry characteristics affect the terms of the financial contract primarily through their effect on the constraint on long-term payments. When the constraint binds, relaxing the constraint increases the effective maturity of the financial contract. For instance, increases in the value of innovation, patent protection, value of existing assets, probability of innovation, creditor liquidation rights, and expected cash flows relax the constraint and, as a result, increase the duration of the contract. On the other hand, an increase in the value of the innovation when not patented tightens the constraint and hence decreases the duration of the contract. When the probability of innovation is sufficiently large, the variables that relax (tighten) the constraint also reduce (increase) interest rates.

We consider two policy tools, i.e., improving creditor rights and patent protection, that highlight the important role played by the innovation's value when *not* patented. Specifically, improving patent protection has a larger impact on feasibility when the value of an unpatented innovation is higher, whereas the opposite is true for improving creditor rights. Empirically, our model predicts that improving patent protection results in higher growth in firms with higher values of unpatented innovations. These predictions could be tested by using shocks to patent protection over time (see Lerner, 2002; Gallini, 2002), as well as cross-sectional differences in the value of unpatented innovations, which could be proxied by variables suggested in the literature that analyze tradeoffs between trade secrets versus patenting (see Hall et al., 2014, for a review).

Our focus is primarily on debt financing of entrepreneurial firms with limited cash flows and for whom future innovations may be difficult to verify. A natural setting to study such issues comes from the literature on incomplete contracting or enforcement, where debt-like contracts that assign investors property rights over the firm's assets emerge (Aghion and Bolton, 1992; Hart and Moore, 1994, 1998). Our model can be thought of as an extension of Hart and Moore (1998) to a setting where a long term asset can be endogenously created. A recent theoretical contribution to this literature is Huang et al. (2019), who, like us, study the trade-off between the termination threat and the desire to avoid early liquidation in a multi-period setting. Garleanu and Zwiebel (2009) emphasize the role of the design of the contractual assignment of property rights through loan covenants. Neither of these papers consider how the liquidation threat itself, or the value of property rights, may be endogenous to the extent that they rely on the existence of assets the lender can seize.

Our model can be classified within the broad literature on debt overhang (Myers, 1977; Holmstrom and Tirole, 1997).[5] From this perspective, our paper is related to Hackbarth et al. (2014), who study how a debt overhang problem influences firms' innovation decisions, and how this incentive varies between integrated and non-integrated firms. We do not consider the question of firm boundaries explored in Hackbarth et al. (2014), and instead focus on the implications arising from the endogeneity of firms' decisions concerning the patenting of innovations.

Our work is perhaps closest to the literature on asset redeployability and lending, which has found that firms with more redeployable assets are less financially constrained, receive larger loans with longer maturities, and have lower cost of external financing (Benmelech et al., 2005; Almeida and Campello, 2007; Benmelech, 2009; Benmelech and Bergman, 2008, 2009; Campello and Giambona, 2013; Marquez and Yavuz, 2013; Kim and Kung, 2017). There is also a parallel in the literature studying the incentives for firms to distort the creation or maintenance of collateral in the face of upcoming debt renegotiations. For instance, Gilje et al. (2020) show that highly levered firms will adjust the timing and value of their investments prior to renegotiating terms with their lenders. In particular, they show that being able to enhance the value of collateral allows firms to obtain better terms when raising

[5] Recently, Diamond et al. (2020, 2021) analyze how firms' decisions on the pledgeability of their cash-flows interacts with debt overhang. In particular, higher outstanding debt reduces a firm's incentive to raise the pledgeability of its cash flows. Instead, we focus on the extent to which property rights over the asset being financed can be enforced by the lender, which is crucial for analyzing firms' investments in innovative activities but is assumed to be frictionless in Diamond et al. (2020). Since our focus is on small innovative firms, whose cash flows from existing assets are not sufficient to ensure financing, increasing the firm's ability to make pledges against those cash flows would not obviate the frictions we study.

new financing. In our case, after raising funding, the firm decides whether to create an asset through patenting, thus increasing its value to the lender and making redeployability itself endogenous.

This aspect of our model is similar to the mechanism in Donaldson et al. (2018), who study how a bank's "warehousing" services can help improve the pledgeability of an entrepreneur's output and thus reduce financial frictions. In Donaldson et al. (2018), a farmer that produces grain can either store it himself, in which case part of the grain is lost, or he can store it in a warehouse (e.g., a bank) and avoid the loss. The main friction in that setting is that the farmer's output is not pledgeable. However, depositing in the warehouse creates a claim that the warehouse/bank can seize if loan repayment is not made, thus making property rights over the consumption good re-assignable through the creation of a receipt at a warehouse. Much like in our setting, the contract has to be structured to provide incentives for the farmer to actually deposit his output at the bank, and this constraint puts a limit on how much the farmer can be forced to repay or, equivalently, how much the bank will be willing to lend. In our setting, patenting by a firm is analogous to the grain deposit in Donaldson et al. (2018) in that it is similarly a solution to the problem of contracting with non-contractible cash flows. In our model, however, rather than making any of the firm's future cash flows contractible, patenting allows property rights over the *means of production* – the innovation – to be assigned to the investor. This distinction is important in our setting because the entrepreneur never wants to pay by liquidating the patent since doing so is inefficient, and this inefficiency further tightens the constraint on lending and feasibility. In our model, by making the property rights over the asset re-assignable through patenting, the firm is better able to commit to repay the investor from existing cash flows. This allows the firm, which has better use of the innovation, to retain control rights over its patented innovation whenever it has sufficient other resources it can use to repay the investor, without having to liquidate the patent in those cases.

The growth of innovative firms is often constrained by the availability of external funding

(Carpenter and Petersen, 2002), which may be difficult to obtain from traditional lenders (Hall and Lerner, 2010) because innovative firms invest in specialized assets (Williamson, 1988) that are difficult to verify (Glaeser et al., 2020), possess, and liquidate (Dell'Ariccia et al., 2021). It is also well documented that funding by venture capital funds plays an important role in the financing of innovation (Kortum and Lerner, 2000; Lerner and Nanda, 2020). Consequently, much of the financial contracting literature that studies innovative firms focuses on venture capital and the convertible contracts that are often used in such financing. Convertible debt has been viewed as helping solve incentive problems (Sahlman, 1990; Kaplan and Strömberg, 2003; Schmidt, 2003; Ewens et al., 2022) and/or enabling the efficient allocation of control rights during exit (Berglöf, 1994; Hellmann, 2006). Our finding that the endogenity of the patenting decision imposes a constraint on how much the firm can credibly promise to repay also applies to convertible debt contracts, as we discuss in more detail below.

Despite the importance of VC financing for innovative firms, many firms are unable to obtain such financing, and there is growing evidence that debt financing also plays an important role in the financing of innovation (Robb and Robinson, 2014). Hochberg et al. (2018) show that young innovation-oriented companies are often financed by debt and salability of patent collateral is one of the factors that facilitates such financing. Ibrahim (2010) estimates that financiers lend about $5 billion to start-ups annually. There is an active lending market for patent-producing startups in innovative industries such as medical devices, semiconductors, and software (Hochberg et al., 2015).[6] Our contribution is to show that when equity financing is not available or feasible, and when the entrepreneur does not have substantial personal assets, debt financing may nevertheless be possible through the reassignment of property rights over future innovations.

The paper proceeds as follows. Section 2 introduces the model. Section 3 solves for the

[6] There is also evidence that equity providers, such as venture funds that maintain significant control rights, use debt financing bundled with warrants between equity financing rounds (Gonzalez-Uribe and Mann, 2017).

participation constraint and the optimal contract terms. Section 4 analyzes the endogeneity of patenting and project feasibility. Section 5 discusses predictions regarding project feasibility and Section 6 focuses on predictions regarding contract terms. Section 7 concludes.

2 The model

A firm has a two-period project that requires an initial investment of $I > 0$ at $t = 0$, and may deliver an innovation at $t = 1$. The firm has no internal funds but has assets in place which produce a cash flow of $\tilde{C}_1 \sim U[0, \bar{C}_1]$ at $t = 1$ and, if allowed to continue, a further cash flow of $\tilde{C}_2 \sim U[0, \bar{C}_2]$ at $t = 2$. All cash flows are observable but not verifiable and, hence, cannot be contracted upon. The firm may use these cash flows, as well as the value of its innovation (described below) to repay investors, with any excess being consumed by the firm. The firm's existing assets have a liquidation value L_1 at $t = 1$.

We assume that the liquidation value of the existing assets and expected cash flows at time 1 are insufficient to cover the cost of the investment, i.e., $L_1 + E[C_1] < I$, and in particular the firm is cash-constrained at time 1: $\bar{C}_1 < L_1 < E[C_2]$. As we show later, these two assumptions simplify the model by ensuring that the lender cannot rely only on first period cash flows or the liquidation value of existing assets for payment. These assumptions also imply that continuing the firm's existing activities is always optimal, even in the absence of innovation. Partial liquidation of assets is not possible and the liquidation value of the assets in place decreases over time – for simplicity, we assume it is equal to zero at $t = 2$.[7]

The investment has a probability $p > 0$ of producing an innovation which arrives at time 1. The innovation is observable but not verifiable in court and hence cannot be contracted upon. If successful, the innovation generates a positive cash flow at some later date (beyond $t = 2$). As above, future cash flows of the innovation are not directly contractible. We denote by V the total value to society that is generated by the innovation and if the firm protects

[7] In the appendix, we extend the analysis to consider the case where the firm may have some long-lived assets, with positive value $A > 0$ at $t = 2$. We show that as long as the value of these assets under liquidation is not sufficient to fully guarantee the needed financing, our results are qualitatively unchanged.

its innovation by patenting it between $t = 1$ and $t = 2$, it can capture a portion $\omega \leq 1$ of V. The parameter ω allows us to analyze the effect of patent protection on financial contracting while keeping the value of the innovation constant.

Investment is efficient for the firm, i.e., $I \leq p\omega V$. Once patented, the innovation represents an asset that in principle can be liquidated at $t = 2$ for a fraction $\beta \leq 1$ of its value. The parameter β captures the portion of the value that a creditor with a claim against that asset can accrue if the asset gets liquidated. For example, β may proxy for the strength of creditor rights in the bankruptcy process. In addition, assigning a patent to its second best user may result in lower value, or secondary markets for patents could be illiquid. In the absence of patenting, or if no innovation occurs, there are no more assets that can be liquidated at $t = 2$.

If the firm does not patent its innovation, there is a chance that the innovation may be copied and revenues will be lost by the firm: without patenting the value of the innovation to the firm is αV. The parameter α captures the effect of the unpatented value of the innovation independently from changes in the value of the innovation when patented, ω, as well from the value of the innovation itself, V. As we discuss below, αV determines the outside option of the firm and hence plays an important role.[8] The two separate variables ω and α allows us to capture the possibility that different factors may affect the value of the innovation to the firm when patented versus not. For instance, extending the duration of patent protection, or allowing for stricter enforcement of patents, should increase the value of the innovation when patented (i.e., increase ω), with no consequent implications for the value of the innovation when there is no patent, αV. Conversely, the value when not patented should be a function of the likelihood a rival may copy the innovation, which should not affect the value of the patented innovation.

[8]If the innovation is not patented, it is natural to consider what happens if the firm chooses to default on its promised payments and thus gets liquidated. In this case, the entrepreneur could set up another firm and use the unpatented innovation to generate future cash flows. Hence, deciding not to patent an innovation is costly to the firm, but does not lead to complete loss even under liquidation, whereas a lender would be unable to obtain any value from liquidating a firm whose innovations have not been patented.

We assume that the potential gain from patent protection is larger than the loss from liquidation of the patent, i.e., $\omega V - \alpha V > \omega V - \beta\omega V$ or, equivalently, $\beta\omega > \alpha$. This implies that patenting is optimal, and is consistent with the idea that the purchaser of the patent also gets protection from competition even if there is some loss of value due to liquidation. We assume that the maximum possible cash flows at time 2 are larger than the social gain of patenting, i.e., $\bar{C}_2 > (\omega - \alpha)V$. As we discuss further below, this ensures that any constraints on what the firm can repay are not driven by $\bar{C}_2$.

Non-verifiability of cash flows forces an investor to rely on control rights and liquidation threats to ensure repayment (e.g., Bolton and Scharfstein, 1990, 1996; Aghion and Bolton, 1992; Hart and Moore, 1994). We focus therefore on debt-like contracts with liquidation rights for the investor if a promised payment is not made. A recent contribution by Donaldson et al. (2018) presents a mechanism where a third party, such as a "warehouse," can be used to verify the existence of a cashflow and, hence, represents a solution to the problem of contracting on something that is not easily verifiable. Patenting in our model plays an analogous role of allowing for contracting based on the outcome of an innovation, which is itself not contractible.

The financial contract has a short-term promised payment of D_1 to be made at $t = 1$ and a long-term payment of D_2 to be made at $t = 2$. If any of these payments are not made the lender is entitled to liquidate the firm's assets to recover the promised payment $D_1 + D_2$. The credit market is competitive at time zero.

We allow for renegotiation of the original contract to improve efficiency. Specifically, if the firm repays its time 1 obligation D_1 in full, it can then make a take-it-or-leave-it offer to the investor for the remaining amount. In particular, the firm may want to repay some of its long-term debt early when it has the resources to do so. On the other hand, if the firm is unable to repay its obligation, or if it simply refuses to do so even if able (i.e., if the firm strategically defaults), then the lender is entitled to make a take-it-or-leave-it offer to the firm, with the limitation that the lender cannot ask for more than the total payment $D_1 + D_2$

promised in the original contract. This allows for the possibility that the lender may prefer not to force liquidation, but may instead opt to roll over some of the shortfall in repayment, or may even find it optimal to forgive a portion of the loan.[9] The renegotiation process also captures the notion that when the firm defaults, a lender's bargaining power likely improves. If an offer is accepted, the new contract replaces the existing contract. If rejected, the original contract remains in place and the investor liquidates if the promised payment was not made in full. As discussed below, this renegotiation mechanism results in agents agreeing on socially efficient outcomes whenever possible at time 1. We also discuss results under alternative renegotiation mechanisms in the Appendix. We allow for renegotiation at time 2; however, since the game ends at time 2 there is no room for either debt forgiveness or deferral of payments and, therefore, for simplicity we assume that only the firm can make a take-it-or-leave-it offer to the lender.

Figure 1 shows the timeline of the model. At $t = 1$, the cash flow C_1 is realized and both parties learn whether or not an innovation has occurred. After contract renegotiation, and if not liquidated, the firm decides whether to patent or not when it has innovated. Finally, if the project is continued the time 2 cash flows are realized, agents again may renegotiate the payment, and the lender may liquidate if the promised payment is not made.

3 Analysis

We now solve the model by backwards induction given a financing contract (D_1, D_2). Later, we use the payoffs obtained to determine the ex-ante optimal financial contract.

3.1 No innovation

We first consider the case when there is no innovation. The analysis is fairly simple. If the first period cash flows are sufficient to pay back the lender, the firm pays D_1, continues

[9]We assume that when indifferent, the lender chooses to make the offer that is preferred by the firm. In other words, the lender chooses a Pareto-improving offer when indifferent.

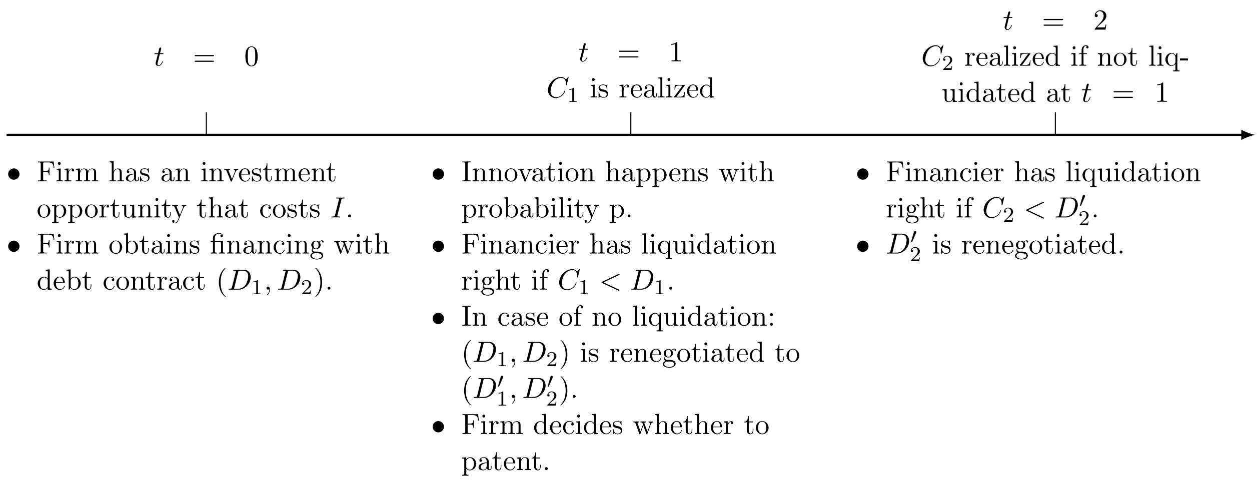


Figure 1: Timeline

to $t = 2$ and receives the second period cash flows. However, if the first period cash flows are not sufficient to pay back D_1, then the lender liquidates to obtain L_1. This is because the liquidation value of assets is zero in the second period and cash flows are not verifiable, making it impossible for the lender to get paid anything in the second period. We have the following result (all proofs can be found in the Appendix).

Claim 1 *Ex-ante, the expected payoff for the lender in case of no innovation can be written as a function of D_1 as follows*

$$\mathcal{F}_0(D_1) = L_1 \Pr(C_1 < D_1) + D_1 \Pr(C_1 > D_1) = L_1 \frac{D_1}{\bar{C}_1} + D_1 \left(1 - \frac{D_1}{\bar{C}_1}\right). \tag{1}$$

The ex-ante payoff to the firm when there is no innovation is equal to

$$\mathcal{H}_0(D_1) = E[C_1] + (E[C_2] - D_1) \Pr(C_1 > D_1) = E[C_1] + (E[C_2] - D_1) \left(1 - \frac{D_1}{\bar{C}_1}\right). \tag{2}$$

Note that the firm's payoff, $\mathcal{H}_0(D_1)$, decreases in D_1. This is because liquidation is inefficient even when there is no innovation. On the other hand, the expected payoff of the lender, $\mathcal{F}_0(D_1)$, increases when D_1 increases. For completeness, we also summarize here the joint payoff – the sum of the payoffs to the lender and the firm – as it will be useful later.

Here, the joint payoff is

$$\mathcal{G}_0(D_1) = \mathcal{F}_0(D_1) + \mathcal{H}_0(D_1) = E[C_1] + E[C_2] - (E[C_2] - L_1)\frac{D_1}{\bar{C}_1}.$$

Like the firm's payoff $\mathcal{H}_0(D_1)$, the joint payoff $\mathcal{G}_0(D_1)$ is also decreasing in D_1. This is because an increase in D_1 raises the probability of liquidation, which is inefficient. Therefore, increasing D_1 creates a trade-off between feasibility of financing and the total payoff when there is no innovation.

3.2 Innovation with endogenous patenting

3.2.1 Second period

Denote the renegotiated terms of the contract as D_1', D_2'. First, suppose that the firm has patented its innovation after paying the lender D_1' at time 1. After the realization of the time 2 cash flows C_2, the firm will pay $\min\{D_2', \beta\omega V\}$ to the lender if $C_2 \geq \min\{D_2', \beta\omega V\}$ to avoid liquidation of the project. The lender will accept this payment given that it is equivalent to what it would get in the event of liquidation. However, if $C_2 < \min\{D_2', \beta\omega V\}$, then the firm will be unable to repay the promised amount and the lender will liquidate to receive $\min\{D_2', \beta\omega V\}$. In other words, the lender's payoff at time 2 if there is a patented innovation is always equal to $\min\{D_2', \beta\omega V\}$, and the lender's payoff varies with D_2' only when $D_2' < \beta\omega V$. This gives us the following preliminary result.

Lemma 1 *The payoffs of the firm and the lender from a contract with $D_2' > \beta\omega V$ are equivalent to the corresponding payoffs from a contract with $D_2' = \beta\omega V$.*

Given Lemma 1, it is without loss of generality to assume that agents always agree on a time 2 payment which is no greater than the liquidation value of the patent, i.e., $D_2' \leq \beta\omega V$. Therefore, at $t = 2$, after the firm has paid D_1' and patented its innovation, the lender's payoff is always $D_1' + D_2'$.

Suppose instead now that the firm has paid D_1' at time 1 to the lender, but decided not to patent the innovation. In this case, the lender will not obtain any payment at time 2 because there are no assets that can be liquidated or threatened to be liquidated to force the firm to pay. Hence, the lender only obtains D_1' and the firm's payoff is

$$C_1 + E[C_2] + \alpha V - D_1'. \tag{3}$$

3.2.2 First period and the incentive to patent

Given the set of possible outcomes and payoffs at time 2, we now analyze what happens at time 1, starting with the patenting decision.

The firm's decision to patent at time 1 depends on its payoff when it patents versus when it does not. Suppose that the firm can make the time 1 promised payment. In this case, if the firm chooses to patent its expected payoff is:

$$C_1 + E[C_2] + \Pr(C_2 \geq D_2')\omega V + \Pr(C_2 < D_2')\beta\omega V - (D_1' + D_2'). \tag{4}$$

This payoff reflects that, if the firm makes its time 1 payment and patents, its payoff equals the time 1 plus the expected time 2 cash flows, as well as the patented value of the innovation when able to meet its obligation at time 2, but only the liquidation value $\beta\omega V$ when not, minus the payments made to the lender, $D_1' + D_2'$. The firm will patent if and only if (4) is at least as large as (3), or in other words if

$$(\omega \Pr(C_2 \geq D_2') + \omega\beta \Pr(C_2 < D_2'))V - \alpha V - D_2' \geq 0. \tag{5}$$

Note that the value of D_2' that satisfies (5) is less than $\bar{C}_2$ because $\bar{C}_2$ is larger than the gain from patenting, i.e., $\bar{C}_2 > \omega V - \alpha V$. Consequently, the left-hand side of (5) is decreasing in D_2'. Given that C_2 is uniform in $[0, \bar{C}_2]$, we can now establish the following result.

Lemma 2 *Let D_2^{max} be the maximum value such that (5) holds, and is given by*

$$D_2^{max} = \frac{\omega - \alpha}{\frac{1}{V} + \frac{(1-\beta)\omega}{\bar{C}_2}}. \tag{6}$$

The firm finds it optimal to patent its innovation only if $D_2' \leq D_2^{max}$.

Lemma 2 puts an upper bound on the amount that can be credibly pledged to the lender, which amounts to an incentive compatibility constraint related to patenting, with the result that any contract that leads to patenting must have a promised time 2 payment no greater than D_2^{max}. This upper bound is analogous to the "pledgeable expected income" in Holmstrom and Tirole (1997), who analyze an investment decision subject to a moral hazard problem. In that setting, the borrower must retain a sufficiently large stake in the project's return in order to help resolve the underlying moral hazard problem. This friction then translates into a limit on how much the borrower can credibly commit to repay its lender, much like D_2^{max} in Lemma 2. One key difference, however, is that, unlike in most standard agency problems, here there is no direct cost for the firm to patent its innovation, and in a frictionless market it would always choose to do so. Rather, the cost of patenting to the firm emerges endogenously as a result of the financial contract used, which requires a transfer of control over the patented innovation in the event of default. This is also analogous to what happens in Donaldson et al. (2018), where depositing grain at a warehouse allows for contracting over cash flows that otherwise could not be pledged. In that setting, the contract has to be structured to provide sufficient incentives for the farmer to actually deposit his output at the bank, thus making cash flows verifiable. Doing so, however, puts a constraint on how much the lender can expect to get paid, which in turn limits how much financing the farmer can obtain. The cost of depositing with the bank at the interim date emerges endogenously as a result of the deposit contract used, which transfers control over the grain to the bank in the event of default.

It is worth noting that the incentive constraint on D_2^{max} depends on two economic quantities: $\omega - \alpha$, which is the value of protecting the innovation, and $(1-\beta)\omega$, which is the inefficiency due to liquidation in period 2. The social gain from patenting, $\omega - \alpha$, plays an important role in the amount that the firm can credibly pledge. However, the effect of changes in α and ω on the ability to obtain financing and on the financial contract that is optimal are not perfectly negatively correlated. This is because a higher value of ω has an additional effect, which leads to an increase in the inefficiency caused by liquidation in period 2. This additional effect decreases the firm's incentive to patent. On the other hand, changes in α do not produce any effect on the liquidation value of the patent.

Note that, as argued above, there is another upper bound on the payment that can be made at time 2. Specifically, the time 2 payment has to be less than the liquidation value of the patent, i.e., $D_2' \leq \omega\beta V$, as otherwise the firm could always renegotiate the payment down to $\omega\beta V$. As a result, we will focus only on the cases when $D_2^{max} \leq \omega\beta V$ going forward.

3.3 Participation constraint and the optimal contract

In this section, we derive the lender's participation constraint and the optimal contract. To do so, we first assume that a feasible contract exists and calculate the lender's payoff. We discuss feasibility in more detail below.

The actions and payoffs of agents differ depending on whether D_2 or D_2^{max} is bigger, and the different payoff structures result in two optimization problems and two (potentially) different optimal contracts, which need to be compared. We show later in Theorem 1 that a contract with $D_2 > D_2^{max}$ can never be optimal. Thus, in the following we only describe the case when $D_2 \leq D_2^{max}$. We have the following result.

Claim 2 *Assume $D_2 \leq D_2^{max}$ and $D_1 + D_2 > D_2^{max} > L_1$, and that an innovation has occurred. The lender's payoff, f_1, as a function of the terms of an original feasible contract is*

$$f_1(C_1, D_1 + D_2) = \min\{D_2^{max} + C_1, D_1 + D_2\}.$$

The lender's expected payoff is then

$$\mathcal{F}_1(D_1 + D_2) := \frac{1}{\bar{C}_1}\int_0^{\bar{C}_1} f_1(C_1, D_1 + D_2)dC_1 = (D_1 + D_2) - \frac{(D_1 + D_2 - D_2^{max})^2}{2\bar{C}_1}. \qquad (7)$$

The lender's payoff depends on the realization of first period cash flows. If $C_1 \geq D_1$, the firm is able to make the time 1 payment and afterward finds it optimal to patent the innovation given that $D_2 \leq D_2^{max}$. In this case, the outside option of the lender is $D_1 + D_2$. The firm prefers to pay the maximum amount possible at time 1, C_1, to minimize the probability of liquidation at time 2. Thus, in this case, the renegotiated contract is $(D_1', D_2') = (C_1, D_1 + D_2 - C_1)$.

However, if $C_1 < D_1$, then the firm cannot make the payment and the lender has an option to make a take-it-or-leave-it offer to the firm. Being aware that the firm will only patent if $D_2' \leq D_2^{max}$, the lender will offer the renegotiated contract $(D_1', D_2') = (C_1, \min\{D_2^{max}, D_1 + D_2 - C_1\})$, with payoff to the lender of $C_1 + \min\{D_2^{max}, D_1 + D_2 - C_1\}$. The lender will find it optimal to offer this contract when this payoff is greater than L_1, and will roll over a portion of the promised payment rather than forcing early liquidation. Importantly, when $D_2^{max} < D_1 + D_2 - C_1$, the lender also prefers to forgive a portion equal to $D_1 + D_2 - C_1 - D_2^{max} > 0$ of the remaining obligation rather than asking for full repayment as this would lead the firm not to patent its innovation (detailed discussion is in the Appendix).

Figure 2 displays the lender's payoff, f_1, as a function of the terms of an original feasible contract for the case where $D_2 \leq D_2^{max}$ and $D_1 + D_2 > D_2^{max}$. For $C_1 < D_1 + D_2 - D_2^{max}$, the lender's payoff is increasing in C_1, while it is constant and equal to $D_1 + D_2$ for larger values of C_1.

Participation constraint

We can now derive the participation constraint of the lender. Given that financing is competitive at time 0, the lender participates as long as its expected payoff from the contract is

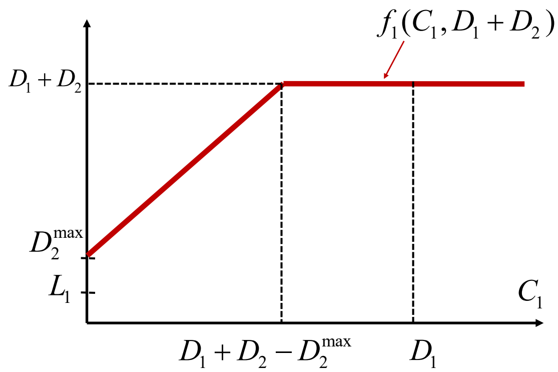


Figure 2: Investor's payoff, conditional on innovation, as a function of C_1 when $D_2 \leq D_2^{max}$.

equal to the amount of the loan, which is equal to the investment, I. The expected payoff of the lender can be written as

$$(1-p)\mathcal{F}_0(D_1) + p\mathcal{F}_1(D_1 + D_2) = I,$$

where $\mathcal{F}_0(D_1) = L_1 \frac{D_1}{\bar{C}_1} + D_1(1 - \frac{D_1}{\bar{C}_1})$ is the expected payment to the lender when there is no innovation (Claim 1), and $\mathcal{F}_1(D_1, D_2)$ is the expected payment to the lender when there is an innovation (Claim 2). Thus, we can rewrite the participation constraint as

$$(1-p)\left(L_1 \frac{D_1}{\bar{C}_1} + D_1(1 - \frac{D_1}{\bar{C}_1})\right) + p\left((D_1 + D_2) - \frac{(D_1 + D_2 - D_2^{max})^2}{2\bar{C}_1}\right) = I. \quad (8)$$

The short and long-term components of the financial contract (D_1, D_2) are linked to each other through this participation constraint. In particular, there is a one-to-one correspondence between D_1 and D_2 for all contracts (D_1, D_2) with $D_2 \leq D_2^{max}$ that satisfy (8).

For a given D_1, denote by $\Delta(D_1)$ the smallest value such that

$$(1-p)\mathcal{F}_0(D_1) + p\mathcal{F}_1(D_1 + \Delta(D_1)) = I.$$

Since both $\mathcal{F}_1(.)$ and $\mathcal{F}_0(.)$ are strictly increasing functions, $\Delta(D_1)$ is a decreasing function of D_1. This is intuitive as when the short-term payment increases, one needs a lower long-term payment to satisfy the participation constraint. Given this, it is useful at this point to convert the constraint that $D_2 \leq D_2^{max}$ to a lower bound constraint on D_1. Let

$$D_1^{min} := \max\{\Delta^{-}(D_2^{max}), 0\}, \text{ where } \Delta^{-} \text{ is the inverse function of } \Delta.$$

In Appendix A.5 we provide a closed-form solution for $\Delta(D_1)$ and D_1^{min}. The important thing to note is that a contract satisfying the incentive constraint, $D_2 \leq D_2^{max}$, is equivalent to a contract satisfying $D_1 \geq D_1^{min}$ and the participation constraint. This helps us to characterize the optimal contract, as discussed next.

Optimal contract

Given that the credit market is competitive, the optimal contract for the firm maximizes joint surplus while ensuring that the lender's participation constraint is satisfied. In the Appendix, we show that the contract that maximizes joint surplus is equivalent to the contract that minimizes the expected inefficiency due to liquidation, and we derive the following minimization problem for the firm that pins down the optimal contract.

$$\begin{aligned}
\min_{D_1,D_2} \quad & (1-p)\frac{D_1}{\bar{C}_1}(E[C_2]-L_1) + p\omega(1-\beta)V\,\Pr(C_1+C_2 < f_1(C_1, D_1+D_2)) \\
\text{subject to} \quad & (1-p)\mathcal{F}_0(D_1) + p\mathcal{F}_1(D_1+D_2) = I, \qquad (9)\\
& D_1 \leq \bar{C}_1, \\
& D_1 \geq D_1^{min}.
\end{aligned}$$

The first and second terms in the objective function represents the expected social loss from liquidation when there is no innovation and when there is innovation, respectively. With this, we have the following characterization of the optimal contract.

Theorem 1 *A contract with $D_2 > D_2^{max}$ cannot be optimal. Let*

$$D_1^* = \frac{L_1}{2} - \frac{\bar{C}_2}{2\omega(1-\beta)V}(\frac{\bar{C}_2}{2} - \bar{C}_1 - L_1).$$

The optimal contract can be characterized as $D_2 = \Delta(D_1)$, and D_1 is determined as follow:

- *If $D_1^* \geq \bar{C}_1$, then $D_1 = \bar{C}_1$*
- *If $D_1^{min} < D_1^* < \bar{C}_1$, then $D_1 = D_1^*$*
- *If $D_1^* \leq D_1^{min}$, then $D_1 = D_1^{min}$*

The theorem establishes that any contract with $D_2 > D_2^{max}$ is dominated, so that the optimal contract always has $D_2 \leq D_2^{max}$. The term D_1^* is a solution to the firm's optimization problem when there are no constraints. The optimal first period payment D_1 depends on how D_1^* compares to the maximum time 1 cash flow and minimum time 1 payment. Moreover, the expression for $D_2 = \Delta(D_1)$, found in (29) in the appendix, shows that the promised long-term component of the total payment is affected by the endogeneity of patenting even when the constraint on long-term payments does not bind ex-ante (i.e., even when $D_2^* < D_2^{max}$). This occurs because even if ex-ante the constraint is not binding, ex-post it may nevertheless bind if the realization of time 1 cash flows is low and the promised time 1 payment needs to be rolled over to time 2. The possibility that the lender gets less than $D_1 + D_2$ when the constraint binds ex-post reduces the expected payoff of the lender so that the total promised payment needs to go up to satisfy the lender's participation constraint.

3.4 Contracts when patenting occurs automatically

For comparison, we present the case when patenting occurs automatically, meaning that any innovation is always patented if the agents continue to time 2. First, notice that the case of no innovation does not change and the analysis remains the same as in Section 3.1.

As discussed above, agents always agree on a time 2 payment which is no greater than the liquidation value of the patent, i.e., $D_2 \leq \beta\omega V$. Therefore, at $t = 2$, after the firm has paid D_1 and patented its innovation, the lender's payoff is always $D_1 + D_2$.

Now consider what happens at date 1. We start with the case where $C_1 \geq D_1$. If the firm refuses to pay, or pays less than D_1, the lender has the right to make a take-it-or-leave-it offer. In that case, the lender can simply offer again the same original contract, which maximizes its payoff if the firm decides to pay. The firm will then either pay D_1 or let the lender liquidate. In case of liquidation, the firms gets C_1. If the firm pays D_1 and continues, it gets at least $C_1 + E[C_2] + \omega\beta V - D_1 - D_2$. Because $D_1 \leq \bar{C}_1 < E[C_2]$ and $D_2 \leq \omega\beta V$, the firms always prefers to pay D_1 and continue. Alternatively, the firm can pay D_1 and retain the right to make a renegotiation offer. If the firm just pays D_1, then it needs to pay D_2 at time 2, and the lender will liquidate the patent in states where $C_2 < D_2$. To reduce the possibility of liquidation at time 2, the firm prefers to pay as much as possible at time 1. Therefore, the firm offers renegotiated payments $\{D_1^{'}, D_2^{'}\}$ that maximize its payoff: $D_1' = C_1$ and $D_2' = D_1 + D_2 - C_1$.

Now consider the case where $C_1 < D_1$. In this case, the lender either liquidates or makes a take-it-or-leave-it offer. If the lender liquidates it only gets L_1. Instead, the payoffs of both agents can be improved if the lender offers the following contract: $D_1' = C_1$ and $D_2' = D_1 + D_2 - C_1$. This contract will be accepted by the firm since rejecting it would lead to liquidation by the lender and a lower payoff for the firm.

We can now write the lender's payoff when there is an innovation for a given cash flow realization C_1 as,

$$C_1 + \min\{\omega\beta V, D_1 + D_2 - C_1\} = \min\{\omega\beta V + C_1, D_1 + D_2\}.$$

With this we can calculate the expected payoff of the lender based on an initial feasible contract when the firm has successfully innovated.

Claim 3 *The lender's expected payoff in case of innovation is:*

$$\frac{1}{\bar{C}_1}\int_0^{\bar{C}_1} \min\{\omega\beta V + C_1, D_1 + D_2\} dC_1. \tag{10}$$

Note that, unlike the case where patenting is endogenous, the lender's expected payoff does not depend on the value of the innovation without patenting. Given that the lender's expected payoff will always be equal to I, a contract that maximizes the total surplus or, equivalently, minimizes inefficiencies is again optimal. Therefore, we can write the optimization problem as

$$\begin{aligned} \min \quad & (1-p)(E[C_2] - L_1)\Pr(C_1 < D_1) + p\omega(1-\beta)V Pr(C_1 + C_2 < D_1 + D_2) \\ s.t \quad & (1-p)\mathcal{F}_0(D_1) + p\frac{1}{\bar{C}_1}\int_0^{\bar{C}_1} \min\{\omega\beta V + C_1, D_1 + D_2\} dC_1 = I \\ & 0 \le D_1 \le \bar{C}_1. \end{aligned} \tag{11}$$

The optimization problem in (11) and the financial contract that emerges differ from the case where patenting is endogenous. Given that there is no need for a constraint that $D_2 \le D_2^{max}$, the time 2 promised payment is only bounded by $\omega\beta V$, because of renegotiation in the second period. So as not to further complicate the analysis and to retain our focus on the case where patenting is endogenous, rather than explicitly providing a solution for the optimal contract in this case, we simply note the main takeaway, which is that, quite generally, the optimal contract in this case will have a larger D_2 and smaller D_1 compared to the case where patenting is endogenous.

3.5 Renegotiation and state contingent contracting

An important feature of our analysis is that agents are allowed to renegotiate the original contract based on the realization of first period cash flows and whether an innovation occurred. In our model, renegotiation is designed to achieve the highest joint payoff by assigning an agent the right to make a take-it-or-leave-it offer so as to maximize the social payoff. We focus on cases when patenting is socially optimal. As a result, in equilibrium whenever there is an innovation, patenting always takes place regardless of the realization of first period cash flows, as discussed above. The threat to not patent affects renegotiation and the terms of contracting, but under the optimal contract we consider it never actually leads to an inefficient patenting decision.

An implication of the efficiency of the patenting decision is that the ability to renegotiate makes the use of state-contingent contracts unnecessary and superfluous, in the sense that allowing for state contingencies does not improve welfare relative to the contracts we consider. Intuitively, a state-contingent contract could be useful by requiring a higher payment if the firm does not patent and then lowering the payment if the firm does in fact patent. However, renegotiation already achieves the socially optimal outcome through early payments, rollovers, and haircuts. For example, consider the case when $C_1 < D_1$. In this case, the lender prefers to roll over the remaining portion of the date 1 promised repayment instead of liquidating the firm. However, if $D_1 + D_2 - C_1 > D_2^{max}$, the firm would not patent the innovation under the terms of the original contract. In this case, the lender finds it optimal to also forgive a portion rather than ask for full repayment. This is the region in Figure 2 where the lender's payoff increases in C_1. On the other hand, punishment by requiring higher payments in case the firm fails to patent an innovation is not possible given that the innovation is not verifiable and firm's verifiable assets are limited. As a result, state contingencies of this type become redundant, and do not serve to loosen the firm's financing constraints (a formal proof is provided in the appendix).

An alternate way of introducing state contingencies is often employed in the context of venture capital financing, where convertible contracts are commonplace (e.g., debt contracts with fixed promised repayments, but which can convert to equity when specific conditions are met). With slight changes to our model, convertible debt could similarly be feasible in our setting if patenting, in addition to making the asset verifiable, also makes the future cash flows from the innovation verifiable. This would ensure that, after patenting, a financier with an equity claim could force the firm to pay, which is not possible when cash flows are not verifiable. In this case, whenever an innovation arises, the firm would need to take into account that the financier can convert its claim, debt, into equity after the innovation is patented. While in principle making cash flows verifiable should help loosen the firm's financing constraints, it would nevertheless be true that in order to satisfy the incentive compatibility condition for the firm to patent there would need to be a limit in terms of how much upside could be promised to the investor. In other words, there would be an upper bound on the amount of equity into which the instrument could convert, thus still limiting the firm's ability to obtain financing for its innovative projects. As a result, we believe that similar issues as those highlighted in our model of debt contracting would continue to apply.

4 The endogeneity of patenting and project feasibility

The constraint on long-term payments resulting from the endogeneity of patenting introduces inefficiencies by limiting which projects are feasible and increasing the probability of liquidation. In what follows, we contrast our findings with the benchmark case where patenting is automatic as well as to the case where the investment is internally financed. This allows us to isolate the effect coming from the endogeneity of patenting rather than from the more widely studied frictions arising from the need to obtain outside financing.

4.1 Feasibility

When the project is internally financed, all that matters is that the expected value of the innovation be greater than the initial investment. The firm will therefore undertake the project if

$$p\omega V \geq I. \tag{12}$$

This is also the condition for the first best because there is no inefficient liquidation. We have assumed throughout that the project is positive NPV for the firm, meaning that $p\omega V > I$ so that if the firm had enough cash at time zero it would always undertake the project. The need for outside financing can make the project infeasible because it introduces inefficiencies. First, it may not be possible to satisfy the participation constraint of the lender. Second, the possibility of liquidation introduced by outside financing may make the project negative NPV from the perspective of the firm. We focus on the former possibility.

We can compare (12) to the case when patenting occurs automatically but the firm requires outside financing to undertake the investment. We have the following result.

Lemma 3 *Assume the firm can commit to patent any innovation. Then, there exists a feasible financing contract if and only if*

$$E[C_1] + \omega\beta V \geq \frac{I - (1-p)L_1}{p}. \tag{13}$$

Feasibility of financing depends on the sum of the expected maximum payment from time 1 cash flows, $E[C_1] = \frac{\bar{C}_1}{2}$, and the time 2 repayment, $\omega\beta V$. Not all projects satisfying the first best, (12), will be taken because of the non-contractibility of future cash flows, which prevents the firm from committing to repay, and the associated inefficient liquidation. To simplify the comparison, we assume $\bar{C}_1 = L_1 = 0$, which eliminates the expected payoff of the lender from time 1 cash flows as well as liquidation at time 1 when there is no innovation. The participation constraint simplifies and becomes

$$p\omega\beta V \geq I. \tag{14}$$

For this case, the only limit on the time 2 promised payment is the liquidation value of the patent, $\omega\beta V$. By comparing (12) to (14) we can see that financing is not feasible for a greater number of projects as β gets smaller, i.e., as the inefficiency introduced by liquidation becomes larger or, equivalently, as creditor liquidation rights worsen.

Next we analyze the condition for feasibility when the patenting decision is endogenous. The main difference is that the expected payment at time 2 is constrained by the incentive compatibility constraint on patenting.

Theorem 2 *When patenting is endogenous, there exists a feasible financing contract if and only if*

$$E[C_1] + D_2^{max} \geq \frac{I - (1-p)L_1}{p}, \tag{15}$$

where D_2^{max} is given by (6). Moreover, when financing is feasible, there will be no liquidation at time 1 when an innovation has occurred: The firm always pays all time 1 cash flows C_1 to the lender and patents the innovation.

The condition for feasibility, (15), shows that the sum of the maximum expected payment from time 1 cash flows, $E[C_1]$, and the maximum possible payment from time 2 cash flows, D_2^{max}, has to be greater than the initial investment, adjusted for the fact that the lender does not get the promised payment when there is no innovation and instead the firm may be liquidated. Theorem 2 also establishes that, for any feasible contract, the firm will never be liquidated at time 1 if it successfully innovated. This is because liquidation is never efficient and, for any feasible contract, renegotiation ensures that both parties will always prefer to continue rather than liquidate the project. In the proof of Theorem 2 we also show that feasibility requires that $D_2^{max} > L_1$, as otherwise the total expected repayment to the lender would be insufficient to cover the cost I of the investment.

It is interesting to note, from Theorem 2 and Lemma 3, that the time 2 cash flows do not play a direct role on feasibility, only affecting feasibility through the constraint on the time 2 promised payment. In other words, time 2 cash flows matter for feasibility only because of the endogeneity of the patenting decision. When the firm can commit to patenting, C_2 becomes irrelevant for the feasibility of financing since the time 2 promised payment can always be made by liquidating the patented innovation.

As we did for the case above where patenting occurs automatically, it is easiest to compare the conditions for feasibility by letting $\bar{C}_1 = L_1 = 0$. Substituting for D_2^{max}, we obtain the following combined condition for participation and positive NPV:

$$pD_2^{max} = p\frac{\omega - \alpha}{\frac{1}{V} + \frac{(1-\beta)\omega}{\bar{C}_2}} \geq I. \tag{16}$$

The same condition when patenting occurs automatically is $p\omega\beta V \geq I$. To further simplify the comparison, consider the case where $\beta = 1$, so that there is no inefficiency in the liquidation of the patent. In that case, the condition for financing to be feasible when patenting occurs automatically coincides with the first best, i.e., $p\omega V \geq I$. By contrast, when patenting is endogenous, (16) reduces to $p(\omega - \alpha)V \geq I$. Clearly, this is a more stringent condition than both the first best case and the case where patenting is automatic. This also illustrates that the inefficiency introduced by the endogeneity of the patenting decision does not disappear when the inefficiency from liquidation disappears. This is because the inefficiency from the endogenity of patenting is not driven entirely by the liquidation that may take place, but rather arises from the ex-post hold-up resulting from the incentive compatibility constraint for patenting. The condition also highlights how the incremental value to the firm from patenting, $\omega - \alpha$, is an important driver of whether financing is feasible. In particular, as α increases, projects become more difficult to finance externally, with no projects being feasible as $\alpha \to \omega$.

4.2 When does the constraint bind?

In this section we discuss when the constraint that $D_2 \leq D_2^{max}$ binds ex-ante. This will prove useful later when deriving empirical predictions.

It is easiest to illustrate when the constraint binds through an example where we vary α and V and plot the regions where the constraint binds and when it does not. Changes in α vary the strictness of the constraint since α affects the incentive to patent, but it does not directly affect the total payoff given that the firm always chooses to patent in equilibrium.

Changes in V, by contrast, directly relate to the value of the project, and also affect feasibility by relaxing the constraint on the long-term payment.

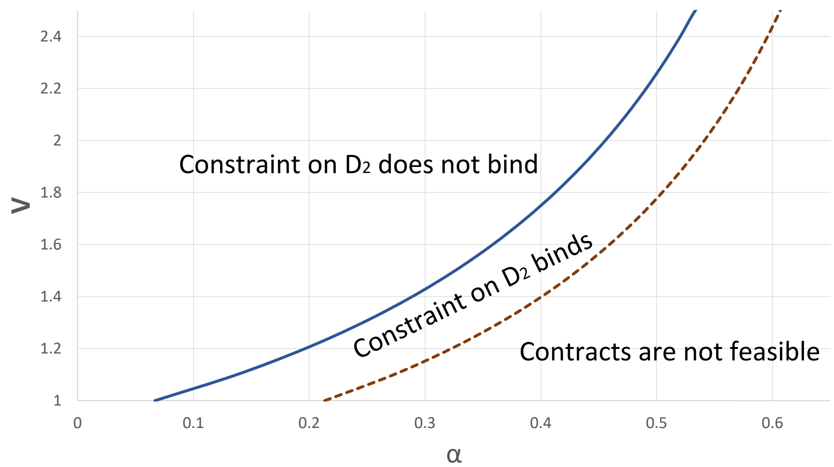


Figure 3: The figure displays the region where the constraint on the long-term payment, D_2^{max}, binds ex-ante. The area below the orange curve, of low project values (V) coupled with large values for the innovation even when not patented (α), represents cases where financing is not feasible. Above the solid curve, where V is relatively large and/or α is relatively low, D_2^{max} does not bind ex-ante and the optimal contract has an interior solution: $D_1^* > D_1^{min}$ and $D_2 < D_2^{max}$. In between the two curves, the constraint binds ex-ante: $D_1^* = D_1^{min}$ and $D_2 = D_2^{max}$. Other parameters are set to: $\omega = 1, \beta = 0.8, I = 0.7, L_1 = \bar{C}_1 = 0.2, \bar{C}_2 = 1, p = 0.9$

Figure 3 plots the region where the constraint binds as a function of α and V where other parameters are set as follows: $\omega = 1$, $\beta = 0.8$, $I = 0.7$, $L_1 = \bar{C}_1 = 0.2$, $\bar{C}_2 = 1$, $p = 0.9$. The bottom right corner represents projects where the constraint on time 2 payments is very strict and the value of the innovation is relatively low. Therefore, financing is not feasible. At the opposite corner, we have a set of projects whose value is very high. As a result, the constraint on long-term payments does not bind ex-ante and the short-term promised payment is equal to D_1^*. In between, we have a region where financing is feasible but the constraint binds ex-ante, so that $D_1 = D_1^{min}$ and $D_2 = D_2^{max}$.

For the set of parameters given above, this latter region is quite large. For example, if we fix $\alpha = 0.4$, then we need to increase the value of the patented innovation by about 25% to move from the line of feasibility to the line where the constraint on the long-term payment no longer binds ex-ante. Hence, the mass of projects where the constraint on D_2 binds ex-ante is significant, and not merely a knife-edge case.

5 Feasibility and empirical predictions

In this section we analyze how feasibility of the project changes when we change important parameters of the model. Using (15) we can rewrite the feasibility condition as

$$pE[C_1] + p\frac{\omega - \alpha}{\frac{1}{V} + \frac{(1-\beta)\omega}{\bar{C}_2}} + (1-p)L_1 - I \geq 0 \tag{17}$$

We can now use (17) to obtain a number of predictions related to when projects are feasible, as described below. Following each implication of the model, we discuss how to interpret and test the prediction empirically. Throughout, we focus primarily on implications related to ω, the value of the innovation to the firm when it is patented, and on α, the value of unpatented innovations. The latter variable, in particular, plays a novel and important role for feasibility, but only when patenting is endogenous. To the best of our knowledge, this is a unique aspect of our model since α does not directly affect the value of the innovation, but rather matters only through its effect on the constraint on long-term payments. As a result, this variable would typically play no role in models where the patenting decision is not endogenous.

Prediction 1 *As $\bar{C}_1, \bar{C}_2, L_1, V, p, \omega$, or β increase, or as α decreases, the constraint on feasibility becomes more relaxed.*

A relaxation of the feasibility constraint implies that it is easier for a firm to get financing. Under the assumption that these firms are financially constrained, the prediction translates into a larger amount of investment. In other words, for a given distribution of parameters, as the constraint on feasibility is relaxed a larger proportion of projects should obtain financing and be undertaken. Empirically, this prediction could be tested in either the cross section or time series by looking at how R&D investment and financing by financially-constrained innovative firms change with proxies for the model's parameters, as described next.

While the value of an unpatented innovation, α, is not directly observed, there are several potential proxies suggested by the literature that analyzes the trade-off between keeping innovations as trade secrets versus patenting (see Hall et al. (2014) for an extensive review). One general principle is that the value of an innovation without patenting will be relatively higher if the innovation is difficult for competitors to replicate. For example, the value of an innovation without patenting would be higher for large, important, or complex innovations, process (rather than product) innovations, if the innovator has more lead time, or if the innovation is cumulative or sequential in nature. Empirical proxies for many of these variables exist and have been used elsewhere. For example, Bena and Simintzi (2022) classify firms as process versus product innovators based on textual analysis of patents, and such measures could be used to identify firms more focused on process innovations and with higher unpatented values for their innovations. On the other hand, the value of an innovation without patenting should be lower if there are more competitors who could potentially copy or preemptively patent the same innovation. A simple proxy for competition is the Herfindahl index for the industry, while more elaborate measures can be constructed using textual analysis of managements' mentions of competition in their 10-K filings or other documents (see, e.g., Li et al., 2013).

The interactions of ω and α with other variables provide additional cross-sectional or time-series heterogeneity that can potentially be tested. Focusing on α, Figure 4 illustrates how feasibility depends on the interaction between α and β for the three different cases of (1) endogenous patenting, (2) automatic patenting, and (3) the first best. As α increases the value of the innovation V has to be larger in order for the project to be feasible. The figure also shows that the effect of α on the feasibility of financing is larger for lower β. In addition, the graph shows that the inefficiency resulting from the endogeneity of patenting does not disappear as β gets larger, as discussed above.

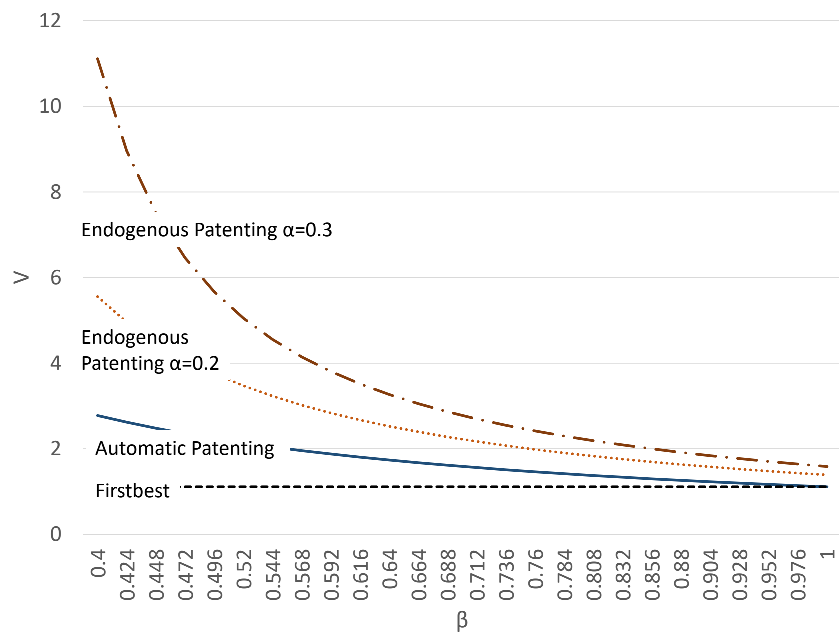


Figure 4: Feasibility as a function of β. The vertical axis represents V. The project is feasible if V is above the curve for each scenario. Other parameters are set to: $\omega = 1, I = 1, p = 0.9, \bar{C}_2 = 2, L_1 = \bar{C}_1 = 0$.

A similar pattern holds for changes to ω, where *reductions* in ω make financing more difficult to obtain. We omit presenting a figure between ω and β given it is very similar to Figure 4. There is also an interaction effect between ω and β: the positive impact of ω on feasibility is larger for lower β. The primary difference is that reductions to ω also affect the graphs for the case where patenting occurs automatically, and for the first best. We summarize these predictions below.

Prediction 2 *The negative effect of α and the positive effect of ω on feasibility is stronger with smaller β.*

Empirically, this finding implies that the negative impact on R&D investment associated with the value of the innovation when not patented should be greater when the liquidation value of the patent is low (i.e., when β is small) or, more generally, when creditors have a poor ability to exercise their liquidation rights over the posted collateral (the patent). A similar finding holds true for the positive effect of increases in ω, the patented value of the innovation.

Given that this prediction is about the interaction of α and ω with β, one could test this using exogenous changes in β. Focusing on creditor rights as a proxy for β, Mann (2018b) uses court decisions that improve creditor rights to show that innovative companies raised more debt and spent more on R&D investment after creditor rights were improved. Our prediction is that the effect of creditor rights on R&D investment should be higher for firms or industries where α is higher or ω is lower. Therefore, one could test whether the findings in Mann (2018b) vary with proxies for α and ω, as listed above. Alternatively, one could also use the Bankruptcy Reform Act (see, e.g., Hackbarth et al., 2015) as a shock to creditor rights.

Figure 5 illustrates how the inefficiency arising from the endogeneity of patenting varies with ω, the value of patenting to the firm. As the value of patent protection increases, the region where the project is feasible increases. In addition, there is an interaction between the values of the innovation with and without patenting, as described below.

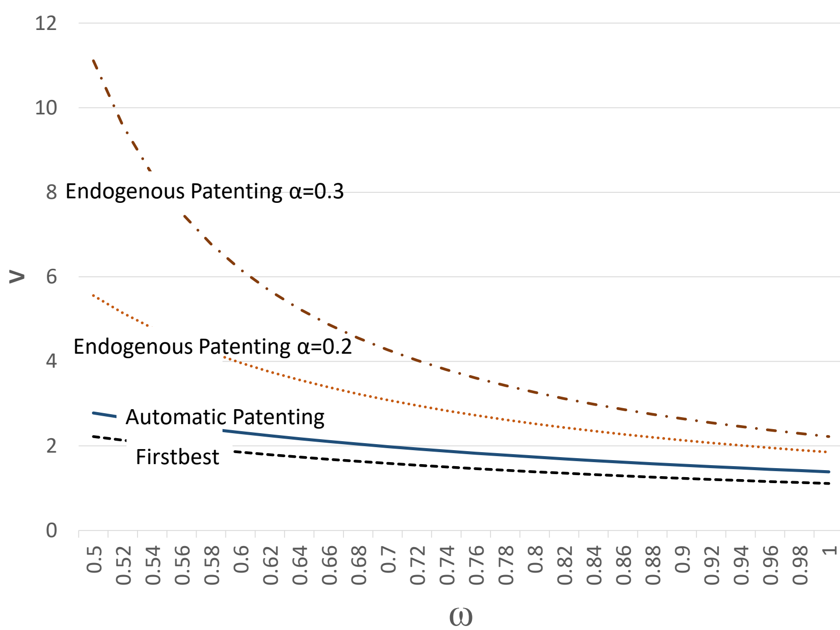


Figure 5: Feasibility according to ω. Vertical axis represents V. The project is feasible if V is above the curve of each scenario. Other parameters are set to: $\beta = 0.8, I = 1, p = 0.9, \bar{C}_2 = 2, L_1 = \bar{C}_1 = 0$.

Prediction 3 *The negative effect of α on feasibility is stronger with smaller ω.*

As discussed above, a tighter feasibility constraint implies lower R&D investment and reduced ability to obtain financing. This implication could be tested by analyzing how the effect of a change in patent protection on R&D investment and financing varies with proxies for the unpatented value of innovations. Significant variation in ω over time and across countries has been documented (see Lerner (2002) for 150 years of changes). Gallini (2002) summarizes changes in patent protection over time in the US, and argues that patent protection has been strengthened in three major ways: 1) extending patent protection to new subject matter; 2) giving greater power to patent holders in infringement lawsuits; and 3) lengthening the term of patents. Such events can be used to test whether R&D investment and external financing increases more for industries/firms with higher values for unpatented innovations.

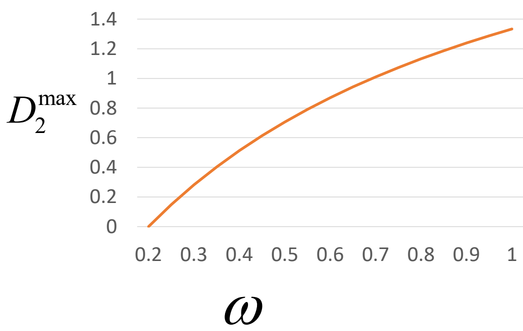
Our model also has implications for how policy variables, such as the degree of patent protection, ω, or the extent of creditor rights, β,[10] impact a firm's ability to invest in innovative activities. Focusing on predictions that we believe are unique to our model, we find that not only does D_2^{max} increase as ω or β changes, but that D_2^{max} is concave in ω but convex in β (see Figure 6 and formal results in the appendix). This means that it is more efficient to improve patent protection (ω) when such protection is low, while the reverse is true for the liquidation value (β) of patents.[11] We summarize the various comparative statics related to policy variables in the prediction below.

Prediction 4 *The following comparative statics on D_2^{max}, the upper limit on long-term payments, hold:*

$$\frac{\partial^2 D_2^{max}}{\partial \beta^2} > 0, \quad \frac{\partial^2 D_2^{max}}{\partial \omega^2} < 0, \frac{\partial^2 D_2^{max}}{\partial \beta \partial \omega} > 0.$$

[10] The efficiency of the secondary market for the firm's assets should influence β as well, which may vary by industry or geographic region. We view this aspect as providing important cross-sectional variation, separately from policy initiatives that could be taken to increase the liquidation value β of the firm's patents.

[11] As above, one could use changes to the bankruptcy code Hackbarth et al. (2015) or court decisions Mann (2018b) to test these predictions.



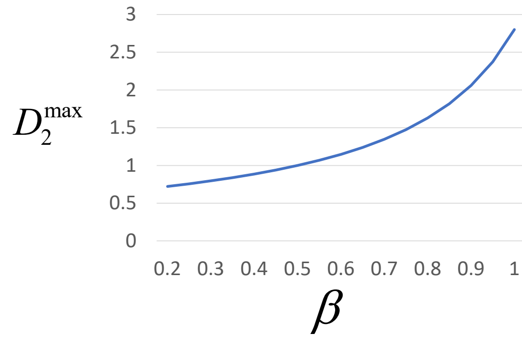


Figure 6: D_2^{max} as a function of ω (when $\beta = 0.65$) and β (when $\omega = 0.9$). For both cases, $\alpha = 0.2, V = 4, \bar{C}_2 = 1$.

Note that, from (17), creditor rights β and patent protection ω affect feasibility only through their effects on D_2^{max}. As a result, we can view these comparative statistics as predictions on feasibility. For example, the finding that $\frac{\partial^2 D_2^{max}}{\partial\beta\partial\omega} > 0$ implies that improving creditor rights and patent protection together makes financing feasible for a greater range of projects. In other words, the two policies are complementary. This prediction could be tested across countries using either changes in patent protection or creditor rights.

Our final prediction concerning feasibility is about how increases in the probability of an innovation or in its value affect feasibility depending on values of α and ω.

Prediction 5 *The effect of an increase in p and V on feasibility is greater for firms with lower α and higher ω.*

5.1 Project risk and feasibility

An aspect that we have so far not emphasized is how the distribution of a project's payoffs affects the feasibility of obtaining financing. Since the project's payoff is V with probability p and 0 with probability $1 - p$, to study this issue we fix the project's expected payoff to be constant, $pV := \mathcal{V}$, and vary p.[12] For simplicity, we also maintain the assumptions that

[12]Otherwise increasing p both reduces volatility and increases expected payoff at the same time.

$\bar{C}_1 = L_1 = 0$. For a given probability of innovation p, the variance of the project's payoff is

$$\mathcal{V}^2 \frac{(1-p)}{p}, \tag{18}$$

which is a strictly decreasing function of p. Thus, a higher p corresponds to a project with lower risk.

The feasibility conditions for both the first best, $\omega p V = \omega \mathcal{V} \geq I$ (see (12)), and for the benchmark case where patenting occurs automatically, $\omega \beta p V = \omega \beta \mathcal{V} \geq I$ (see (14)), only depend on the expected payoff $\mathcal{V}$ of the project. By contrast, when patenting is endogenous, the condition for feasibility, (16), can be written as

$$\frac{\omega - \alpha}{\frac{1}{\mathcal{V}} + \frac{(1-\beta)\omega}{p\bar{C}_2}} \geq I.$$

When $\mathcal{V} = pV$ is fixed, the left-hand side is an increasing function of p. Therefore, for projects with the same expected payoff, only those that are less risky (i.e., higher p) will be feasible.

Prediction 6 *When $\mathcal{V} = pV$ is fixed, feasibility is an increasing function of p.*

Figure 7 shows the feasibility region for different scenarios. For either the first best case or the case where patenting occurs automatically, the variance of the project's payoff, (18), does not affect which projects are feasible since only the expected project payoff matters – the graphs for both of these cases are flat and, hence, independent of p. By contrast, when patenting is endogenous, the figure shows that it is easier to obtain financing for projects that are safer and have a higher probability of delivering an innovation, even if that innovation is less valuable when successful, and even though investors have no particular preference for financing safer projects.

The result is different from standard risk-shifting considerations that often arise as a result of debt financing, where creditors worry that shareholders will take higher-risk projects than

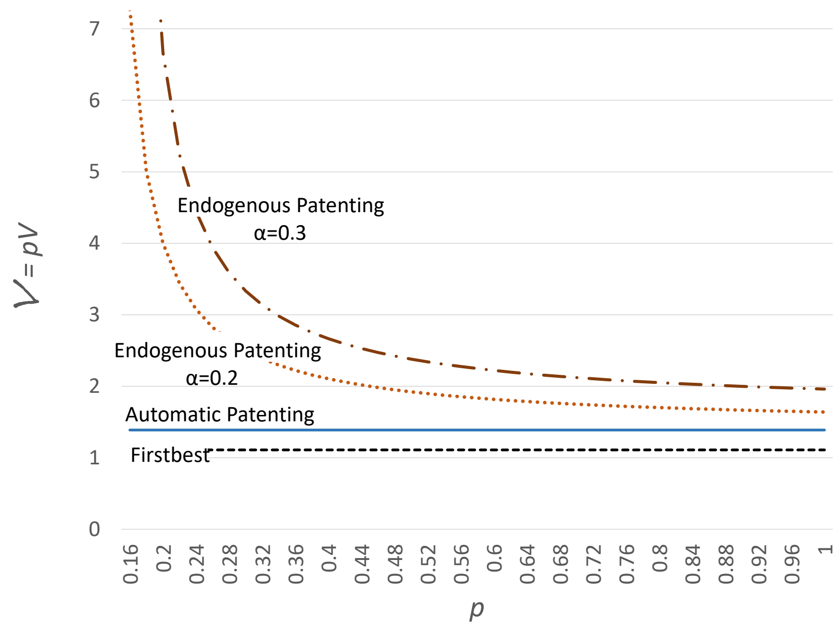


Figure 7: Feasibility according to project expected payoff (pV) and success probability (p). The vertical axis represents $\mathcal{V} = pV$. The horizontal axis represents the probability of innovation p. The area above each curve represents feasible projects for each scenario. Other parameters are set to: $\omega = 0.9, \beta = 0.8, I = 1, \bar{C}_2 = 2, L_1 = \bar{C}_1 = 0$.

what creditors prefer. Here, higher risk (i.e., lower p) projects will be unable to get financing even if p is fixed and known ex ante, so that there is no scope for risk-shifting by the firm. This effect is therefore similar to credit rationing (Stiglitz and Weiss, 1981; Besanko and Thakor, 1987), although the mechanism is very different; here, the constraint on long term payments limits the degree to which a lender can be compensated for risk.

6 Predictions on contract terms

In this section we outline predictions of the model related to the terms of financial contracts. In particular we focus on loan duration and promised interest payments. One advantage of focusing on these terms is that they can potentially be observed, allowing us to provide testable empirical predictions. We focus on cases where the incentive compatibility constraint on the time 2 payment, D_2^{max}, binds ex-ante, since it is for these firms that concerns related to patenting matter most in shaping the financial contracts they will use. We define contract duration as $1 \cdot \frac{D_1}{D_1+D_2} + 2 \cdot \frac{D_2}{D_1+D_2}$ and, since there is no discounting across periods, we measure the promised interest rate as $\frac{D_1+D_2-I}{I}$.

When the constraint on long-term payments binds ex-ante, the optimal contract is given by $D_1 = D_1^{min}$ and $D_2 = D_2^{max}$ from Theorem 1, where $D_2^{max} = \frac{\omega-\alpha}{\frac{1}{V}+\frac{(1-\beta)\omega}{\bar{C}_2}}$. Plugging D_2^{max} into the participation constraint (8) and simplifying yields

$$\frac{(1-p)}{p}\left(L_1\frac{D_1}{\bar{C}_1} + D_1(1-\frac{D_1}{\bar{C}_1})\right) + \left(D_1 - \frac{(D_1)^2}{2\bar{C}_1}\right) + D_2^{max} = \frac{I}{p}. \tag{19}$$

We can use (19) to study how duration and interest rates change as we vary model parameters.

Prediction 7 *When the constraint on long-term payments binds ex-ante, increasing ω, β, V, $\bar{C}_2$, and decreasing α, results in higher $D_2 = D_2^{max}$, lower D_1, and therefore a longer duration.*

The main implication of this prediction is that increases in D_2^{max} allow the firm to offer larger long-term payments and, as a consequence, reduce the promised early repayment, D_1. This makes the overall repayment more back-loaded, increasing the duration of the contract.

While the focus above was on increases in D_2^{max}, there are some variables that do not have an effect on D_2^{max} but may nevertheless relax the constraint. Changes in these variables will be balanced by changes in D_1 alone, as described next.

Prediction 8 *When the constraint on long-term payments binds ex-ante, so that $D_2 = D_2^{max}$, increasing L_1, p, or decreasing I results in lower D_1 but no change in D_2. As a result, the duration of the contract increases.*

Since there is no change to the maximum repayment that can be promised, D_2^{max}, when these variables change, it is straightforward to see that whenever D_1 increases, the contract's duration must decrease.

We can also study how changes in the same set of variables affect the loan's interest rate.

Prediction 9 *When the constraint on long-term payments binds ex-ante, increasing L_1 or p results in lower interest rate: $\frac{D_1+D_2-I}{I}$ decreases.*

For brevity and simplicity, above we focus on variables that have a clear-cut effect on contract duration and interest. However, in some cases the implications are less straightforward since variables that relax the constraint on long-term payments allow D_2 to increase but at the same time lead D_1 to go down. Hence, the overall implication on the total interest that is paid is unclear. Nevertheless, an additional minor condition allows us to provide predictions in some cases.

Prediction 10 *Assume the constraint on long-term payments binds ex-ante. For p, the probability of innovation, large enough, increasing ω, β, V, $\bar{C}_2$, and decreasing α results in lower interest rates.*

While the prediction assumes p is large enough, this is not necessarily restrictive since p must be sufficiently large in order for the project to be feasible. Intuitively, an increase in D_2 matters more for feasibility when the probability of innovation is larger. Therefore, an increase in D_2 can be balanced by a larger reduction in D_1, reducing the interest rate. The prediction that increases in β lead to decreases in interest rates finds support in the findings of Hackbarth et al. (2015).

7 Conclusion

Studying financing of small innovative firms is important given that they contribute disproportionately to major innovations, facilitate creative destruction, and drive economic growth (Rosen, 1991; Akcigit and Kerr, 2018). Growing empirical evidence shows that debt financing plays an important role in funding the growth of small and innovative firms (Chava et al., 2013; Cornaggia et al., 2015; Kerr and Nanda, 2015; Chava et al., 2017; Mann, 2018a), despite a perception that such financing is not suitable for firms that invest in specialized assets with low liquidation values (Williamson, 1988). One reason why debt financing for innovative firms is difficult is because of their paucity of assets that could be useful as collateral. While recent research has shown that intellectual property assets, in the form of patents, can potentially be used as collateral (Mann, 2018a; Ma and Wang, 2019), many small, innovative firms may not have such assets at the time of financing. Moreover, the decision to patent future innovations is endogenous, and firms may sometimes prefer not to patent their innovations for strategic reasons, hoping to reduce the ex-post bargaining power of their lenders in the event of renegotiation. This key friction – that patenting is itself an endogenous choice for firms – shapes financial contracts and has implications for what types of projects can be financed through debt instruments.

We show that the difference between the patented value of an innovation and its value in the absence of patent protection is particularly important since it determines how large of a payment the firm can credibly commit to make to a lender. To the best of our knowledge, the result that the value of innovation without patenting plays a role in determining a firm's ability to obtain financing as well as its terms of contracting is novel.

Our results can be extended to other types of intellectual property as well as to actions by the firm over which the lender and the firm's interests are not perfectly aligned. For example, intellectual property (IP) violations may need to be litigated and even though both parties value the IP, firms that are in financial distress may care less about the long term value of the intellectual property that is collateral to lenders.

Our analysis also derives unique implications for the effects of improving patent protection and creditor rights. In particular, we show that improving patent protection and creditor rights have different implications on firm investment in R&D depending on the value of the innovation without patenting. Moreover, the model is flexible in that it does not assume there are large benefits to patenting, but rather offers predictions based on the size of any such benefits, and their interaction with other variables of interest. These implications we believe are new to the literature, and offer a novel perspective on aspects that may be important to consider when revising policies related to patent protection, as well as to the ease with which patents may be liquidated by creditors.

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Appendix

A Proofs

A.1 Proof of Claim 1

At time 2, there are no assets to liquidate and therefore the payoff of the lender is zero while the firm gets C_2. If at time 1 $C_1 < D_1$, then the firm does not have the resources to repay and, moreover, cannot credibly promise to repay anything at time 2. The lender then has the right to liquidate and will choose to do so, obtaining L_1. If the lender liquidates, the total amount $D = D_1 + D_2$ becomes due and the investor's payoff is equal to $\min\{L_1, D_1 + D_2\} = L_1$ since the liquidation value of the assets is low relative to the investment $(L_1 < I)$ and at the same time the total promised payment has to be larger than or equal to the initial investment $(D_1 + D_2 \geq I)$ for the lender to break even.

If $C_1 \geq D_1$, then the firm can either pay D_1 and continue or allow liquidation. In the former case, the firm gets $C_1 + E[C_2] - D_1$, while in the latter case, it gets C_1, with the proceeds from liquidation, L_1, going to the lender. Given that $E[C_2] > \bar{C}_1$ and the maximum payment that can be made out of time 1 cash flows is $\bar{C}_1$, we can rule out the possibility that the firm prefers liquidation.

Consider now the possibility that $D_1 > \bar{C}_1$, the highest possible time 1 cash flow. Since the most the lender can get under liquidation is L_1, and the probability that C_1 is less than D_1 would be equal to 1, such a contract could always be replaced with a contract that has $D_1 = \bar{C}_1$ and which would keep the total payment to the investor unchanged. As we show later, a similar argument applies to the case where there is an innovation, so that it is without loss of generality to assume that $D_1 \leq \bar{C}_1$, and we make this assumption going forward.

The lender's payoff is therefore

$$= \begin{cases} L_1 & \text{for} \quad C_1 < D_1 \\ D_1 & \text{for} \quad C_1 \geq D_1 \end{cases}$$

Ex-ante, the expected payoff for the lender in case of no innovation can be written as a function of D_1:

$$\begin{aligned} \mathcal{F}_0(D_1) &= L_1 \Pr(C_1 < D_1) + D_1 \Pr(C_1 \geq D_1) \\ &= L_1 \frac{D_1}{\bar{C}_1} + D_1 \left(1 - \frac{D_1}{\bar{C}_1}\right). \end{aligned}$$

Now consider the renegotiation of the original contract at time 1. When $C_1 \geq D_1$, the firm offers to pay D_1, nothing at $t = 2$, and continue, which is exactly equal to the lender's outside option. On the other hand, when $C_1 < D_1$ the firm will refuse to pay anything given that the firm will be liquidated regardless, and there is no welfare-improving renegotiation offer that is possible since $\bar{C}_1 < L_1$. Thus, ex-ante the joint payoff when there is no innovation is equal to:

$$\begin{aligned} \mathcal{G}_0(D_1) &= \frac{1}{\bar{C}_1} \int_0^{D_1} (C_1 + L_1) dC_1 + \frac{1}{\bar{C}_1} \int_{D_1}^{\bar{C}_1} (C_1 + E[C_2]) dC_1 \\ &= E[C_1] + E[C_2] - (E[C_2] - L_1) \frac{D_1}{\bar{C}_1}. \end{aligned}$$

From this, we obtain the ex-ante payoff to the firm when there is no innovation is equal to:

$$\mathcal{H}_0(D_1) = \mathcal{G}_0(D_1) - \mathcal{F}_0(D_1) = E[C_1] + (E[C_2] - D_1) \left(1 - \frac{D_1}{\bar{C}_1}\right).$$

A.2 Proof Of Claim 2

Given the firm's patenting decision described in Section 3.2.1, we can now characterize the payoffs of both parties at time 1.[13]

Let's first consider the case where $D_2 \leq D_2^{max}$ and $C_1 \geq D_1$, so that the firm is able to make the time 1 payment and afterward finds it optimal to patent the innovation. In this case, the outside option of the lender is $D_1 + D_2$, as described above. The firm also can make a take-it-or-leave-it offer and finds it optimal to pay the maximum amount possible at time 1, C_1, to minimize the probability of liquidation at time 2. Therefore, the firm offers a new contract such that $D_1' = C_1$ and $D_2' = D_1 + D_2 - C_1$. This contract keeps the payoff of the lender the same but increases the payoff to the firm by reducing the probability of liquidation at time 2.

Next consider the case where $D_2 \leq D_2^{max}$ but $C_1 < D_1$. In this case, the firm cannot make the time 1 payment, and, under the original contract, the lender would liquidate. If the lender liquidates the total amount $D = D_1 + D_2$ becomes due and the investor's payoff is equal to $\min\{L_1, D_1 + D_2\} = L_1$.

However, when the firm fails to make the promised payment, the lender has the option to make a take-it-or-leave-it offer to the firm. Being aware that the firm will only patent if $D_2' \leq D_2^{max}$, the lender will offer the contract $(C_1, \min\{D_2^{max}, D_1 + D_2 - C_1\})$, with payoff to the lender of $C_1 + \min\{D_2^{max}, D_1 + D_2 - C_1\}$. The lender will find it optimal to offer this contract when this payoff is greater than L_1, rolling over a portion of the promised payment rather than forcing early liquidation. Importantly, when $D_2^{max} < D_1 + D_2 - C_1$, the lender also prefers to forgive a portion of the remaining debt obligation equal to $D_1 + D_2 - C_1 - D_2^{max} > 0$ rather than asking for full repayment as this would create disincentives for the firm to patent its innovation. With the assumption that $D_2^{max} > L_1$, the lender is always better off offering this new contract.

[13] As per the discussion for the no innovation case, we assume without loss of generality that $D_1 \leq \bar{C}_1$. If not, so that $D_1 > \bar{C}_1$, we could replace this contract with another one where $D_1^{new} = \bar{C}_1$ and $D_2^{new} = D_1 + D_2 - D_1^{new}$, which would be payoff-equivalent for both the firm and the investor.

Overall, the lender's payoff, f_1, as a function of the terms of an original feasible contract is

$$f_1(C_1, D_1 + D_2) = \min\{D_2^{max} + C_1, D_1 + D_2\}.$$

The lender's expected payoff is then

$$\mathcal{F}_1(D_1 + D_2) := \frac{1}{\bar{C}_1}\int_0^{\bar{C}_1} f_1(C_1, D_1 + D_2)dC_1 = (D_1 + D_2) - \frac{(D_1 + D_2 - D_2^{max})^2}{2\bar{C}_1}.$$

A.3 Proof of Theorem 1

Inefficiency is caused by liquidation. There are two possible causes of liquidation. First, the firm may get liquidated at time 1 when there is no innovation and the firm's cash flows are less than the promised payment: $C_1 < D_1$. In this case, the expected total surplus under liquidation and no liquidation are $C_1 + L_1$ and $C_1 + E[C_2]$, respectively. Thus, the expected inefficiency of time 1 liquidation is

$$(1-p)\Pr(C_1 < D_1)(E[C_2] - L_1) = (1-p)\frac{D_1}{\bar{C}_1}(E[C_2] - L_1).$$

Second, the patent may get liquidated at time 2. In this case, the expected total surplus under liquidation and no liquidation are $C_1 + C_2 + \omega\beta V$ and $C_1 + C_2 + \omega V$, respectively. Hence, the expected inefficiency associated with the liquidation of the patent is

$$p\omega(1-\beta)V\Pr(\text{time 2 liquidation}),$$

where the probability of liquidation is endogenous and depends on the contract.

Case 1: $D_2 \leq D_2^{max}$

We first characterize the optimal contract with the assumption that $D_2 \leq D_2^{max}$.

The payment to the lender is $f_1(C_1, D_1 + D_2) = \min\{C_1 + D_2^{max}, D_1 + D_2\}$, as in Figure 2.

At equilibrium, conditional on innovation, the first payment is C_1, and the time 2 payment is $f_1(C_1, D_1+D_2)-C_1$. Thus, the probability of liquidation at time 2 conditional on innovation is

$$\Pr(\text{time 2 liquidation}) = \Pr(C_2 < f_1(C_1, D_1 + D_2) - C_1) = \Pr(C_1 + C_2 < f_1(C_1, D_1 + D_2)). \tag{20}$$

This probability is a function of the realization of both time 1 and 2 cash flows. We compare these cash flows to $D_1 + D_2$ if the constraint on the time 2 payment does not bind and to $C_1 + D_2^{max}$ if the constraint does bind.

We can now express the optimization problem of the firm as

$$\begin{aligned} \min_{D_1,D_2} \quad & (1-p)\frac{D_1}{\bar{C}_1}(E[C_2] - L_1) + p\omega(1-\beta)V \Pr(C_1 + C_2 < f_1(C_1, D_1 + D_2)) \\ \text{subject to} \quad & (1-p)\mathcal{F}_0(D_1) + p\mathcal{F}_1(D_1 + D_2) = I, \\ & 0 \leq D_1 \leq \bar{C}_1, \\ & 0 \leq D_2 \leq D_2^{max}. \end{aligned} \tag{21}$$

The firm aims to minimize the total inefficiency due to external financing subject to the lender's participation constraint, the constraint that the short-term payment does not exceed the maximum amount of cash the firm could possibly have at time 1, and the incentive compatibility constraint on the long-term payment, that it does not exceed D_2^{max}.

As shown in the main text, we convert the constraint $D_2 \leq D_2^{max}$ to the constraint $D_1 \geq D_1^{min}$. In the remaining of the proof, we will convert this optimization problem to a single variable D_1 and will solve it with the constraint $D_1^{min} \leq D_1 \leq \bar{C}_1$.

Observe that according to (20),

$$\begin{aligned}
\Pr(\text{time 2 liquidation}) =& \Pr(C_2 < f_1(C_1, D_1 + D_2) - C_1) \\
=& \frac{1}{\bar{C}_1}\int_0^{\bar{C}_1}\frac{1}{\bar{C}_2}\int_0^{\bar{C}_2}(f_1(C_1, D_1+D_2) - C_1)dC_2 dC_1 \\
=& \frac{1}{\bar{C}_1\bar{C}_1}\int_0^{\bar{C}_1} f_1(C_1, D_1 + D_2)dC_1 - \frac{1}{\bar{C}_1\bar{C}_1}\int_0^{\bar{C}_1} C_1 dC_1 \\
=& \frac{1}{\bar{C}_1\bar{C}_2}\int_0^{\bar{C}_1} f_1(C_1, D_1 + D_2)dC_1 - \frac{\bar{C}_1}{2\bar{C}_1}.
\end{aligned}$$

Thus, we obtain

$$\Pr(\text{time 2 liquidation}) = \frac{\mathcal{F}_1(D_1 + D_2)}{\bar{C}_2} - \frac{\bar{C}_1}{2\bar{C}_2}.$$

Using the participation constraint, (8), this probability is the same as

$$\Pr(\text{time 2 liquidation}) = \frac{I - (1-p)\mathcal{F}_0(D_1)}{p\bar{C}_2} - \frac{\bar{C}_1}{2\bar{C}_2}. \tag{22}$$

Thus, the total inefficiency is

$$(1-p)\frac{D_1}{\bar{C}_1}(E[C_2] - L_1) + p\omega(1-\beta)V\left(\frac{I - (1-p)\mathcal{F}_0(D_1)}{p\bar{C}_2} - \frac{\bar{C}_1}{2\bar{C}_2}\right).$$

We can write the total inefficiency as a quadratic function of D_1.

$$\begin{aligned}
\mathcal{T}(D_1) \quad &:= \quad (1-p)\big(D_1\frac{E[C_2] - L_1}{\bar{C}_1} - \frac{\omega(1-\beta)V}{\bar{C}_2}\mathcal{F}_0(D_1)\big) + \omega(1-\beta)V\frac{(2I - p\bar{C}_1)}{2\bar{C}_1} \\
&= \quad (1-p)(D_1\frac{E[C_2] - L_1}{\bar{C}_1} - \frac{\omega(1-\beta)V}{\bar{C}_2}(L_1\frac{D_1}{\bar{C}_1} + D_1(1 - \frac{D_1}{\bar{C}_1}))) + \omega(1-\beta)V\frac{(2I - p\bar{C}_1)}{2\bar{C}_1}
\end{aligned}$$

$$= (1-p)(D_1(\frac{E[C_2] - L_1}{\bar{C}_1} - \frac{\omega(1-\beta)V}{\bar{C}_2}\frac{L_1}{\bar{C}_1} - 1) + \frac{\omega(1-\beta)V}{\bar{C}_1\bar{C}_2}D_1^2) + \omega(1-\beta)V\frac{(2I - p\bar{C}_1)}{2\bar{C}_1} \tag{23}$$

From this, we can characterize the equilibrium because it is a function of only one variable,

D_1. The derivative is 0 at

$$\begin{aligned} D_1^* &= \frac{\bar{C}_2}{2\omega(1-\beta)V}(\bar{C}_1 + L_1 - E[C_2]) + \frac{L_1}{2} \\ &= \frac{L_1}{2} - \frac{\bar{C}_2}{2\omega(1-\beta)V}(\frac{\bar{C}_2}{2} - \bar{C}_1 - L_1). \end{aligned} \tag{24}$$

With the constraint that $D_1^{min} \leq D_1 \leq \bar{C}_1$, this establishes the result under the assumption that $D_2 \leq D_2^{max}$.

Case 2: $D_2 > D_2^{max}$

Next, we consider a contract with $D_2 > D_2^{max}$. We will show that one can modify that contract with $D_2^{new} \leq D_2^{max}$ so that the participation constraint is maintained and has a better total payoff.

Let (D_1, D_2) be a feasible contract with $D_2 > D_2^{max}$. First, we show that it must be that $D_1 \geq D_1^{min}$.

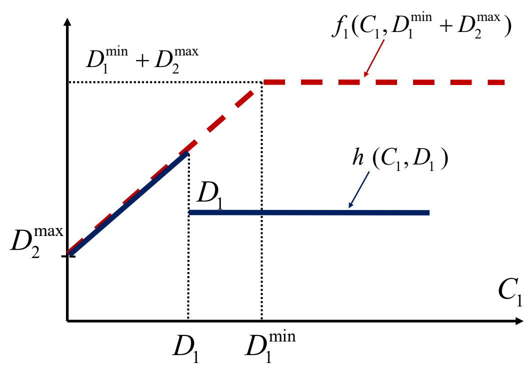

Figure 8: Payment to the lender when $D_1 < D_1^{min}, D_1 > D_2^{max}$

Assume to the contrary that $D_1 < D_1^{min}$. Then the payment to the lender conditional on innovation is

$$h(C_1, D_1) = \begin{cases} D_2^{max} + C_1 & \text{if} \quad C_1 \leq D_1 < D_1^{min} \\ D_1 & \text{if} \quad C_1 \geq D_1 \end{cases}$$

Notice that $h(C_1) \leq f_1(C_1, D_1^{min} + D_2^{max})$, which is the payment function under the contract

(D_1^{min}, D_2^{max}). Thus, the expected payment of such a contract conditional on innovation is at most that of the contract (D_1^{min}, D_2^{max}) (see Figure 8). Furthermore, when there is no innovation, the expected payment is a strictly increasing function of D_1. Therefore, the total expected payment of the contract (D_1, D_2) is less than that of the contract (D_1^{min}, D_2^{max}). This shows that (D_1, D_2) cannot be feasible because (D_1^{min}, D_2^{max}) satisfies the participation constraint with equality. Hence, for (D_1, D_2) to be feasible, it must be that $D_1 \geq D_1^{min}$.

Now let $D_2^{new} := \Delta(D_1)$, as defined in (29). Because $D_1^{min} \leq D_1 \leq \bar{C}_1$, $D_2^{new} \leq D_2^{max}$ and the participation constraint of the contract (D_1, D_2^{new}) binds. Next, we will show that the probability of liquidation at time 2 under the contract (D_1, D_2^{new}) is smaller than that of (D_1, D_2). Because both contracts satisfy the participation constraint with equality, this implies that the firm's payoff is larger under (D_1, D_2^{new}) and thus the contract (D_1, D_2) with $D_2 > D_2^{max}$ cannot be optimal.

According to (22), the probability of liquidation at time 2 for the contract (D_1, D_2^{new}) is

$$\Pr(\text{time 2 liquidation}) = \frac{I - (1-p)\mathcal{F}_0(D_1)}{p\bar{C}_2} - \frac{\bar{C}_1}{2\bar{C}_2}, \tag{25}$$

where $\mathcal{F}_0(D_1) = L_1 \frac{D_1}{\bar{C}_1} + D_1(1 - \frac{D_1}{\bar{C}_1})$.

To compute the probability liquidation at time 2 for the contract (D_1, D_2), we start from the participation constraint. Recall that for $D_2 > D_2^{max}$, we defined the payoff to the lender when there is innovation as $h(C_1, D_1)$. Thus, the expected payoff of the lender conditional on innovation is

$$\int_0^{D_1} (C_1 + D_2^{max}) dC_1 + \int_{D_1}^{\bar{C}_1} D_1 dC_1 = \frac{D_1}{\bar{C}_1}(D_2^{max} + \frac{D_1}{2}) + D_1(1 - \frac{D_1}{\bar{C}_1}) = \frac{D_1}{\bar{C}_1}(D_2^{max} + \bar{C}_1 - \frac{D_1}{2}).$$

Hence, the participation constraint becomes

$$(1-p)(L_1 \frac{D_1}{\bar{C}_1} + D_1(1 - \frac{D_1}{\bar{C}_1})) + p\frac{D_1}{\bar{C}_1}(D_2^{max} + \bar{C}_1 - \frac{D_1}{2}) = I \tag{26}$$

$$= (1-p)\mathcal{F}_0(D_1) + p\mathcal{F}_1'(D_1),$$

where we define $\mathcal{F}_1'(D_1) = \frac{D_1}{\bar{C}_1}(D_2^{max} + \bar{C}_1 - \frac{D_1}{2})$ as the lender's expected payoff conditional on innovation under this contract. Notice that

$$\Pr(\text{time 2 liquidation}) = \Pr(C_1 < D_1 \& C_2 < D_2^{max}) = \frac{D_1}{\bar{C}_1}\frac{D_2^{max}}{\bar{C}_2} = \frac{\mathcal{F}'(D_1)}{\bar{C}_2} - D_1\frac{2\bar{C}_1 - D_1}{2\bar{C}_1\bar{C}_2}.$$

Using the participation constraint, (26), to substitute for $\mathcal{F}'(D_1)$, we have

$$\Pr(\text{time 2 liquidation}|(D_1, D_2)) = \frac{I - (1-p)\mathcal{F}_0(D_1)}{p\bar{C}_2} - D_1\frac{2\bar{C}_1 - D_1}{2\bar{C}_1\bar{C}_2}. \tag{27}$$

From (27) and (25), we obtain

$$\Pr(\text{time 2 liquidation}|(D_1, D_2)) - \Pr(\text{time 2 liquidation}|(D_1, D_2^{new})) =$$

$$= \frac{\bar{C}_1}{2\bar{C}_2} - D_1\frac{2\bar{C}_1 - D_1}{2\bar{C}_1\bar{C}_2} = \frac{(\bar{C}_1 - D_1)^2}{2\bar{C}_1\bar{C}_2} \geq 0.$$

Equality is only obtained when $D_1 = \bar{C}_1$, in which case the two contracts are the same. This shows that a contract with $D_2 > D_2^{max}$ cannot be optimal.

A.4 Proof of Theorem 2

To prove Theorem 2, we first establish the following preliminary result, which also establishes the second part of the theorem, that liquidation does not occur in equilibrium when financing is feasible.

Lemma 4 *Financing is feasible only if $D_2^{max} > L_1$. Moreover, when financing is feasible, there will be no liquidation at time 1 when there is innovation: The firm always pays all time 1 cash flows C_1 to the lender and patents the innovation.*

A.4.1 Proof of Lemma 4

The expected payment the investor receives conditional on no innovation is

$$L_1\frac{D_1}{\bar{C}_1} + D_1(1-\frac{D_1}{\bar{C}_1}) \leq L_1 + D_1(1-\frac{D_1}{\bar{C}_1}) \leq L_1 + \frac{\bar{C}_1}{4} < L_1 + E[C_1].$$

Given that $L_1 + E[C_1] < I$ by assumption, the expected payment to the lender when there is no innovation is clearly less than I.

Consider now the case where there is innovation. If $D_2^{max} \leq L_1$, and assume that $D_2 \leq D_2^{max}$. Then the payment conditional on innovation, as a function of C_1, is

$$f_1(C_1, D_1 + D_2) = \begin{cases} L_1 & \text{if} \quad C_1 \leq L_1 - D_2^{max} \\ D_2^{max} + C_1 & \text{if} \quad L_1 - D_2^{max} < C_1 \leq D_1 + D_2 - D_2^{max} \\ D_1 + D_2 & \text{if} \quad C_1 \geq D_1 + D_2 - D_2^{max} \end{cases} \tag{28}$$

If, on the other hand, $D_2 > D_2^{max}$, then the payment conditional on innovation is only D_1 when $C_1 > D_1$ as the firm, which can make a take-it-or-leave-it offer, would always offer $D_2' = 0$. The lender would accept since that is what they would get under the original contract which, since $D_2 > D_2^{max}$, would not lead to patenting. The payment conditional on innovation is, therefore, at most $f_1(C_1, D_1 + D_2)$.

Hence, like in the no innovation case, the payment to the lender is at most $f_1(C_1, D_1 + D_2) \leq L_1 + C_1$. Therefore, the expected payment is at most $L_1 + E[C_1] < I$. Thus, for any feasible contract, it must be the case that $D_2^{max} > L_1$.

Finally, we show that if financing is feasible, there will be no liquidation conditional on innovation. We focus on the case where $D_2 \leq D_2^{max}$. If $C_1 < D_1$, then the firm patents only if $D_1 + D_2 - C_1 \leq D_2^{max}$. Otherwise, the firm would not patent and the lender's outside option would be equal to L_1 under the original contract. However, if $D_2^{max} \geq L_1$, then the firm could promise to pay up to D_2^{max} even when the time 1 cash flow, C_1, equals 0 and avoid liquidation. The firm clearly finds it optimal to do so, so that liquidation never occurs when financing is feasible.

A.4.2 Proof of Theorem 2

As argued earlier, we may restrict the analysis to contracts where $D_1 \leq \bar{C}_1$. Next, we establish that a contract with $D_2 = D_2^{max}$ yields a higher payoff for the lender than a contract with $D_2 > D_2^{max}$, for the same D_1. To see this, consider what happens under the original contract when $D_2 > D_2^{max}$: if $C_1 > D_1$, the firm can repay D_1, and has the right to make a take-it-or-leave-it offer D_2' to the lender for the time 2 payment. If the lender rejects the new offer, the firm will prefer not to patent the innovation, and hence the payment to the lender at time 2 will be zero. Hence, the lender's outside option is equal to zero. By contrast, if $D_2 = D_2^{max}$ under the original contract, any offer D_2' less than D_2^{max} would be rejected by the lender since, with the original contract in place, the firm will prefer to patent the innovation, leading to a payment of $D_2 = D_2^{max}$ for the lender.

Likewise, the payoff to the lender for any contract with $D_2 < D_2^{max}$ can again be increased by instead setting $D_2 = D_2^{max}$, for the same D_1. To see this, consider the case where $C_1 < D_1$, so that the firm cannot make the time 1 payment. This allows the lender to make a take-it-or-leave-it offer to the firm, where the optimal such offer is $(C_1, \min\{D_2^{max}, D_1 + D_2 - C_1\})$. The payoff from this contract is weakly increasing in D_2 for $D_2 < D_2^{max}$, and is strictly increasing for C_1 close to D_1. Hence, the expected payment to the lender is maximized for $D_2 = D_2^{max}$, given a fixed value for D_1.

We now show that, given D_2, the payoff to the lender is maximized for $D_1 = \bar{C}_1$. To see this, suppose that $D_1 < \bar{C}_1$, and assume first that there is no innovation. When $C_1 < D_1$, the lender can liquidate to obtain $\min\{L_1, D_1 + D_2\} = L_1$. However, when $C_1 \geq D_1$, the firm can simply make the time 1 payment D_1 and pay nothing at time 2. Hence, the lender's payoff can be strictly increased by raising D_1, as long as $D_1 < \bar{C}_1$.

Consider then the case of innovation. As argued above, for any given C_1, and assuming $D_2 \leq D_2^{max}$, the lender's payoff is $D_1 + D_2$ when the firm has the power over renegotiation and is $C_1 + \min\{D_2^{max}, D_1 + D_2 - C_1\}$ when instead the lender can propose the renegotiated contract. Either way, the lender's payoff is increasing in D_1, for a given D_2. Together with the argument above, this implies that the contract that yields the highest expected payoff for the lender is $(D_1, D_2) = (\bar{C}_1, D_2^{max})$.

Given this, we can now substitute the contract $D_1 = \bar{C}_1$, $D_2 = D_2^{max}$ into the lender's participation constraint to obtain

$$(1-p)L_1 + p(D_2^{max} + \frac{\bar{C}_1}{2}) \geq I,$$

which, after some rearranging, yields the expression in the statement of the Theorem.

A.5 Closed form for $\Delta(D_1)$ and D_1^{min}

$$(1-p)\mathcal{F}_0(D_1) + p\mathcal{F}_1(D_1 + \Delta(D_1)) = I.$$

Replacing $F_0(D_1), F_1(D_1 + \Delta(D_1))$ from Claim 1,2 and using algebra, we obtain

$$\Delta(D_1) = \bar{C}_1\left(1 - \sqrt{1 - \frac{2A}{\bar{C}_1}}\right) + D_2^{max} - D_1, \tag{29}$$

where

$$A := \frac{I}{p} - \frac{(1-p)}{p}\left(L_1\frac{D_1}{\bar{C}_1} + D_1(1 - \frac{D_1}{\bar{C}_1})\right) - D_2^{max}.$$

Notice that both $\mathcal{F}_1(.)$ and $\mathcal{F}_0(.)$ are strictly increasing functions. Thus, there is a one-to-one correspondence between D_1 and D_2 for all contracts (D_1, D_2) with $D_2 \leq D_2^{max}$ that satisfy the participation constraint $(1-p)\mathcal{F}_0(D_1) + p\mathcal{F}_1(D_1 + D_2) = I$.

To shorten the notation, let $D_1^0 = \Delta^-(D_2^{max})$, we have

$$(1-p)\mathcal{F}_0(D_1^0) + p\mathcal{F}_1(D_1^0 + D_2^{max}) = I,$$

D_1^0 can be determined from

$$(1-p)\left(L_1\frac{D_1^0}{\bar{C}_1}+D_1^0(1-\frac{D_1^0}{\bar{C}_1})\right)+p\left(D_1^0+D_2^{max}-\frac{(D_1^0)^2}{2\bar{C}_1}\right)=I,$$

which can be rewritten as a quadratic equation of D_1^0:

$$\left((1-p)\frac{L_1}{\bar{C}_1}+1+p\right)D_1^0-\frac{2-p}{2\bar{C}_1}(D_1^0)^2=I-pD_2^{max}. \tag{30}$$

There are two values of D_1^0 that satisfy (30). But $D_1^0=\Delta^-(D_2^{max})$ by definition is the smallest value satisfy (30), thus

$$D_1^0=\bar{C}_1\frac{(1-p)\frac{L_1}{\bar{C}_1}+1+p-\Omega}{2-p},$$

where

$$\Omega=\sqrt{((1-p)\frac{L_1}{\bar{C}_1}+1+p)^2-2(2-p)\frac{I-pD_2^{max}}{\bar{C}_1}}.$$

And hence,

$$D_1^{min}=\max\{0,D_1^0\}.$$

A.6 Calculation for Prediction 4

The lemma can be established from simple differentiation of D_2^{max}, as follows:

$$\frac{\partial^2 D_2^{max}}{\partial\beta^2}=2V^3\omega^2\overline{C}_2\frac{\omega-\alpha}{\left(V\omega+\overline{C}_2-V\beta\omega\right)^3}>0$$

$$\frac{\partial^2 D_2^{max}}{\partial\omega^2}=2V^2\overline{C}_2\left(\beta-1\right)\frac{V\alpha+\overline{C}_2-V\alpha\beta}{\left(V\omega+\overline{C}_2-V\beta\omega\right)^3}<0$$

$$\frac{\partial^2 D_2^{max}}{\partial\beta\partial\omega}=-V^2\frac{\overline{C}_2}{\left(V\omega\left(1-\beta\right)+\overline{C}_2\right)^3}\left(\overline{C}_2\left(\alpha-2\omega\right)-V\alpha\omega\left(1-\beta\right)\right)>0.$$

A.7 Calculation for Prediction 10

To study the effect of changing D_2^{max} on interest rate, we will show that under a certain condition, if D_2^{max} increases by ϵ, then D_1 need to decrease more than ϵ so that (19) still holds. This means that when D_2^{max} increases and the long-term payment binds ex-ante $D_2 = D_2^{max}$, the interest rate $(D_1 + D_2 - I)/I$ will decrease.

Note that the prediction above is derived from the first order derivative of the left-hand side of (19) according to D_1:

$$\frac{1-p}{p}\left(\frac{L_1}{\bar{C}_1} + 1 - \frac{2D_1}{\bar{C}_1}\right) + 1 - \frac{D_1}{\bar{C}_1}$$

If this derivative is smaller than 1 a small increase in D_2 results in a larger decline in D_1 reducing the promised interest payment.

$$\frac{1-p}{p}\left(\frac{L_1}{\bar{C}_1} + 1 - \frac{2D_1}{\bar{C}_1}\right) < \frac{D_1}{\bar{C}_1}$$

which is equivalent to

$$D_1 > \frac{1-p}{2-p}(L_1 + C_1).$$

This holds when p is close to 1.

B Contingent Contracts

Assume a state-contingent contract of the following form: payment D_1 at date 1 and $D_2^P < D_2^{NP}$ at date 2 in case of patenting or non-patenting, respectively. The rest of the model remains the same. However, we will show that D_2^{NP} does not affect the payoff to the lender nor the incentive constraint.

Consider now the various cases. If $C_1 \geq D_1$ and the firm decides not to patent, then it will pay D_1 at date 1 to avoid liquidation and pay nothing at date 2 since, without patenting, the asset is not verifiable. Therefore, the firm's payoff is $C_1+C_2+\alpha V-D_1$, which is independent of D_2^{NP}, and the payment to the lender is also independent of D_2^{NP}.

Suppose instead that the firm decides to patent. In that case, it will pay C_1 at date 1, and renegotiate the date 2 payment, which is the minimum of either $D_1 + D_2^P - C_1$ or the largest amount that makes patenting incentive compatible.

Given that the payoff of the firm without patenting, as described above, is independent of D_2^{NP}, the promised repayment D_2^{NP} does not play any role in ensuring the incentive compatible constraint is satisfied. Therefore, it does not play any role in the renegotiation of the date 2 payment when the firm decides to patent.

Now consider the case where $C_1 < D_1$. In this case, the lender is entitled to make a take-it-or-leave-it offer to the firm. If the lender liquidates, neither the lender's nor the firm's payoff depends on D_2^{NP} because the existing assets are not sufficient to pay back the lender.

If the lender wants to continue to date 2, it offers to receive C_1 at date 1 and then for the minimum of either the remaining payment $D_1 + D_2^P - C_1$ or the greatest amount that makes patenting incentive compatible. Hence, similar to the previous case, this amount is independent of D_2^{NP}.

We can now conclude that the promised payment D_2^{NP} in the event that the firm does not patent has no effect on the payment to the lender. It also does not affect the incentive constraints and, thus, the payoff of the firm in equilibrium. Therefore, a state-contingent contract of this type cannot improve efficiency relative to the contract studied in the paper, which already allows for efficient renegotiation and leads to patenting whenever an innovation is realized.

C Robustness

In this section, we consider a variation of the renegotiation game where the firm has all the bargaining power, and as a result, always makes the offer in the bargaining process. Our main goal is to show that the endogeneity of patenting remains a significant friction for lending. Specifically, we show the following result.

Theorem 3 *Under the assumption that the firm always has the right to make a take-it-or-leave-it offer to the lender, a feasible contract exists if and only if* $pD_2^{max} \geq I$.

To prove this result, first, consider the case when there is no innovation. If $C_1 < D_1$, the lender gets L_1, and the firm liquidates at time 1 with probability 1. If $C_1 \geq D_1$, the lender gets D_1 and the firm does not liquidate. This is the same as before.

When there is innovation, the outcome of the contract depends on whether $D_2 \leq D_2^{max}$ or $D_2 > D_2^{max}$.

- If $D_2 \leq D_2^{max}$:
  - If $C_1 < D_1$, the lender gets L_1 and the firm gets liquidated at time 2 with $\Pr(C_2 < L_1 - C_1)$;
  - If $C_1 \geq D_1$, the lender gets $D_1 + D_2$ and the firm gets liquidated at time 2 with $\Pr(C_2 < D_1 + D_2 - C_1)$.
- If $D_2 > D_2^{max}$:
  - If $C_1 < D_1$, the lender gets L_1 and the firm gets liquidated at time 2 with $\Pr(C_2 < L_1 - C_1)$;
  - If $C_1 \geq D_1$, the lender gets D_1 and the firm does not patent.

Thus, when $D_2 > D_2^{max}$ the payoff of the lender is

$$\frac{D_1}{\bar{C}_1} L_1 + (1 - \frac{D_1}{\bar{C}_1}) D_1 \leq L_1.$$

Therefore, such a contract is not feasible.

Now, when $D_2 \leq D_2^{max}$, the expected payment to the lender is

$$p\left(\frac{D_1}{\bar{C}_1}L_1 + (1 - \frac{D_1}{\bar{C}_1})(D_1 + D_2)\right) + (1-p)\left(\frac{D_1}{\bar{C}_1}L_1 + (1 - \frac{D_1}{\bar{C}_1})D_1\right)$$

$$= \frac{1}{\bar{C}_1}\big((L_1 + \bar{C}_1 - pD_2)D_1 - D_1^2\big) + pD_2. \tag{31}$$

Lemma 5 *If (D_1, D_2) is feasible, then the expected payment to the lender in (31) is a decreasing function of D_1.*

Proof. We need to show that $L_1 + \bar{C}_1 - pD_2 \leq 0$, which implies that (31) is a decreasing function of D_1. Assume that $L_1 + \bar{C}_1 - pD_2 > 0$. Then (31) is maximized when $D_1 = \min\{\frac{L_1+\bar{C}_1-pD_2}{2}, \bar{C}_1\}$, and the maximum value is either

$$\frac{1}{\bar{C}_1}(\frac{L_1 + \bar{C}_1 - pD_2}{2})^2 + pD_2 \text{ when } \frac{L_1 + \bar{C}_1 - pD_2}{2} \leq \bar{C}_1$$

or

$$L_1 \text{ when } \frac{L_1 + \bar{C}_1 - pD_2^{max}}{2} > \bar{C}_1.$$

In the former case, that value is at most

$$(\frac{L_1 + \bar{C}_1 - pD_2}{2}) + pD_2 = (\frac{L_1 + \bar{C}_1 + pD_2}{2}) \leq L_1 + \bar{C}_1 < I$$

So in both cases, our assumption rules out feasibility.

This implies that for a contract to be feasible, $L_1 + \bar{C}_1 - pD_2 \leq 0$, which establishes the result. ■

Because of Lemma 5, the expected payoff to the lender is maximized when $D_1 = 0, D_2 = D_2^{max}$, and therefore the maximum value is pD_2^{max}. This proves Theorem 3.

D Existing assets

In this section, we briefly study the role of existing assets the firm may have which can be used to support the financing that is needed, and how they affect the firm's incentive to patent. We model this by assuming the firm has additional assets which have value A to the firm at time 2, and λA to the lender if liquidated, where $0 \leq \lambda \leq 1$. For simplicity, we assume that A is small, so that it cannot fully support the needed financing I, and that in the event of default both the asset and the patent, if any, need to be liquidated.

Incentive compatibility of patenting

The firm's decision to patent at time 1 depends on its payoff when it patents versus when it does not. Suppose that $C_1 > D_1$, so that the firm can make the time 1 promised payment. In this case, if the firm chooses to patent its expected payoff is

$$C_1 + E[C_2] + \Pr(C_2 \geq D_2)(\omega V + A) + \Pr(C_2 < D_2)(\beta\omega V + \lambda A) - (D_1 + D_2). \tag{32}$$

This payoff reflects that, if the firm makes its time 1 payment and patents, its payoff equals the time 1 plus the expected time 2 cash flows, as well as the patented value of the innovation and the asset when able to meet its obligation at time 2, but only the liquidation value of the patent and the additional asset, $\beta\omega V + \lambda A$, when not. The firm will patent if and only if (32) is at least as large as the value of not patenting, which is

$$C_1 + E[C_2] + \alpha V + \lambda A - D_1 - \min\{D_2, \lambda A\}. \tag{33}$$

We consider the case $\lambda A \leq D_2$, consistent with our assumption that the value of additional assets is relatively small. Then the condition for endogenous patenting is

$$(\omega V + A)\Pr(C_2 \geq D_2) + (\omega\beta V + \lambda A)\Pr(C_2 < D_2) - \alpha V \geq D_2. \tag{34}$$

The left-hand side of (34) is decreasing in D_2, while the right-hand side is increasing in D_2.

Furthermore, at $D_2 = \bar{C}_2$, the LHS is $\omega\beta V + \lambda A$, which we take to be less than $\bar{C}_2$ (by assumption, $\omega\beta V < \bar{C}_2$, so that this condition holds for A small). Hence, there is a D_2^{newmax} of D_2 that satisfies (34) with equality.

We can now establish the following result.

Lemma 6 *Let D_2^{newmax} be the maximum value of D_2 such that (34) holds. The firm finds it optimal to patent its innovation only if $D_2 \leq D_2^{newmax}$, where*

$$D_2^{newmax} = \frac{\omega - \alpha + \frac{A}{V}}{\frac{1}{V} + \frac{(1-\beta)\omega}{\bar{C}_2} + \frac{(1-\lambda)}{\bar{C}_2}\frac{A}{V}}.$$

When $A = 0$, $D_2^{newmax} = D_2^{max}$, the threshold value of D_2 below, which patenting is optimal, derived in the main text. For $A > 0$, we have that $D_2^{newmax} > D_2^{max}$, so that the incentive compatibility constraint associated with patenting is loosened, and the firm has a greater incentive to patent. Nevertheless, as long as the value of any additional existing assets is not so large that the firm can fully collateralize its loan (i.e., as long as $\lambda A < I$), the firm's patenting decision will still be endogenous, with the same implications derived above.