The Impacts of Prediction Technologies on Relational Contracts

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Abstract

The recent advances in data science and artificial intelligence have led to better and cheaper prediction machines, which improve business decisions in virtually all industries. We show that these advances, however, can do harm to the economic efficiency of supply chains that rely on relational contracts. This is because better predictions undermine the willingness of parties to commit to and trust each other long term. Using a model of relational contracts between suppliers and retailers who have different prediction abilities on market shocks, we identify and analyze an inherent trade-off between the new prediction technologies and the old trust-building mechanism in relational contracts.

1 Introduction

In a recent book titled "Prediction Machines: The Simple Economics of Artificial Intelligence" [Agrawal et al., 2018], the authors convincingly argue that the recent advances in data science and artificial intelligence (AI) amount to nothing more than better prediction machines, yet their consequences for the economy and society are profound and far-reaching. These consequences are not all positive. Issues with privacy, security, and fairness as a result of advances in data science and artificial intelligence are currently among the most important topics debated among scholars and policy makers. Nevertheless, there seems to be a universal agreement on the overall economic benefits of better predictions: when predictions become faster, more accurate, and cheaper, they are used more, lead to better decisions, and improve efficiency. The current debates, therefore, are largely centered around "who gets which share of the improved surplus?" For example, are firms better off at the expense of consumers' privacy and society's fairness?

In this paper, we show that better and cheaper predictions can, in fact, make a market less efficient. This negative effect could potentially disrupt and change the market structure of certain industries, leading to undesirable consequences for firms and consumers for reasons beyond privacy and fairness issues.

Industries that depend crucially on relational contracts are the ones that are likely to be affected by this negative effect. Relational contracts are a form of long-term contract that arises to prevent mis-coordination, build trust, and mitigate risks. Unlike complete long-term contracts, which are often too complex to be implemented and enforced, relational contracts are self-sustained without formal enforcement because parties have incentives to keep the on-going relationship to maintain value from future interactions. While relational contracts are as old as human civilization, they still remain the backbones of many modern supply chains. They are prominent in a variety of environments, from agricultural supply chains of developing countries, where law enforcement is weak, to high tech industries such as computer, textile, and automobile industries [Cohen et al., 2003, Bloom and Van Reenen, 2007, McFarlan et al., 2007, Antras and Foley, 2015].

Relational contracts are sustained by the ability of parties to commit to and trust each other in the long run. Uncertainty about the future provides incentives for agents to do so, therefore, better predictions can undermine this ability. In particular, a common insight from prior works in the relational contract literature (e.g., [MacLeod and Malcomson, 1998, Baker et al., 2002, Levin, 2003]) is that a relationship can be self-enforcing if the difference between the expected value of trade in the relational contract going forward and the value of the outside option is greater than the current incentive to cheat. The ability to use computation on a vast amount of data for better forecasting and prediction substantially changes these underlying contractual costs. Specifically, better prediction creates a trade-off for parties involved in relational contracts. On the one hand, prediction identifies better alternative trade opportunities that could improve welfare. On the other hand, when parties deviate for better trade, it undermines their ability to commit in relational contracts. This reduces the expected value of the on-going relationship and makes it harder for the contract to be self-sustained. Ex-ante, it is unclear which effect will dominate.

Our paper studies this trade-off in a game-theoretical model of relational contracts between suppliers and retailers, where suppliers need to exert costly and unverifiable effort to produce goods that the retailers sell to the market. The value of the good to the market and the cost of production depend on both the level of effort and a market shock. Through the use of data analytics and historical market information, the retailer and/or the supplier may be able to analyze these exogenous factors and predict the state of the market shock for the near future.

Our model is a novel extension of the classical relational contract model developed in [MacLeod and Malcomson, 1998]. The extension incorporates different prediction abilities for the involved parties, which allows us to systematically study the effects of such predictions on the retailers' and suppliers' welfare, as well as the suppliers' efforts (and consequently the production quality). Specifically, we analyze and compare different market scenarios capturing combinations of different parties making predictions.

We gain three main insights into the effects of prediction technologies on relational contracts: (1) Prediction from either suppliers or retailers can make the market outcome Pareto-inferior, and this negative impact is amplified when both parties use the prediction technology, (2) Even though the prediction technology can worsen their outcome, neither suppliers nor retailers can credibly commit not to use the technology once they have access to it, and (3) The negative impacts of prediction persist even when both suppliers and retailers have the same information about future states. This shows that our main findings are not driven by information asymmetry, but by the fact that the prediction technologies lead to the suppliers' and retailers' inability to credibly commit to each other long term.

Our paper starts with the formal model and assumptions in Section 2. The main results are summarized as follows:

- Section 3 characterizes the equilibrium of the benchmark case, called Case I, where neither party has access to predictions.

- Section 4 studies Case II, in which only suppliers predict their costs of producing the good. We show that compared with Case I, the retailers' utility and the effort exerted by suppliers (i.e. production quality) are always strictly lower, and there are ranges of parameters where the suppliers are also worse off (Theorem 1).

- Section 5 considers Case III, where only retailers predict the market value of the good. We analyze the main trade-off of prediction. On the one hand, prediction leads to the retailers' frequent contract breaking, which disincentives the suppliers from exerting effort. In Theorem 2, we show that this negative effect can result in lower good quality and make both parties worse off. On the other hand, prediction allows the retailers to find better suppliers and improve welfare. In Theorem 3, we quantify this positive effect by varying the value of information gained by prediction.

- Section 6 analyzes Case IV, in which both retailers predict the market value, and suppliers predict production costs. Comparing with Case III, we show that both retailers and suppliers can be worse off (Theorem 4). This shows that the negative effects of prediction identified in the previous cases will persist and are in fact amplified, leading to highly inefficient outcomes.

- Finally, in Section 7, we break the asymmetric information barrier between suppliers and retailers by giving them access to the same predictions (of either the production cost or the market value). We show that in this case (Case V), the negative effects demonstrated in Cases II-IV persist (Theorem 5). This shows that the main driving force behind the effects of prediction on relational contracts is not asymmetric information, but rather it is a direct consequence of prediction abilities.

- We show that in all the scenarios we consider, neither suppliers nor retailers can credibly promise not to use predictions, even if they will be worse off when they do (Lemmas 2, 4, and 6). This is because using prediction is a dominant strategy for both suppliers and retailers. In other words, if the contract is drawn up with the assumption (or promise) that the retailer or supplier does not use predictions, the firm(s) with access to predictions will be better off if they deviate and use predictions. Thus, predictions will be used whenever available, in spite of their possible negative consequences.

Related Literature

Prediction technologies, and more generally, artificial intelligence, are affecting many industries and reshaping the business landscape. A fast-growing body of literature that cuts across disciplines attempts to study and predict these effects. Varian [2018] gives a broad introduction to the various effects these technologies can have on businesses; Athey [2018] explores the effects of AI on economics; Chen and Xiao [2012] study the effects of prediction technologies on supply chain, and Agrawal et al. [2019] and Frank et al. [2019] study the effects of AI on labor markets. Our paper is the first to study the impacts of prediction technologies on relational contracts, and as such, is at the intersection of the two largely separate lines of literature: one studies the impacts of prediction technologies on complete contracts, the other analyzes the incentives of self-enforced (incomplete) relational contracts.

The majority of the first line of research, which studies the impacts of prediction technologies on supply chain contracts (e.g., [Aviv, 2001, Taylor and Xiao, 2010]), consider static models of trade in which contracts are complete. Our paper focuses on relational contracts, which depend on the future value of trade; the way prediction influences the ability of suppliers and retailers to commit to long-term relational contracts is very different from complete contract settings.

The second line of research on relational contracts is extensive in both economics (e.g., [Bull, 1987, MacLeod and Malcomson, 1998, Levin, 2003]) and supply chain literatures (e.g., [Plambeck and Taylor, 2006, Kim et al., 2007, Bondareva and Pinker, 2018]). However, this research has largely ignored the impact of changing technologies in forecasting and prediction. Our main contributions are the novel insights on the impacts of these technologies on long-term relationships, which we gain by systematically comparing different market scenarios.

From methodological perspectives, our paper is different from most of the literature of relational contracts in supply chains, which considers game-theoretical models between a *single* principle (retailers) and one or many agents (suppliers) (see for example, [MacLeod and Malcomson, 1989, Levin, 2003, 2002, Board, 2011]). We analyze the game with a continuum of suppliers and retailers. This is an important deviation because it allows us to endogenize both retailers' and suppliers' outside options, which are key to analyzing the impact of prediction on the aggregate market behavior.[1] The approach of using a large market model also provides predictions that are robust against individual deviation. This is unlike models with a single retailer, where equilibrium behavior is sensitive to the details of the history of play and reputation effects.

Prediction technologies in our model can be viewed as a form of private information. Relational contracts with private information have also been studied extensively in the literature (e.g., [Malcomson, 2015, 2016]). However, this literature focuses on the principal-agent framework, while we embed relational contracts in a model of a market with a continuum of agents. Moreover, our paper compares different market scenarios to study the impact of prediction on welfare, and thus our focus is different from the dynamic principle-agent literature.

[1] Shapiro and Stiglitz [1984], MacLeod and Malcomson [1998], Board and Meyer-ter Vehn [2014], study continuum models of relational contracts, but do not consider private information and prediction.

Our paper is also related to the study of the influence of increasing competition on relational contracts. It is a well-studied phenomenon that competition can harm the ability of parties to credibly commit to relational contracts, (e.g., [Aghion et al., 2005, Macchiavello and Morjaria, 2019]). The model we consider is general and can also capture the degree of competition via the ratio between the number of retailers and suppliers; however, our paper's main focus differs from this literature.

2 Model

We consider an economy where retailers hire suppliers through relational incentive contracts. Our model builds on the classical model of Shapiro and Stiglitz [1984], MacLeod and Malcomson [1988]. The market consists of a continuum of retailers and suppliers measured R and S, with $S > R$; for simplicity, we assume further that $S > 2R$.[2] Suppliers and retailers are expected utility maximizers and discount future payoffs by a factor δ.[3]

At each time step, a supplier that is matched to a retailer can choose to exert effort $e \in \mathbb{R}_{\geq 0}$ in providing a good/service. This effort is non-observable and non-contractible by the retailer; however, retailers can imperfectly monitor this effort. In particular, the retailer can verify whether her supplier has exerted effort e with probability α. The supplier incurs a cost $c_{\mathcal{S}_c}(e)$ for this effort, where $\mathcal{S}_c$ denotes the state of the cost in the current period. This state is exogenous and contract-specific (that is, it depends on the identities of both supplier and retailer), reflecting non-effort related factors which may affect the supplier's cost, such as difficulty in obtaining raw material depending on the state of the market, effect of quality of raw material on production effort, etc. We assume that the state is binary: $\mathcal{S}_c \in \{H, L\}$, denoting high and low cost, respectively. In each period, the cost state is H with probability β_c. The cost function $c_{\mathcal{S}_c}(\cdot)$ is non-decreasing, differentiable, and convex, with $c_H(e) \geq c_L(e)$ for all e, and $c_{\mathcal{S}_c}(0) = 0$.

[2]This guarantees that a retailer can always be matched immediately upon breaking an existing match, regardless of other retailers' actions.

[3]We will be using male pronouns for suppliers and female pronouns for retailers for clarity.

The supplier's effort leads to a production value of $v_{\mathcal{S}_v}(e)$ for the retailer. The value of production also depends on a contract-specific state $\mathcal{S}_v \in \{H, L\}$.[4] This state reflects non-effort related factors which may affect the value of production to the retailer, such as the state of the market, demand for the produced goods, etc. In each period, the production state is H with probability β_v. The production function $v_{\mathcal{S}_v}(\cdot)$ is non-decreasing, differentiable, and concave, with $v_H(e) \geq v_L(e)$ for all e.

2.1 Relational contracts

Following MacLeod and Malcomson [1989], Levin [2003], we consider self-enforcing relational contracts in which retailers hire suppliers directly, and offer them a compensation package consisting of a *wage* $w^* \in \mathbb{R}$ and a *bonus* payment $b^* \in \mathbb{R}$,[5] for exerting a pre-specified effort $e^* \in \mathbb{R}_{\geq 0}$. The timeline of events in one period of a relational contract is shown in Figure 1, and proceeds as follows.

The retailer pays her supplier the wage upfront. The supplier then decides how much effort $e \in \mathbb{R}_{\geq 0}$ to exert, and incurs cost $c_{\mathcal{S}_c}(e)$. Retailers can (imperfectly) evaluate their suppliers' effort, detecting shirking with probability α. Subsequently, the supplier may also receive his promised bonus if no shirking is detected by the retailer, whether because the supplier has not shirked, or because shirking has gone unnoticed by the retailer. At this point, the states of supplier cost and value of production are revealed, with the match yielding high value $v_H(e)$ with probability β_v and high cost $c_H(e)$ with probability β_c. Further, if prediction is used, the state(s) of the contract for the next period will become known to the retailer and/or supplier.

Following these transactions, the current contract may continue to the next period or

[4] Note that we abuse the notation and use the same notation of H, L for the two different state spaces that retailers and suppliers predict.

[5] We do not want to limit the space of contracts, and so do not restrict the sign of the wage and bonus. Further, we use the terms wage and bonus to be consistent with the terminology used in the literature. In a supply chain setting, wage can be thought of as the fixed, enforceable payments promised to suppliers. Bonuses on the other had are unenforceable payments, which are promised by the retailers to their supplier as a supplemental pay in case of good performance.

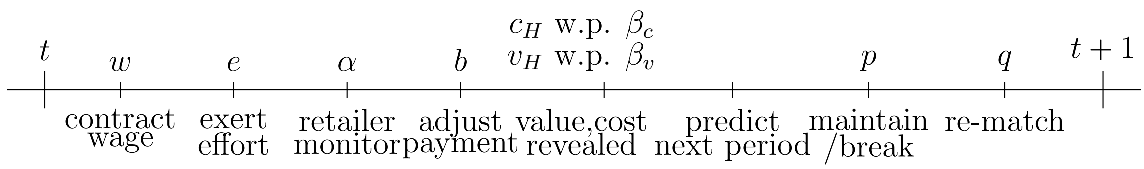


Figure 1: Timeline of events/decisions in each period of a relational contract.

break. Specifically, the relationship breaks if the supplier is caught shirking or if the retailer reneges on paying the promised bonus. Furthermore, we assume, a match might break with probability p due to exogenous, non-contract related reasons such as relocation or change of business. When prediction is used, the retailer and/or supplier may also choose to terminate the contract based on predictions about the future state(s) (as described in the following sections).

If the relationship is terminated for any of these reasons, the retailer has a vacancy to fill prior to the next period. We assume that the retailers with vacancies randomly pick matches from the pool of $S - R$ suppliers that were not matched in this period. Note that we assume the suppliers whose contracts have broken in this period are not part of the rematch pool due to potential search frictions. These recently unmatched suppliers will join the unmatched pool and seek a match in the next period.

We denote the probability that an unmatched supplier gets selected by a retailer during matching by q. This probability depends on the way prediction is used to adjust contracts, and will be addressed in more detail later.

Because there is continuum measure of retailers and suppliers, the probability of re-encounters between suppliers and retailers is negligible. Furthermore, we assume that information about past contracts can not be shared across participants; neither suppliers nor retailers can be assigned a rating or reputation. Thus, the decisions regarding maintaining/breaking of contracts are based solely on the current period's events, and remain unaffected by the history of either parties' previous contracts. This assumption abstracts away reputational effects and allows us to isolate the effects of the prediction technologies.

We study the design of optimal relational incentive contracts by the retailers, where the contracts consist of a prescribed effort level and the corresponding compensation package (wage and promised bonus payments). The retailer's goal is to maximize her expected profit. We summarize the notation in Table 1.

S, R	measure of suppliers, retailers
δ	discount rate
e	effort level
$c_L(e), c_H(e)$	cost of effort at given state
$v_L(e), v_H(e)$	market value of production at given state
β_c, β_v	probability of high state for suppliers, retailers
p	exogenous probability of contract breaking
w, b	contract terms: wage and bonus
α	probability of detecting shirking
q	probability of rematch

Table 1: Summary of notation.

An example: relational contracts in the textile industry

While our model is stylized, we give an example to better illustrate how it relates to relational contracts in the real world. Retailers such as Walmart and Sears have relational contracts with textile factories in developing countries [Bondareva and Pinker, 2018]. There are tens of thousands of textile factories in developing countries, and even though they are not homogeneous, we assume that there is a sufficiently large subset that is roughly equivalent in the eyes of the retailers, and they are indifferent between them. This is the set S in the model. At the beginning of the period, we assume that retailers that currently do not have a supplier are matched at random to a supplier from this subset. The retailer pays a wage at the beginning of the period. As noted elsewhere, e.g., MacLeod and Malcomson [1989], this assumption is purely for convenience: as the wage is legally enforceable, it can be made at any time. While some of the cost is known up-front to the supplier, a portion of it is not, and is only revealed after the supplier has made the decision about how much effort to exert (in this case, for example, acquiring high-quality materials, training and/or self-inspections). The retailer in this case will inspect the supplier with some probability – this could be on-site, or inspection of the goods. If no shirking is detected, a bonus is paid upon receiving the goods. The retailer then finds out the value of the garments produced, when the garment is made available in the stores. If a bonus was agreed upon but is withheld, the contract breaks. The contract may also break for exogenous reasons, for example, if safety issues in the factory are reported by a news outlet (unfortunately, this is all too common, e.g., Emont [2018]). If the contract breaks, the retailer can immediately match with a new supplier (e.g., via a B2B marketplace). We assume that the supplier is not matched immediately, due to the time it takes to register to these marketplaces and become available, although this assumption is made for technical convenience, and our results hold *mutatis mutandis* if this assumption is removed, i.e., if the supplier can immediately match with a new retailer.

2.2 Using predictions in relational contracts

We consider the availability of the prediction technology to different market participants. We make the following assumption.

Assumption 1. *When available, prediction is perfect, i.e., the retailer and/or supplier predict the true future state(s) with probability 1.*[6] *In addition, for simplicity, we assume $\mathcal{S}_c$ and $\mathcal{S}_v$ are independent from each other, and independent across periods.*

We analyze and compare the following five scenarios:

- Case I: Prediction technology is available to neither retailers nor suppliers.
- Case II: Prediction technology is available only to suppliers, who can predict their cost, $\mathcal{S}_c$.

[6] The assumption of perfect prediction of the states is for simplicity. Similar qualitative results hold when we relax this assumption.

- Case III: Prediction technology is available only to retailers, who can predict their value, $\mathcal{S}_v$.
- Case IV: Prediction technology is available to both retailers and suppliers; retailers can predict their value $\mathcal{S}_v$ and suppliers can predict their cost $\mathcal{S}_c$.
- Case V: Prediction technology is available to both retailers and the suppliers; both retailers and suppliers can predict the suppliers' cost $\mathcal{S}_c$.

We also analyze a sixth case, when prediction technology is available to both retailers and suppliers, with both predicting the retailer's value $\mathcal{S}_v$, but as we show in Section 7.1, this case is identical to Case III.

Note that we only assume that the prediction technology is available or not available to a certain agent. An agent can decide to opt out of the technology. However, we will show that agents will typically have incentive to use such technology when it is available.

For each of these scenarios, we study the prescribed effort, wage, and bonus, and suppliers' and retailers' expected payoffs. The effort prescribed in the contract in turn determines the average quality of the products for consumers of the goods sold by the retailer. We note that Cases II, III, and IV involve information asymmetry, while in Cases I and V, the retailers and suppliers have access to the same information. Comparing Cases I, II, and V will allow us to assess the role which information asymmetry plays in our results.

2.3 Solution concept and intuitions

Our solution concept will be that of *stationary contracts*, which consist of a required effort level and promised wage and bonus, that are independent of time.[7] We say a supplier *follows the terms of the contract* if he exerts the level of effort prescribed in the contract. A retailer

[7] In a large economy, it is standard to consider stationary contracts to analyze the aggregate behavior of the market. More complex contracts that depend on time and state of the economy are intractable and out of the scope of the paper.

follows the terms of the contract if she pays the promised bonus when no shirking is detected. Our main solution concept is the following.

Definition 1. *A stationary contract (e, w, b) is at equilibrium if no party can improve their utility by deviating from the terms of the contract given that all other parties follow them.*

In the following sections, we will characterize optimal stationary contracts under the five prediction scenarios in Cases I-V. To do so, we derive two types of constraints on the contract terms in each scenario, by contrasting the gain each party can obtain by deviating from the contract in the current period against the loss of breaking the long-term contract as a result. The first type of constraints guarantee that the suppliers participates in the market and do not shirk given the contract terms and their private information. The other ensures that retailers will pay the promised bonus payments.

The intuitions behind these constraints are tied to the fundamental properties of the two types of payment: wage and bonus. In particular, wages and bonuses together constitute the total payment that can be used to ensure the supplier will not shirk. The retailer can choose how to allocate the required total payment between wages and bonuses. As the wages are paid to the suppliers at the beginning of each period, a potential risk for the retailers is that suppliers can take the wage and then shirk. On the other hand, bonuses are paid only if no shirking is detected. This makes bonuses more efficient from the retailers' perspective and, unlike wages, bonus payments can be tied to the suppliers' effort decisions. However, bonus payments are less preferred by suppliers, as retailers can refuse to pay even when suppliers have exerted the prescribed effort. Thus, the promised bonus at the optimal contract needs to be credible, as determined by the retailer's incentive constraint. In summary, the two basic incentive constraints together determine the total payment, as well as its partitioning into wage and bonus.

3 The Benchmark Case: No Prediction

We begin with relational contracts in the absence of state predictions by either party. We consider this scenario as the benchmark for comparison of our results in subsequent scenarios. Below, we provide the main characterization of the optimal contract and some of its properties.

Let $\bar{c}(e) := \beta_c c_H(e) + (1-\beta_c)\, c_L(e)$ and $\bar{v}(e) := \beta_v v_H(e) + (1-\beta_v)\, v_L(e)$ be the average cost and valuation of the good given an effort level e, respectively. The following result characterizes the optimal contract.

Lemma 1. *Let* $\Psi_p = \frac{1-\delta(1-p)}{\alpha\delta(1-p)}$ *and* $\Phi_p = \frac{1}{\alpha\delta(1-p)}\left(1-\delta+\frac{\delta p S}{S-R}\right)$.[8] *The optimal relational contract* $\{e, w\}$ *in the absence of prediction is found by the unique solution to the following optimization problem*

$$e := \arg\max_{e' \geq 0} \quad \bar{v}(e') - (1+\Psi_p)\bar{c}(e') \tag{1a}$$

$$w := (1+\Phi_p)\,\bar{c}(e) \tag{1b}$$

We make three observations based on the above characterization.

Observation 1. *Without prediction, retailers cannot credibly offer a positive bonus payment to suppliers; that is,* $b = 0$.

Note that this differs from the principal-agent case (e.g., MacLeod and Malcomson [1989]) where the individual rationality and incentive constraints are independent of the division of the wage profile. That is, in the principal-agent case, the contract is enforceable for any such division, whereas in our model, there exists a unique division (with $b = 0$) for which the contract is self-sustaining. The intuition behind this is simple: at the end of a period,

[8] These two constants will appear in all of the contracts. Both constants are parameterized by the probability that the contract breaks. We will show that in Case II, this probability is identical to Case I (hence the constants are identical); in Cases III and IV, the probability is higher, which affects the value of these constants.

retailers will not have the incentive to pay a bonus to suppliers because a retailer can find a new supplier immediately if the current relation is broken. Thus, a promise of bonus payment will not be credible in the contract. We will see that this will not necessarily be the case for contracts with prediction.

Observation 2. *Due to unobservability of suppliers' effort, the equilibrium is not efficient. In particular, the contract's prescribed effort level is lower than the market clearing one.*

The market clearing effort is that at which the marginal value of production is equal to the marginal cost of effort, i.e., $\bar{v}'(e) = \bar{c}'(e)$. However, to keep suppliers from shirking, retailers need to give rent to suppliers, as captured by (1a). As a result, the optimal contract faced with this constraint results in a less efficient outcome and a lower level of equilibrium effort. See Figure 2 for an illustration.

The main difference between the optimal outcome in this scenario and the market clearing solution is the term Ψ_p in (1a). This term is a rent multiplier, the magnitude of which captures the inefficiency of the market. Observe that Ψ_p is decreasing in α and δ, and increasing in p. Because of this, we obtain the following observation that summarizes the intuition for the behavior of this market outcome when these parameters change.

Observation 3. *The effort prescribed in the relational contract is more efficient (i.e., closer to the market clearing effort) when,*

(i) Monitoring technologies improve; here, as α increases.

(ii) The parties care more about their future payoffs; here, as δ increases.

(iii) Contracts break less often due to non-contract-related reasons; here, as p decreases.

4 Case II: Suppliers Predict

Next, we consider the case where only suppliers have the ability to predict. We first note that suppliers cannot commit to not use prediction technologies or act upon the resultant

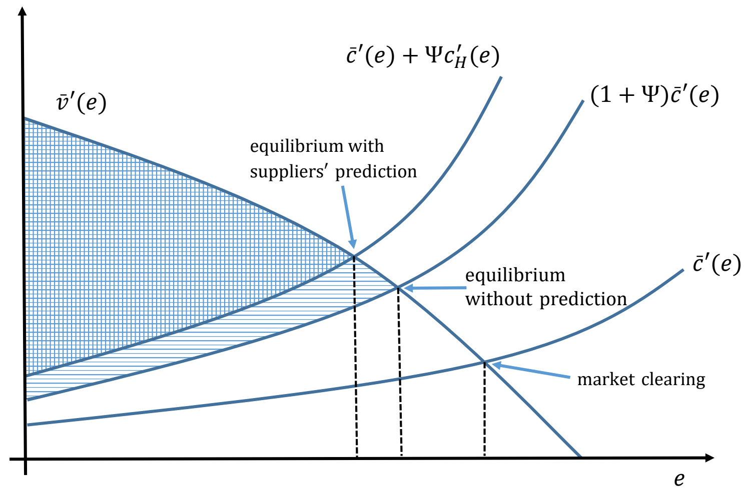


Figure 2: Effort prescribed in relational contracts is inefficient (compared to the market clearing effort) due to the rent paid as a result of unobservability of suppliers' efforts, for $\Psi > 0$.

predictions.

Lemma 2. *If $\beta_c \in (0,1)$, the supplier cannot credibly promise not to use predictions.*

The intuition behind the proof of Lemma 2 is simple. If the supplier can credibly promise not to use predictions, a contract is drawn up based on this promise. The contract is then identical to the contract of Case I, as that is exactly the situation in which neither party predicts. However, in the contract of Case I, a supplier using predictions would obtain a higher utility by shirking when the predicted cost is high.

When a supplier predicts, his incentive compatibility (IC) constraints (i.e., incentives to exert the prescribed effort) are different depending on his realized cost of effort. Therefore, in order to prevent the suppliers from shirking, the retailer should choose a contract that satisfies the IC constraints in both states (H and L). In contrast, without prediction, the retailer only needs to satisfy the supplier's non-shirking constraint under the expected cost of effort. As such, the optimal contract prescribes a reduced optimal effort level compared to Case I, due to the stricter non-shirking constraints. This leads to the reduction of retailers' payoff, and sometimes a reduced supplier payoff as well. The optimal contract can be characterized as follows.

Lemma 3. *Let* $\Psi_p = \frac{1-\delta(1-p)}{\alpha\delta(1-p)}$ *and* $\Phi_p = \frac{1}{\alpha\delta(1-p)}\left(1-\delta+\frac{\delta pS}{S-R}\right)$*. Assume that only the supplier can predict his future cost. The optimal relational contract* (e, w) *is found by the unique solution to the following optimization problem*

$$e := \quad \arg\max_{e' \geq 0} \quad \bar{v}(e') - (\bar{c}(e') + \Psi_p c_H(e')) \tag{2a}$$

$$w := \quad \bar{c}(e) + \Phi_p c_H(e). \tag{2b}$$

Note that the optimal contract in this scenario is similar to the one in Case I, except that the additional cost $\Phi_p \bar{c}(e)$ in (1b) is now replaced by the larger $\Phi_p c_H(e)$ in (2b). We will now show how this stricter constraint influences the equilibrium effort level and the parties' payoffs.

4.1 Comparison of Cases I and II

The supplier's expected utility (when matched) is $\bar{c}(e) - w$, in both Cases I and II. The supplier's utility is therefore $\Phi_p \bar{c}(e)$ in Case I and $\Phi_p c_H(e)$ in Case II. As a result, if the same effort were to be exerted in both contracts, the supplier's utility would be higher in Case II. Interestingly, though, sometimes the ability to predict *reduces* the utility of the supplier. This is because the retailer chooses an effort level that is sufficiently low that it counterbalances the fact that the retailer compensates the supplier for the higher cost state. Using Programs 2 and 2, we can compare the contracts in Cases I and II as follows.

Theorem 1. *Assuming* $c'_H(e) \geq c'_L(e)$ *for all* $e > 0$*,*

(i) The effort of the optimal contract when suppliers predict (Case II) is always strictly lower than in the no-prediction benchmark (Case I).

(ii) The retailer's utility is always strictly lower when suppliers predict (Case II) than in the no-prediction benchmark (Case I).

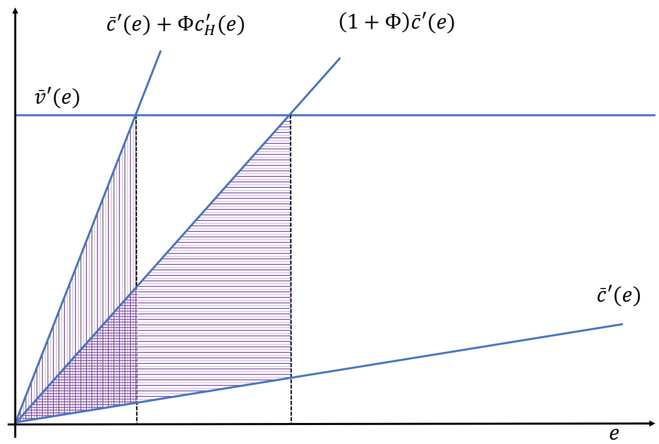


Figure 3: An example with linear retailer value functions and quadratic costs, where prediction reduces the supplier's utility: the areas shaded with horizontal and vertical lines denote the suppliers' utilities in Cases I and II, respectively.

(iii) There exist cost and value functions for which the supplier's utility is lower when suppliers predict (Case II) than in the no-prediction benchmark (Case I)

An example for point *(iii)* of Theorem 1 is given in Figure 3, using linear retailer value functions and quadratic costs.

5 Case III: Retailers Predict

We now consider the case where only retailers can predict their future states. We first show that the retailers will use the prediction technology and fire the suppliers when a low production state is predicted (Lemma 4). In other words, retailers cannot credibly commit to a contract where they do not use the prediction technology, or one where they use it but do not act upon it. We then provide the characterization of the optimal contract in Lemma 5.

With this characterization, we analyze the trade-off between the two opposing impacts of prediction technologies. First is the negative impact on efficiency because of the retailers' inability to continue the relational contract with the supplier when a low state is predicted. Similarly to Case II, we show that for a wide range of parameters, compared with the scenario of no prediction, both retailers and suppliers can be worse off (see Theorem 2, and Figures 4 and 5). The positive impact, on the other hand, comes from the value of information gained by prediction. Prediction helps to increase trade surplus. Furthermore, it leads to strictly positive bonus compensation. This is a sharp contrast to Cases I and II. We show that bonus, when credible, is a better form of compensation than wage to incentivize the suppliers to exert effort. We quantify these insights in Theorem 3.

Our first result regards the retailers' inability to commit to not use prediction technologies or act upon the resultant predictions.

Lemma 4. *The retailer cannot credibly promise not to use prediction or to not fire the supplier if she predicts that the state will be v_L.*

Next, we have the following characterization of the optimal contract for this scenario.

Lemma 5. *Let $p^\dagger = 1 - \beta_v(1-p)$, $\Psi_{p^\dagger} = \frac{1-\delta(1-p^\dagger)}{\alpha\delta(1-p^\dagger)}$, and $\Phi_{p^\dagger} = \frac{1}{\alpha\delta(1-p^\dagger)}\left(1 - \delta + \frac{\delta p^\dagger S}{S-R}\right)$. Assume that only the retailer can predict her future value, and she breaks the contract at the end of time t if predicting state v_L for $t+1$. The optimal relational contract (e, w, b) is given as follows. The effort and bonus are set as the solution to the following optimization problem*

$$\max_{e,b} \quad v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - \bar{c}(e) - \Psi_{p^\dagger}\left(\bar{c}(e) - \alpha b\right) \tag{3a}$$

$$s.t. \quad b = \min\left\{\delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)), \frac{\bar{c}(e)}{\alpha}\right\} \tag{3b}$$

and the wage is set as follows:

$$w = \bar{c}(e) - b + \Phi_{p^\dagger} \max\{\bar{c}(e) - \alpha b, 0\}. \tag{3c}$$

5.1 The negative effect of breaking contracts

Lemma 4 states that when retailers can predict, they will fire suppliers when the low state is predicted. This reduces the continuation value of the relationship, and as a result, increases the incentives needed for the suppliers to exert effort. This effect can be seen in the optimization program in Lemma 5.

To understand these effects more intuitively, consider the case when β_v is close to 1 or to 0, which means that with high probability, the state will remain fixed (hence almost certainly known a-priori). The constraint (3b) under these extreme β_v implies that the bonus b is negligible. Hence, the wage (3c) becomes approximately

$$w \approx \bar{c}(e) + \Phi_{p^\dagger}\bar{c}(e).$$

While in the benchmark case, the wage (1b) becomes

$$w \approx \bar{c}(e) + \Phi_p\bar{c}(e).$$

As $p^\dagger > p$, we have $\Phi_{p^\dagger} > \Phi_p$. Therefore, the wage required to satisfy the suppliers' incentive constraints increases in Case III compared to the benchmark Case I. This in turn negatively impacts the optimal effort level and the contract's efficiency.

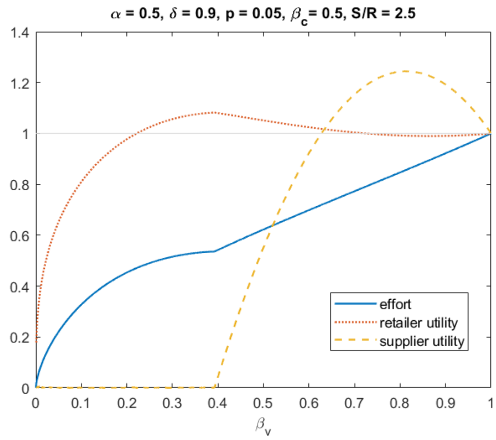
The following result formalizes this intuition, as well as other comparisons between this case and the benchmark Case I.

Theorem 2. *If $v_H(e) - v_L(e)$ is concave and non-increasing for all e, and the optimal effort in Case I is non-zero for all $0 \leq \beta_v \leq 1$,*

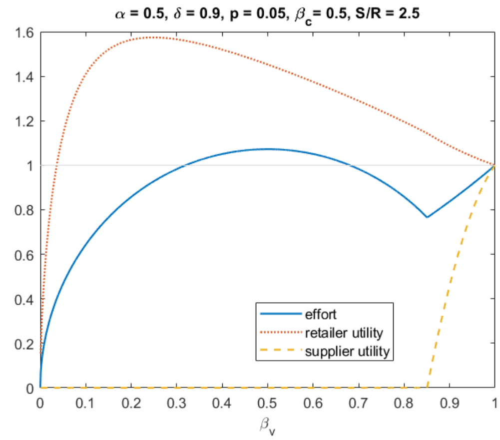
(i) there exists $\beta_v^* > 0$ such that for all $\beta_v \leq \beta_v^*$, the effort when retailers predict (Case III) is lower than the no-prediction benchmark (Case I).

(ii) if $v_H(0) = 0$, there exists $\beta_v^* > 0$ such that for all $\beta_v \leq \beta_v^*$, the retailers' welfare when they have prediction (Case III) is lower than when they do not (Case I).

(iii) there exists $\beta_v^\dagger > 0$ such that for all $\beta_v \geq \beta_v^\dagger$, the effort when retailers predict (Case III) is lower than the no-prediction benchmark (Case I).



(a) $v_H = 2\sqrt{e}, v_L = \sqrt{e}$



(b) $v_H = e + 1, v_L = e$.

Figure 4: Plot of the ratios of the effort and suppliers' and retailers' welfare in Case III over Case I as a function of β_v. (When the ratio is greater than 1 this indicates that the value is higher in Case III than in Case I). In both plots, $c_H(e) = 2e^2, c_L(e) = e^2$. The breaks in the plots correspond to the point at which the bonus stops being sufficient to cover the supplier's rent.

Figure 4 serves as an illustration for Theorem 2, and implies that the results are in fact more general than for the conditions stipulated in the theorem statement: in Figure 4(a), $v_H(e) - v_L(e)$ in increasing in e, in Figure 4(b), $v_H(0) \neq 0$. In this figure, the likelihood of the high state (represented by β_v) is varied, while all other parameters are fixed. The two sub-figures show two different retailer value functions, keeping the supplier's cost functions the same. Figure 4 shows that, perhaps unsurprisingly, the advantage that predictions offer is generally higher when the uncertainty is greater. This is consistent with our findings in part (ii) of Theorem 2, which shows that when the low state predominates, the retailer is worse off when using predictions.

5.2 Credibility of bonuses and their positive effects

Next, we show that prediction can also have a positive impact on the retailers and the total welfare. First, we find that unlike Cases I and II, here retailers can credibly promise bonus payments to suppliers. This is because if the relationship is broken when a high state is predicted, the retailer cannot be guaranteed a high-quality supplier right away. In this case, a positive bonus incentivizes the supplier to exert effort and maintain the relationship.

Formally, from Lemma 5, we make the following observation.

Observation 4. *When retailers predict (and* $v_H(e) - v_L(e) > 0$*), they can credibly promise bonuses to the suppliers; that is,* $b > 0$ *in the optimal contract.*

The ability to (partially) compensate suppliers with bonuses, rather than wages alone, is beneficial to the retailers. This is because the payment of bonuses in contingent on the suppliers not shirking (or at least, not getting caught), whereas wages have to be paid upfront regardless. Therefore, from the retailers' perspective, bonus has an advantage over wage, as it is a promised compensation that only needs to be paid after suppliers make their effort decisions. Mathematically, we can see this from the retailer's objective in (3a), which is increasing in b. This means that in the optimal contract retailers will want to include, and in fact maximize, the bonus.

Given that the retailer now has two forms of payment, bonuses and wages, she faces a trade-off between them when incentivizing suppliers, with higher bonuses implying the need for lower wages, and vice versa. As discussed above, the retailer has a preference for bonuses over wages. However, as bonuses are not committed payments, suppliers need to have faith that the retailers will pay their bonus if they perform. That is, even though by Observation 4 retailers can credibly promise bonuses in this case, and by (3a) they have a preference for bonuses, there is still a limit to *how much* bonus they can credibly promise. Lemma 5 shows that the retailer will maximize the *credible* bonus she can pay, as captured in (3b), and cover the remaining required payments using wages, as captured in (3c).

Note that the retailer's preference for bonuses over wages holds in Cases I and II as well, for the same reasoning.[9] In these other cases, however, a non-contractible bonus is not credible: if the contract were to include a bonus, the (utility maximizing) retailer would renege on the bonus, fire the supplier and hire another one, strictly improving her utility. Therefore, it is the credibility of paying the bonuses once the retailers can predict, which

[9]This can be easily verified formally by considering the proofs of Lemmas 1 and 3.

allows them to shift as much of the required compensation as credible to bonuses.

Lastly, in the next theorem, we show that the size of the credible bonus (and the ensuing positive effects) depends on how valuable the prediction is, which is captured by the difference between v_H and v_L, as also seen in (3b). It is unsurprising that the greater the difference, the more useful the predictions can be to the retailer, leading to increased retailer utility. The other two results are arguably more interesting: when the difference between the two values is higher, this leads to higher contracted effort, and the bonus is higher. Thus, we can see that being able to pay a higher bonus is one of the main driving factors of improving effort and retailer utility.

Theorem 3. *Let S and $\bar{S}$ be markets with identical parameters, except the retailer value functions: v_H, v_L are the value functions in S and $\bar{v_H}$ and $\bar{v_L}$ are the functions in $\bar{S}$. Assume $v_H - v_L$ and $\bar{v_H} - \bar{v_L}$ are concave and furthermore, let β_v, v_H, v_L, $\bar{v_H}$ and $\bar{v_L}$ be such that $v_H(0) = \bar{v_H}(0)$, $v_L(0) = \bar{v_L}(0)$, for every $e > 0$, $\bar{v_H}'(e) > v_H'(e)$ and $\bar{v_L}'(e) < v_L'(e)$ and*

$$\beta_v v_H(e) + (1 - \beta_v) v_L(e) = \beta_v \bar{v_H}(e) + (1 - \beta_v) \bar{v_L}(e);$$

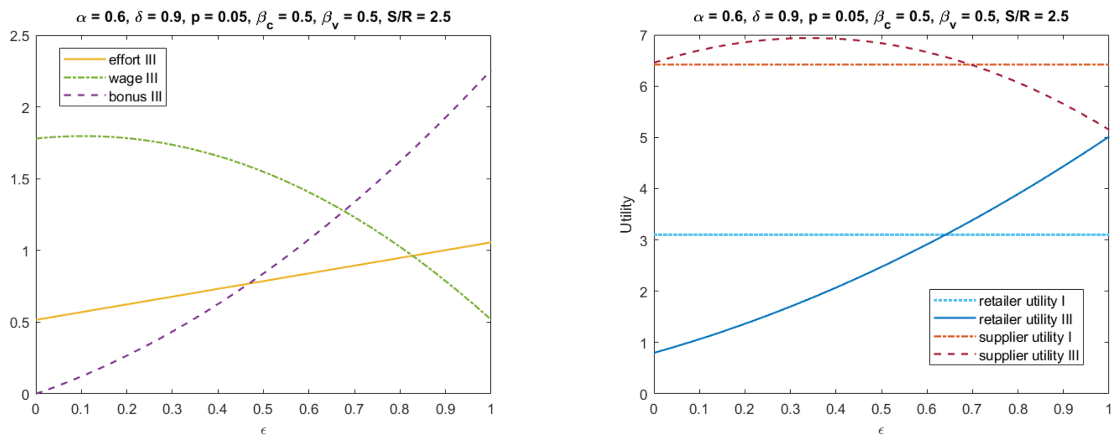
that is, the average retailer payoff is identical in S and $\bar{S}$. Further, assume that $\bar{c}(e)$ is strictly increasing in e. Then for all non-degenerate cases (when the optimal effort of S is not zero),

(i) the optimal effort of $\bar{S}$ is strictly greater than the optimal effort of S.

(ii) the bonus offered in $\bar{S}$ is strictly greater than the bonus offered in S.

(iii) the retailer's utility in $\bar{S}$ is strictly greater than in S.

In this theorem, in order to isolate the effect that the difference between the high and low retailer values plays, we have kept the average utility constant (as opposed, to, say, keeping v_L constant and increasing v_H). The theorem shows that as the difference between v_H and v_L grows, the positive effect of having large bonus payments becomes more dominant



(a) e,w and b as a function of ϵ

(b) utilities as a function of ϵ

Figure 5: Retailer predicts, with average utility kept constant at $\bar{v}(e) = e$ by setting $v_H = (1+\epsilon)e, v_L = (1-\epsilon)e, \beta_v = 0.5$.

than the negative effect of contracts being broken more often: as the difference grows, so do the bonus, and with it the effort and retailer utility. Thus, while the average utility stays constant, the prediction becomes more valuable. The numerical examples in Figure 5 illustrate Theorem 3. In this example, $\beta_v = 0.5$, $v_H = (1+\epsilon)e$ and $v_L = (1-\epsilon)e$, hence the expected value is $\bar{v}(e) = e$ for any ϵ. In Figure 5(a), we see that as ϵ grows, so do the effort and the bonus. Note that the bonus grows at the expense of the wage. In Figure 5(b), we see that the retailer's Case III utility grows with ϵ, and at some point becomes greater than her utility for Case I. Lastly, as shown in Figure 5(b), unlike the effort, bonus, and retailer payoff, the supplier's utility does not show a particular trend similar to those identified in Theorem 3: it can be greater or less than in Case I, and it is not monotone in ϵ.

6 Case IV: Both Suppliers and Retailers Predict

We now consider the case where both the suppliers and the retailers can make predictions. All of the effects identified in the previous two cases will be present and amplify each other in this case. In particular, because of the supplier's ability to predict, the retailer further faces the more stringent incentive compatibility constraints, as she needs to ensure that the supplier exerts effort regardless of his predicted costs. Furthermore, for the retailers, the two effects of prediction – the more frequent contract breaking and the ability to offer bonus – continue to conflict with each other.

First, similar to Case II, the supplier can not credibly promise not to use his prediction. Further, similar to Case III, the retailer has an incentive to use prediction and fire the supplier if she predicts the state will be v_L. The proofs are similar to those of Lemmas 2 and 4, and are omitted.

Lemma 6. *The supplier cannot credibly promise not to use prediction. The retailer cannot credibly promise not to use prediction or to not fire the supplier if she predicts that the state will be* v_L.

Thus, even when the availability of prediction leads to lower production value and/or utility for the retailer or the supplier, neither can commit to not using predictions. With this result, we obtain the following characterization of the optimal contract.

Lemma 7. *Let* $p^\dagger = 1 - \beta_v(1-p)$, $\Psi_{p^\dagger} = \frac{1-\delta(1-p^\dagger)}{\alpha\delta(1-p^\dagger)}$, *and* $\Phi_{p^\dagger} = \frac{1}{\alpha\delta(1-p^\dagger)}\left(1 - \delta + \frac{\delta p^\dagger S}{S-R}\right)$. *Assume both the retailer and the supplier can predict their respective future states, and that the retailer breaks the contract at the end of time* t *if predicting state* v_L *for* $t+1$. *The optimal relational contract* (e, w, b) *is given as follows. The effort and bonus are set as the solution to the following optimization problem*

$$\max_{e,b} \quad v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - \bar{c}(e) - \Psi_{p^\dagger}\left(c_H(e) - \alpha b\right) \tag{4a}$$

$$s.t. \quad b = \min\left\{\delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)), \frac{c_H(e)}{\alpha}\right\} \tag{4b}$$

and the wage is set as follows:

$$w = \bar{c}(e) - b + \Phi_{p^\dagger} \max\{c_H(e) - \alpha b, 0\}. \tag{4c}$$

6.1 Comparison of Cases III and IV

The difference between Cases III and IV is similar qualitatively to the difference between Cases I and II. In particular, the optimization programs in Lemma 5 and Lemma 7 differ in that the cost $\bar{c}(e)$ is replaced by the higher cost $c_H(e)$ in certain expressions. Because of this, we obtain the following theorem.

Theorem 4. *Assume $c'_H(e) \geq c'_L(e)$ and $v_H(e) - v_L(e)$ is concave.*

(i) The effort of the optimal contract is always lower in Case IV than in Case III.

(ii) The retailer's utility is always lower in Case IV than in Case III.

(iii) There exist cost and value functions for which the supplier's utility is lower in Case IV than in Case III.

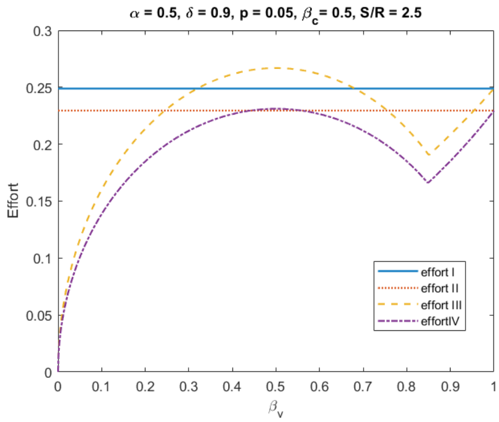
6.2 The combined effects of predictions

The characterization in Lemma 7 and Theorem 4 show that the negative effects of predictions for both retailers and suppliers can be amplified when both sides can predict. This can also be seen in the case when β_v is close to either 1 or 0, that is, when the retailer's value is fixed (either high or low) with high probability. In this case the bonus b is close to 0, and the wage (4c) essentially becomes:

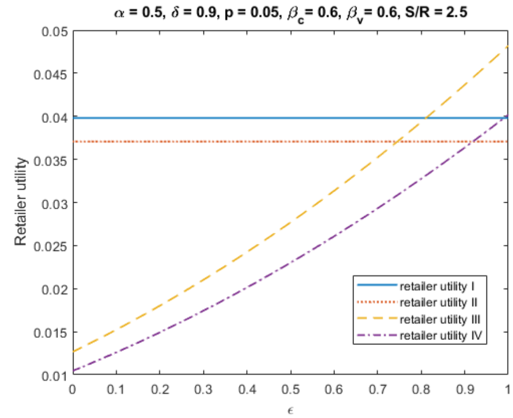
$$w \approx \bar{c}(e) + \Phi_{p^\dagger} c_H(e).$$

Thus, compared with the wage constraint of the benchmark case, (1b), the above constraint is stricter because of two factors: (i) $c_H(e) \geq \bar{c}(e)$ and (ii) $\Psi_{p^\dagger} \geq \Psi_p$. The former corresponds to the fact that retailers need to ensure the suppliers' incentive constraints hold in both states. The latter represents the impacts of more frequent contract breaking because of the retailers' ability to predict.

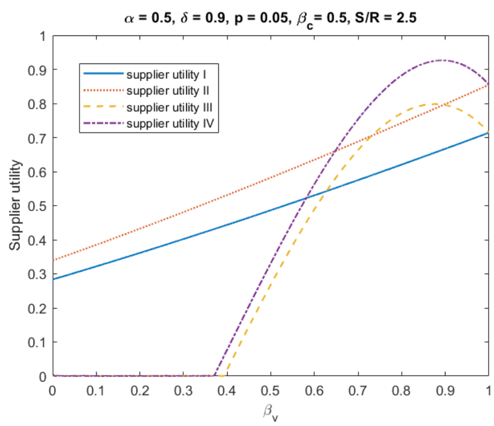
The combined negative effect of prediction can make all parties worse off. However, there are scenarios where, similarly to Case III, when the difference of $v_H(e)-v_L(e)$ is large enough,



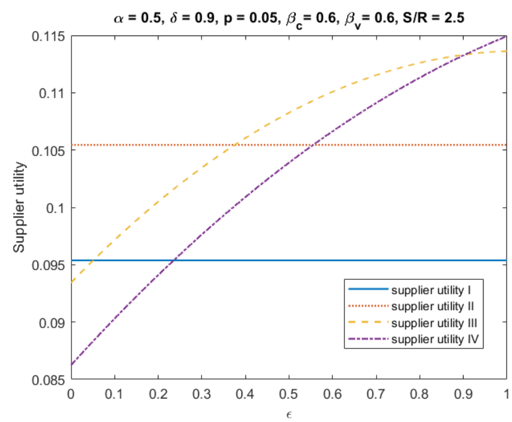
(a) Effort as a function of β_v, $v_H = e + 1, v_L = e$



(b) Retailer utility as a function of ϵ, $v_H = (1 + \epsilon)e, v_L = (1 - \epsilon)e$



(c) Supplier utility as a function of β_v, $v_H = 2\sqrt{e}, v_L = \sqrt{e}$



(d) Supplier utility as a function of ϵ, $v_H = (1 + \epsilon)e, v_L = (1 - \epsilon)e$

Figure 6: Effort, retailer and supplier utility, for Cases I-IV. In all cases, $c_H = 2e^2, c_L = e^2$.

the positive effect from large bonus payments can dominate the negative effects. To illustrate this, Figure 6 compares the equilibrium effort, and the suppliers' and retailers' payoffs, for all four cases of prediction, when the suppliers' costs are quadratic, and for several different forms of the retailers' value functions.

Figure 6(a) compares the effort across all four cases, when retailers have linear value functions. We first note that, consistent with Theorem 4, the effort in Case IV is always lower than that of Case III. The effort of Case II is also consistently lower than that of Case I, as shown in Theorem 1. The relation between the effort of Cases I and III at high and low β_v is consistent with Theorem 2.

Figure 6(b) shows that prediction by the suppliers decreases the retailers' utility (compare Case I vs II, and Case III vs IV), as the retailers now need to satisfy the stricter incentive constraints of their suppliers. This is again consistent with our findings in Theorems 1 and 4. We also observe that the retailer's payoff can be strictly higher when using prediction for most values of β_v. This is due to both increase of bonuses (the more efficient form of payment from retailers' perspective), and the larger profitability of avoiding low production value states by breaking contracts.

Lastly, Figures 6(c) and 6(d) contrast the suppliers' utilities, for two different forms of retailers' value functions. We see that the effect can vary depending on the parameters. In particular, in Figure 6(d), we keep the expected value of production constant, and vary the difference between v_H and v_L, by setting $v_H(e) = (1+\epsilon)e, v_L(e) = (1-\epsilon)e$, $\beta_v = 0.5$, (as in Theorem 3) and plotting the suppliers' utilities as a function of ϵ. We observe that, as stated in Theorem 4, there exists problem instances in Figure 6(d) for which the supplier does worse in Case IV (when he can predict his future costs) compared to Case III (where he could not predict). In contrast, in Figure 6(c) we can see that the supplier is better off when he has access to predictions for all values of β_v.

7 Extension: Symmetric Information

Thus far, we have considered scenarios in which retailers or/and suppliers can predict the information regarding their own production value or cost, while their partners do not have access to this information. This raises a natural question of whether information asymmetry is a "hidden" market friction leading to our main results. In this section, we break this asymmetric information barrier, and show that the negative impacts of prediction identified in our paper are, in fact, driven by the retailers' and suppliers' inability to commit to long-term relational contracts.

To see this, we consider the case when both the retailer and the supplier can predict the supplier's cost or the retailer's value. Let us once again recall the strategic options of the retailer and the supplier once the contract is set. The retailer has two decisions: (i) to pay or withhold the bonus and (ii) to continue or break the contract at the end of the period. The supplier's options are (i) to exert the contractually stipulated effort or shirk and (ii) to continue or break the contract. We first consider the case of both suppliers and retailers predicting the retailer's value, and then consider the case where both parties predict the supplier's cost.

7.1 Both retailers and suppliers predict the retailer's value

This scenario gives some intuition as to why information asymmetry does not play a main role in our findings. When both parties predict the retailer's value state $\mathcal{S}_v$, the optimal contract will be identical to that of Case III (i.e., when only retailer predicts). In particular, while one might think that the supplier could potentially leverage the knowledge of the retailer's future state into obtaining a higher wage when the retailers' value is high, it turns out that this is not the case. To see why, note that under the contract specified by Lemma 5, the supplier will not shirk and is incentivized not to break the contract at the end of the period. The supplier has no bargaining power, and neither of the supplier's available actions can offer a credible threat to the retailer. Therefore, the asymmetry of information plays no role in the effects of prediction in Case III.

7.2 Case V: Both retailers and suppliers predict the supplier's cost

We now turn to the case where both suppliers and retailers can predict the supplier's cost. Comparing the outcomes of this case with that of Case I (no prediction) allows us to further determine to what extent the effects we have seen thus far (in Cases II, III, and IV) are a consequence of prediction and information asymmetry. If information asymmetry was the only driving factor in the results of Case II, then giving the retailer access to the same predictive abilities would revert the contract to Case I, of no prediction. We show that this does not happen, and in fact giving both retailers and suppliers access to predictions about the suppliers' costs gives rise to a fairly complex market scenario.

In contrast to Case II, when only the supplier predicts, the ability to predict gives the retailer the option of firing the supplier when a high cost is predicted. A retailer that can predict the supplier's cost therefore has two options[10]: to terminate the contract when high cost is predicted, or to continue with the contract regardless of the predicted state (assuming that no shirking is detected). Note that these two decisions can simply be mapped to two different contracts. The retailer should therefore compare the two possible contracts, choosing the one that leads to a higher payoff.

We denote the contract for the retailer using predictions and firing when the predicted supplier cost state is high by C^P, and the contract when not using predictions/not acting upon them by C^N. Out of these two contracts, the retailer chooses the one that maximizes her revenue. Note that C^N is identical to the optimal contract of Case II in Lemma 3. The following lemma describes the contract C^P: when the retailer terminates the contract when high cost is predicted.

Lemma 8. *Let* $p^{\ddagger} := \beta_c + (1-\beta_c)p$, $\Psi_{p^{\ddagger}} = \frac{1-\delta(1-p^{\ddagger})}{\alpha\delta(1-p^{\ddagger})}$, *and* $\Phi_{p^{\ddagger}} = \frac{1}{\alpha\delta(1-p^{\ddagger})}\left(1-\delta+\frac{\delta p^{\ddagger}S}{S-R}\right)$. *Assume both the retailer and the supplier can predict the supplier's future cost, and that the retailer breaks the contract at the end of time* t *if predicting state* c_L *for* $t+1$. *The optimal contract* (e, w) *is found by the solution to the following optimization problem*

$$e := \arg\max_{e' \geq 0} \quad \bar{v}(e') - \left(c_L(e') + \Psi_{p^{\ddagger}}\bar{c}(e')\right) \tag{5a}$$

$$w := c_L(e) + \Phi_{p^{\ddagger}}\bar{c}(e) - \beta_c \frac{\delta p^{\ddagger} R}{S-R}\left(c_H(e) - c_L(e)\right) \ . \tag{5b}$$

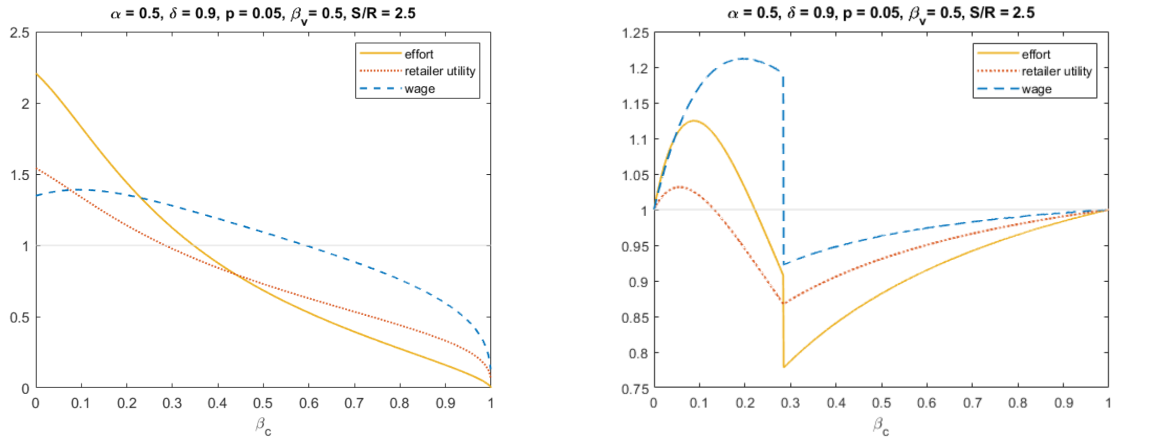
[10]Barring irrational decisions, such as firing only when a low supplier cost is predicted. Furthermore, it is easy to see that it suffices to consider deterministic options: even if firing with some probability $\pi \in (0,1)$ is stipulated in the contract, the retailer, as a payoff maximizer, will chose the payoff maximizing option.

We would like to determine under which conditions the retailer will choose each contract. There is no credible bonus payment in either of these contracts, hence the retailer's choice depends solely on the wage required to incentivize each effort level in the two contracts. There are two oppositing forces at work when determining the wages needed to keep suppliers from shirking in Contract C^P, compared to Contract C^N, for a given effort level e: the lower costs incurred by suppliers reduces the wage needed to keep suppliers from shirking, while the more frequent contract breaks increase these wages.

To more easily contrast the two cases, first consider $\beta_c \to 0$. Note that in this case $p^\ddagger \to p$ and therefore $\Phi_{p^\ddagger} \to \Phi_p$, and also, that $\bar{c}(e) \to c_L(e)$. Thus, in C^P, $w \to c_L(e) + \Phi_p c_L(e)$, whereas in C^N, $w \to c_L(e) + \Phi_p c_H(e)$. This means that when the cost is likely to be c_L, the wage required to incentivize some effort level e is lower in Contract C^P than in Contract C^N. This is due to the fact that C^N still aims to satisfy the supplier's no shirking constraint in the infrequently occurring high cost states, requiring the higher wages. Note also that contract breaks due to high cost predictions in C^P are infrequent in this regime, and therefore there is no considerable increase in wages due to shorter-lasting contracts. However, as β_c increases, this is no longer the case: an increase in β_c increases $p^\ddagger$, and consequently, $\Phi_{p^\ddagger}$. Then, as $\Phi_{p^\ddagger} > \Phi_p$, the wage of C^P may surpass that of C^N at the same effort level e. Figure 7(a) shows the effort, retailer utility, and wage, in C^P and C^N, as a function of β_c.

Comparing Case V and Case I. We now show that for some parameters, the retailer will simply not use the predictions. In these situations, the retailer will always be worse off in Case V than in Case I (similarly to Case II). In other cases, the retailer *will* use predictions, which in some instances allows the retailer to have a higher utility than in Case I. It is clear that information asymmetry plays no role in many of our results, and if it *does* play a role in some results, it does not appear to be a major role. That is, the effects observed are due to access to predictions.

Theorem 5. *Let* $\Phi^* = \frac{1}{\alpha\delta(1-p)}\left(1 - \delta + \frac{\delta S}{S-R}\right)$.



(a) Ratio of the effort, retailer's utility, and wage, in Contract C^P over Contract C^N.

(b) Ratio of the effort, retailer's utility, and wage, in Case V over Case I.

Figure 7: Case V—both supplier and retailer predict supplier's cost, $c_H = 10e^2, c_L = e^2, \bar{v} = 1.5\sqrt{e}$. The break in Figure 7(b) is when the retailer "switches" between C^P and C^N.

(i) If $c_H(e) > (1 + \Phi^*)c_L(e)$ there exists $\beta_c^* > 0$ such that for all $\beta_c \leq \beta_c^*$, the retailers' utility is higher in Case V than in Case I.

(ii) If $p < 1$, there exists $\beta_c^\dagger > 0$ such that for all $\beta_c \geq \beta_c^\dagger$, the retailer's utility is lower in Case V than in Case I.

We show in the proof of Theorem 5 that for sufficiently small (large) β_c, the retailer will choose Contract C^P (Contract C^N). Intuitively, this is because if the retailer were to choose Contract C^P when β_c is large, the shorter-lasting contracts due to frequent contract breaks would increase the required incentives to keep the suppliers from shirking, and would overall reduce the retailer's payoff. Therefore, when β_c is large, the suppliers' prediction capabilities are the only ones driving the contract, even though the retailer has the same predictive abilities. When β_c is small, the retailer can offer a lower wage in Contract C^P than in Contract C^N, for the same effort. This can naturally lead to a higher utility for the retailer, as shown in part (i) of Theorem 5. Note that this does not necessarily mean that the wage offered in the optimal contract will be lower, as it may be possible to now incentivize a higher effort. In particular, Figure 7(a) shows that the wage can be higher in C^P than in C^N because contract C^P can incentivize a higher effort. Nonetheless, choosing

Contract C^P is insufficient by itself to guarantee that the retailer gains an advantage from being able to predict the supplier's cost. Only when the difference between the high and low costs is sufficiently large and β_c is sufficiently small can the retailer leverage her prediction ability to increase her revenue over Case I. Figure 7(b) shows such a scenario.

8 Conclusion

The recent technological advances in data science and artificial intelligence are significantly changing the ways business decisions are made in virtually all industries. The current debates on their consequences have mainly focused on how the technologies might distort the share of surplus between firms and consumers. Our paper suggests that there are other far-reaching consequences of better predictions on the economy: they can negatively influence incentive problems in long-term relational economic partnerships, leading to a less efficient market. This negative effect is robust because we only assume that better prediction technologies are available and parties can still decide whether or not to use them. We show that at equilibrium, firms use these technologies, causing a "tragedy of the commons" type situation.

Two implications can be drawn from our results. First, from a managerial perspective, given the rapid adoption of data science, it will be crucial for firms whose business depends on relational contracts to maintain strong partnership with their suppliers/retailers. Second, the weakening ties in relational contracts could lead to changes in the market structure. Some firms will manage to maintain the relational partnership with either mergers or joint investments as a commitment device. Other firms failing to do so will exit the market or work with suppliers/retailers in short-term contracts, which results in products with lower quality. Such a market segmentation could also negatively impact consumers because the level of competition is reduced for each of the market segments. Furthermore, the prices of high quality products will increase because of the cost associated with mergers or investments in the higher tier markets.

A systematic study of the changes in market structure and their implications requires a substantial extension of our model, which is left for future work. Furthermore, several aspects of the model are stylized and their extensions are also possible directions for future research. For example the prediction technologies can be modeled in a more general way. Prediction could be imperfect; the state spaces could be more general instead of binary; shocks could follow a stochastic process along time periods; contracts could be dynamic; the states and/or information that retailers and suppliers obtain could be correlated.

References

Philippe Aghion, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt. Competition and innovation: An inverted-u relationship. *The Quarterly Journal of Economics*, 120(2):701–728, 2005.

Ajay Agrawal, Joshua Gans, and Avi Goldfarb. *Prediction Machines: The Simple Economics of Artificial Intelligence*. Harvard Business Review Press, USA, 2018. ISBN 1633695670, 9781633695672.

Ajay Agrawal, Joshua S Gans, and Avi Goldfarb. Artificial intelligence: The ambiguous labor market impact of automating prediction. Working Paper 25619, National Bureau of Economic Research, February 2019.

Pol Antras and C Fritz Foley. Poultry in motion: a study of international trade finance practices. *Journal of Political Economy*, 123(4):853–901, 2015.

Susan Athey. *The Impact of Machine Learning on Economics*, pages 507–547. University of Chicago Press, January 2018.

Yossi Aviv. The effect of collaborative forecasting on supply chain performance. *Management Science*, 47(10):1326–1343, 2001.

George Baker, Robert Gibbons, and Kevin J Murphy. Relational contracts and the theory of the firm. *The Quarterly Journal of Economics*, 117(1):39–84, 2002.

Nicholas Bloom and John Van Reenen. Measuring and explaining management practices across firms and countries. *The Quarterly Journal of Economics*, 122(4):1351–1408, 2007.

Simon Board. Relational contracts and the value of loyalty. *American Economic Review*, 101(7):3349–67, 2011.

Simon Board and Moritz Meyer-ter Vehn. Relational contracts in competitive labour markets. *The Review of Economic Studies*, 82(2):490–534, 2014.

Mariya Bondareva and Edieal Pinker. Dynamic relational contracts for quality enforcement in supply chains. *Management Science*, 65(3):1305–1321, 2018.

Clive Bull. The existence of self-enforcing implicit contracts. *The Quarterly journal of economics*, 102(1):147–159, 1987.

Ying-Ju Chen and Wenqiang Xiao. Impact of reseller's forecasting accuracy on channel member performance. *Production and Operations Management*, 21(6):1075–1089, 2012.

Morris A Cohen, Teck H Ho, Z Justin Ren, and Christian Terwiesch. Measuring imputed cost in the semiconductor equipment supply chain. *Management Science*, 49(12):1653–1670, 2003.

Jon Emont. A 2013 collapse killed 1,100 people. bangladesh factories are still unsafe - wsj. `https://www.wsj.com/articles/five-years-after-tragedy-bangladesh-factories-remain-unsafe-1522324800`, March 2018. (Accessed on 12/27/2019).

Morgan R. Frank, David Autor, James E. Bessen, Erik Brynjolfsson, Manuel Cebrian, David J. Deming, Maryann Feldman, Matthew Groh, José Lobo, Esteban Moro, Dashun Wang, Hyejin Youn, and Iyad Rahwan. Toward understanding the impact of artificial intelligence on labor. *Proceedings of the National Academy of Sciences*, 116(14):6531–6539,

2019. ISSN 0027-8424. doi: 10.1073/pnas.1900949116. URL https://www.pnas.org/content/116/14/6531.

Sang-Hyun Kim, Morris A Cohen, and Serguei Netessine. Performance contracting in after-sales service supply chains. *Management Science*, 53(12):1843–1858, 2007.

Jonathan Levin. Multilateral contracting and the employment relationship. *The Quarterly Journal of Economics*, 117(3):1075–1103, 08 2002.

Jonathan Levin. Relational incentive contracts. *American Economic Review*, 93(3):835–857, 2003.

Rocco Macchiavello and Ameet Morjaria. Competition and relational contracts in the Rwanda coffee chain, 2019.

W Bentley MacLeod and James M Malcomson. Reputation and hierarchy in dynamic models of employment. *Journal of Political Economy*, 96(4):832–854, 1988.

W Bentley MacLeod and James M Malcomson. Implicit contracts, incentive compatibility, and involuntary unemployment. *Econometrica: Journal of the Econometric Society*, pages 447–480, 1989.

W Bentley MacLeod and James M Malcomson. Motivation and markets. *American Economic Review*, pages 388–411, 1998.

James M Malcomson. Relational incentive contracts with productivity shocks. *Games and Economic Behavior*, 92:122–137, 2015.

James M Malcomson. Relational incentive contracts with persistent private information. *Econometrica*, 84(1):317–346, 2016.

F Warren McFarlan, William C Kirby, and Tracy Manty. Li & Fung 2006. *Havard University*, 2007.

Erica L Plambeck and Terry A Taylor. Partnership in a dynamic production system with unobservable actions and noncontractible output. *Management Science*, 52(10):1509–1527, 2006.

Carl Shapiro and Joseph E Stiglitz. Equilibrium unemployment as a worker discipline device. *The American Economic Review*, 74(3):433–444, 1984.

Terry A Taylor and Wenqiang Xiao. Does a manufacturer benefit from selling to a better-forecasting retailer? *Management Science*, 56(9):1584–1598, 2010.

H. Thorisson. *Coupling, Stationarity, and Regeneration*. Probability and Its Applications. Springer New York, 2000. ISBN 9780387987798. URL `https://books.google.com/books?id=0Kq6VMGVEUkC`.

Hal Varian. Artificial intelligence, economics, and industrial organization. Working Paper 24839, National Bureau of Economic Research, July 2018. URL `http://www.nber.org/papers/w24839`.

APPENDIX

A Observation: Choice of effort by shirking suppliers

We use the following observation in the subsequent proofs.

Observation 5. *As $c_{\mathcal{S}}(\cdot), \mathcal{S} \in \{H, L\}$ are strictly decreasing and the penalty for shirking is independent of the effort exerted, a supplier that shirks always chooses to exert no effort.*

Proof of Observation 5. Assume that the contract prescribes that the supplier exert e^* effort. If the supplier chooses to shirk and exert effort $e' < e^*$, he may or may not be fired depending on whether deviation is detected. Let U be the expected payoff of an unmatched supplier who is fired when his shirking is detected. Note that U is independent of the deviation effort e' of the supplier, as the penalty for shirking is independent of the effort exerted. In addition, let Γ denote the supplier's expected payoff following an undetected deviation. Note that Γ itself still captures non-shirking related reasons for the contract breaking, i.e., due to exogenous reasons or due the retailer's predictions, but that it is also independent of e'.

The supplier's expected payoff from shirking to effort e' is therefore given by

$$w - c_{\mathcal{S}}(e') + \alpha\delta U + (1-\alpha)(b + \delta\Gamma) \ ,$$

when a supplier can predict, and by

$$w - \bar{c}(e') + \alpha\delta U + (1-\alpha)(b + \delta\Gamma) \ ,$$

for a supplier without prediction.

As $c_{\mathcal{S}}, \mathcal{S} \in \{H, L\}$ and $\bar{c}$ are all non-increasing in e', a deviating supplier's expected shirking utility is maximized when exerting no effort. □

B Missing proofs from Section 3 (Case I)

In this appendix we prove Lemma 1, which relies on the following proposition. Set $\Psi_p := \frac{1-\delta(1-p)}{\alpha\delta(1-p)}$ and $\Omega_p := \frac{\delta p R}{(1-\delta)(S-R)+\delta p S}$.

Proposition 1. *If neither the retailer or the supplier can predict future states, the following hold.*

(i) The retailer will choose $b = 0$.

(ii) The rematch probability for unmatched suppliers is given by $q := \frac{pR}{S-R}$.

(iii) Let $\gamma_x := \frac{1-\delta}{\delta x} + 1$. *The suppliers' expected utility at the beginning of a period when being matched and unmatched, under a symmetric market contract with effort and wage* $\{e^*, w^*\}$, *are*

$$E^* = \frac{\gamma_q}{\delta p} \cdot \frac{w^* - \bar{c}(e^*)}{\gamma_p \gamma_q - 1}, \qquad U^* = \frac{1}{\delta p} \cdot \frac{w^* - \bar{c}(e^*)}{\gamma_p \gamma_q - 1} \ .$$

(iv) For any given choice of effort and wage $\{e, w\}$ *and assumed symmetric contract* $\{e^*, w^*\}$ *elsewhere in the market, the suppliers' no shirking constraint (NSC) is*

$$w \geq (1 + \Psi_p)\, \bar{c}(e) + \Omega_p(w^* - \bar{c}(e^*)) \ .$$

(v) Let E^I *be the expected payoff of matched suppliers in contract* $\{e, w\}$. *Then, given that the (NSC) of Part (iv) is satisfied, the suppliers' participation constraints will be satisfied as well; that is, the suppliers choose to participate in the market (i.e.* $U^* \geq 0$*), and they do not quit (i.e.* $E^I \geq U^*$*).*

(vi) The retailers' expected utility is $\bar{v}(e) - w$.

Proof of Proposition 1. Part (i): We start with the retailer's decision to pay a bonus. We have assumed that $S \geq 2R$, so that all retailers can find a new supplier immediately after match breaks. Let Π^{cont} be the retailer's expected profit at the beginning of the period when continuing her current match, and Π^{new} be her expected profit when starting with a new match. Then, from paying the bonus and continuing the current match, the retailer gets

$$\bar{v}(e) - w - b + \delta(p\Pi^{\mathsf{new}} + (1-p)\Pi^{\mathsf{cont}})$$

On the other hand, from retaining the bonus, the retailer gets

$$\bar{v}(e) - w + \delta\Pi^{\mathsf{new}}$$

Therefore, the retailer will pay the bonus if and only if

$$\delta(1-p)(\Pi^{\mathsf{cont}} - \Pi^{\mathsf{new}}) \geq b \ . \tag{6}$$

As retailers can find a match immediately, and all new matches have the same expected profit as existing matches, we have $\Pi^{\mathsf{new}} = \Pi^{\mathsf{cont}}$. Together with (6), this implies that $b = 0$.

Parts (ii)-(iv): We now evaluate the suppliers' incentives. At any given time, a supplier may be either matched or unmatched. Let the expected utility for matched and unmatched suppliers at the beginning of each period in this case (i.e., when there is no prediction) be E^I and U^I, respectively. Note that E^I depends on the contract offered by a supplier's current retailer, while U^I depends on contracts offered elsewhere in the market. A matched supplier who exerts effort e has an expected utility of

$$u(e) := w - \bar{c}(e) + \delta(pU^I + (1-p)E^I) \ . \tag{7}$$

From Observation 5, if the supplier shirks, it will be to effort zero, and his expected utility is

$$u(0) := w + \delta(1-\alpha)\left(pU^I + (1-p)\,E^I\right) + \alpha\left(\delta U^I\right) \ .$$

Thus, the supplier's *incentive constraint* for exerting the effort e prescribed in the contract) requires that $u(e) \geq u(0)$, which reduces to

$$\alpha\delta(1-p)\left(E^I - U^I\right) \geq \bar{c}(e). \tag{8}$$

Part (ii): Now, assume the incentive constraint (8) is satisfied and denote the expected rematch probability by q. Existing contracts will only break due to exogenous reasons, creating vacancies at a rate pR. Given that there are exactly $S - R$ available unmatched suppliers from the previous period as well, and that retailers with vacancies pick at random from the pool of unmatched suppliers, $q = \frac{pR}{S-R}$.

Part (iii):. Recall that our goal is to find the symmetric market equilibrium contract. Denote the contract offered by the retailer by (e, w), and the assumed symmetric contract offered by other retailers in the market by (e^*, w^*).

Let E^I be the expected utility of a supplier matched in contract (e, w), and let U^* and E^* be the expected utility of unmatched and matched suppliers, respectively, in the assumed symmetric market contract (e^*, w^*).

Given that there is a continuum of retailers and the probability of matching to contract (e, w) is negligible, the utility of an unmatched supplier is:

$$U^* = \delta(qE^* + (1-q)\,U^*) \tag{9}$$

Let $\gamma_x := \frac{1-\delta}{\delta x} + 1$. Then, from (7) and (9), we get $E^* = \gamma_q U^*$.

As the contract (e^*, w^*) must also satisfy the incentive constraint (Equation (8) rewritten with E^* and U^*), a supplier's expected utility from being matched to contract (e^*, w^*) is then exactly $u(e^*)$ as given in (7), and using $E^* = \gamma_q U^*$, we get

$$U^* = \frac{1}{\delta p} \cdot \frac{w^* - \bar{c}(e^*)}{\gamma_p \gamma_q - 1} \ . \tag{10}$$

As $U^I = U^*$, from (7), for contract (e, w) we have

$$E^I = w - \bar{c}(e) + \delta(pU^* + (1-p)E^I)$$

Therefore,

$$E^I = \frac{w - \bar{c}(e) + \delta p U^*}{\delta p \gamma_p} \tag{11}$$

Part (iv): Substituting for U^* from (10) and E^I in (11), we get

$$E^I - U^* = \frac{1}{\delta p \gamma_p} \left(w - \bar{c}(e) - \frac{\gamma_p - 1}{\gamma_p \gamma_q - 1} (w^* - \bar{c}(e^*)) \right) \tag{12}$$

Substituting for the above in the incentive constraint (8), we get the supplier's *non-shirking constraint (NSC)*[11]

$$w \geq \left(\frac{1 - \delta(1-\alpha)(1-p)}{\delta\alpha(1-p)} \right) \bar{c}(e) + \frac{\delta p R}{(1-\delta)(S-R) + \delta p S} (w^* - \bar{c}(e^*)) \ . \tag{13}$$

Given that there cannot be a bonus, this NSC provides a lower bound on the wage required to keep suppliers from shirking.

Part (v): We now note that even if the supplier does not shirk *if* he participates, this does not guarantee that the supplier will indeed participate. This is guaranteed if $E^I \geq U^* \geq 0$; that is, the supplier does not quit when employed and does not prefer to not participate over being in the market. We verify that both of these inequalities hold, thus ensuring the suppliers' participation. The first inequality is already guaranteed by the NSC. By Part (iii), $U^* \geq 0$ is equivalent to having $w^* \geq \bar{c}(e^*)$, which holds by the NSC as well, taking $w = w^*$ and $e = e^*$.

Part (vi): Retailers are matched in every period. The retailer pays only a wage w,

[11] We use the following easily verifiable equality:

$$\frac{\gamma_p - 1}{\gamma_p \gamma_q - 1} = \frac{\frac{\delta p R}{S-R}}{1 - \delta + \frac{\delta p S}{S-R}}.$$

and derives value $\bar{v}(e)$ from the supplier's effort, resulting in the total expected utility of $\bar{v}(e) - w$. □

Proof of Lemma 1. We turn to the choice of the wage and effort. The retailer's goal is to maximize her expected profit, while satisfying the supplier's no shirking constraint (13). Given Proposition 1, the optimal contract will be the solution to

$$\begin{aligned} \max_{e,w} \quad & \bar{v}(e) - w \\ \text{s.t.} \quad & w \geq (1 + \Psi_p)\bar{c}(e) + \Omega_p(w^* - \bar{c}(e^*)) \ , \end{aligned} \tag{14a}$$

where $\{e^*, w^*\}$ is the assumed contract offered elsewhere in the market. As the revenue is maximized when the inequality is tight and because e^* and w^* are determined exogenously, the last term in Equation (14a) is a constant, the program can be rewritten as follows:

$$\begin{aligned} e :=& \arg\max_{e'} \quad \bar{v}(e') - (1 + \Psi_p)\bar{c}(e') \\ w =& (1 + \Psi_p)\bar{c}(e) + \Omega_p(w^* - \bar{c}(e^*)) \ . \end{aligned}$$

At equilibrium $w = w^*$, $e = e^*$, giving Program 1, where

$$\Phi_p := \frac{\Psi_p}{1 - \Omega_p} = \frac{1}{\alpha\delta(1-p)} \left(1 - \delta + \delta p \frac{S}{S - R}\right) .$$

□

C Missing proofs from Section 4 (Case II)

C.1 Proof of Lemma 2

Proof of Lemma 2. Assume towards a contradiction that the supplier enters into a contract with the promise not to use prediction, and that the retailer offers a contract that stipulates an effort e^p and wage w^p; this would be the contract of Case I. If the supplier deviates and uses his prediction ability at time t and realizes that time $t + 1$ has cost c_H, he can choose to (i) quit at time t and enter the unemployment pool, or (ii) stay in the contract and shirk, or (iii) stay in the contract and exert e^p. We compare options (ii) and (iii). The expected utility from not using predictions/ignoring the prediction and exerting effort e^p at time $t+1$ is $w^p - c_H(e^p) + \delta(pU^p + (1-p)E^p)$. The expected utility from using prediction and shirking is $w^p + \alpha\delta U^p + (1-\alpha)\delta(pU^p + (1-p)E^p)$. It holds that

$$
\begin{aligned}
&\alpha\delta U^p + (1-\alpha)\delta(pU^p + (1-p)E^p) - (-c_H(e^p) + \delta(pU^p + (1-p)E^p) \\
=&\alpha\delta U^p - \alpha\delta(pU^p + (1-p)E^p) + c_H(e^p) \\
=&c_H(e^p) - \bar{c}(e^p) > 0 \qquad (15)
\end{aligned}
$$

where Equation (15) holds because at this promised contract the (IC) in Equation (8) binds, so that $\alpha\delta(1-p)(E^p - U^p) = \bar{c}(e^p)$. Together, this implies that a supplier with the ability to predict will use his predictions and benefit from shirking when predicting cost c_H. Therefore, the suppliers can not credibly promise not to use their predictions. □

C.2 Proof of Lemma 3

First, we show that suppliers never break a contract that does not offer a bonus. When the supplier can predict the future state, he has an option to voluntarily break the contract and enter the unmatched pool when predicting a high cost state. We show that shirking always leads to a higher expected utility than quitting. Therefore, as retailers cannot credibly offer a bonus (similarly to Case I, they can always renege on the bonus and be matched to an identical supplier), it will suffice for the retailers to only consider contracts in which suppliers never break contracts.

Lemma 9. *Suppliers will not voluntarily break any contract in which no bonus is offered when predicting a high state.*

Proof. Let $\{e, w\}$ be a contract, and let U^{II} and E^{II} be the expected utility of unmatched and matched suppliers in contract $\{e, w\}$ respectively. In Case II, if a supplier quits when predicting a high state, this must imply that the utility of being unemployed, U^{II}, satisfies $U^{II} \geq u(e, c_H)$ for any $e \geq 0$, where $u(e, c_H)$ denotes the supplier's utility of exerting effort e in the next time period when the cost is high. If $E^{II} < U^{II}$, the supplier would not have entered the contract and therefore cannot break it. We therefore assume that $E^{II} \geq U^{II}$. We show by coupling[12] future events that for any future time, the supplier will have been better off not quitting.

Assume that the supplier predicts that the cost state of time t will be high. The supplier's utility from time period t if is 0 if he quits, and w if he shirks (as $c_H(0) = 0$). As $E^{II} \geq U^{II}$, the supplier's utility if caught and fired is less than if he is not fired, so we can assume w.l.o.g. that the supplier was fired. Thus, we only need to compare the two scenarios: when

[12] See e.g., Thorisson [2000] for an introduction to coupling in probability theory.

the supplier becomes unemployed at time t and when the supplier receive a wage w and shirks at time t and becomes unemployed at time $t+1$. As the equilibrium is stationary, the probability of future events is identical, hence we can couple the future events in a straightforward manner. As the maximum utility gain from any time period is w, we have that the total utility for not quitting is always at least as high as for quitting at any time period. □

Lemma 3 follows directly from the following proposition, in similar fashion to the Proof of Lemma 1. In the proposition, we assume that suppliers never quit, which we can do due to Lemma 9.

Proposition 2. *Set* $\Psi_p := \frac{1-\delta(1-p)}{\alpha\delta(1-p)}$ *and* $\Omega_p := \frac{\delta p R}{(1-\delta)(S-R)+\delta p S}$. *If only the suppliers can predict their cost for one step ahead, the following hold.*

(i) The retailer will choose $b = 0$.

(ii) The rematch probability for unmatched suppliers is given by $q := \frac{pR}{S-R}$.

(iii) Let $\gamma_x := \frac{1-\delta}{\delta x} + 1$. *The suppliers' expected utility at the beginning of a period when being matched and unmatched, under a symmetric market contract with effort and wage* $\{e^*, w^*\}$, *are*

$$E^* = \frac{\gamma_q}{\delta p} \cdot \frac{w^* - \bar{c}(e^*)}{\gamma_p \gamma_q - 1}, \qquad U^* = \frac{1}{\delta p} \cdot \frac{w^* - \bar{c}(e^*)}{\gamma_p \gamma_q - 1} \ .$$

(iv) For any given choice of effort and wage $\{e, w\}$ *and assumed symmetric contract* $\{e^*, w^*\}$ *elsewhere in the market, the suppliers' no shirking constraint is*

$$w \geq \bar{c}(e) + \Psi_p c_H(e) + \Omega_p (w^* - \bar{c}(e^*)) \ .$$

(v) Let E^{II} *be the expected payoff of matched suppliers in contract* $\{e, w\}$. *Then, given that the (NSC) of Part (iv) is satisfied, the suppliers' participation constraints will be satisfied as well; that is, the suppliers choose to participate in the market (i.e.* $U^* \geq 0$*), and they enter into the contract (i.e.* $E^{II} \geq U^*$*).*

(vi) The retailers' expected utility is $\bar{v}(e) - w$.

Proof of Proposition 2. The proofs of Parts *(i)*, *(ii)*, and *(vi)*, are identical to that of Proposition 1, as the supplier's cost (predictions) do not affect the arguments. Proofs for Parts *(iii)*, *(iv)* and *(v)* are as follows.

Let E^{II} and U^{II} denote the expected payoffs of matched and unmatched suppliers at the beginning of a given period. Note that E^{II} depends on the contract offered by a supplier's current retailer, while U^{II} depends on contracts offered elsewhere in the market. We analyze what happens when the supplier predicts the state will be c_H. From exerting effort, the supplier expects a payoff of

$$u(e, c_H) := w - c_H(e) + \delta \left(pU^{II} + (1-p)E^{II} \right) \ . \tag{16}$$

By Observation 5, a shirking supplier will exert no effort, and expects a payoff of

$$u(0, c_H) := w + \alpha\delta U^{II} + (1-\alpha) \left(\delta \left(pU^{II} + (1-p)E^{II} \right) \right) \ .$$

Therefore the incentive compatibility (IC) constraint of the supplier, $u(e, c_H) \geq u(0, c_H)$, reduces to

$$\alpha\delta(1-p) \left(E^{II} - U^{II} \right) \geq c_H(e) \ . \tag{17}$$

Note that as $c_H(e) \geq c_L(e)$, this IC implies the supplier is also incentivized to put in effort when the state is c_L, and when the state is unknown (as is the case with new contracts).

Denote the contract offered by the retailer by (e, w), and the assumed symmetric contract offered by other retailers in the market by (e^*, w^*). Recall that E^{II} is the expected utility of suppliers matched to contract (e, w), and let U^* and E^* be the expected utility of unmatched and matched suppliers, respectively, to the assumed symmetric market contracts (e^*, w^*).

Given the assumed symmetric contract (e^*, w^*) elsewhere in the market, and that the probability of matching to contract (e, w) is negligible, similar to Part (ii) of Proposition 1, we get $E^* = \gamma_q U^*$, where $\gamma_x := \frac{1-\delta}{\delta x} + 1$.

Under (17), a matched supplier will not shirk, and will have an (ex-ante) expected utility of $u(e^*, \bar{c})$, as defined in (16), when matched to to contract (e^*, w^*). Then,

$$U^* = \frac{1}{\delta p} \cdot \frac{w^* - \bar{c}(e^*)}{\gamma_p \gamma_q - 1} \ . \tag{18}$$

Further, for contract (e, w), the (ex-ante) expected utility is $u(e, \bar{c})$ as defined in (16), i.e.,

$$E^{II} = w - \bar{c}(e) + \delta(pU^* + (1-p)E^{II})$$

Therefore,

$$E^{II} = \frac{w - \bar{c}(e) + \delta p U^*}{\delta p \gamma_p} \tag{19}$$

For contract (e, w), given outside contracts (e^*, w^*), the incentive constraint (17) becomes

$$\alpha\delta(1-p)\left(E^{II} - U^*\right) \geq c_H(e) \ .$$

Substituting for U^* from (18) and E^{II} from (19) in the above gives the supplier's no-shirking constraint (NSC)

$$w \geq \bar{c}(e) + \left(\frac{1-\delta(1-p)}{\delta\alpha(1-p)}\right) c_H(e) + \frac{\delta p R}{(1-\delta)(S-R)+\delta p S}(w^* - \bar{c}(e^*)) \ .$$

Part (v): The proof that $E^{II} \geq U^* \geq 0$ is identical to the proof of Proposition 1. In addition, we need to verify that the supplier will not choose to quit when predicting that the future state is high. We prove this is Lemma 9. □

C.3 Proof of Theorem 1

Proof of Theorem 1. The proof of the first and second points is visualized in Figure 2: The derivatives of the value function and the wage functions for Cases I and II, as well as the derivative of the average cost function, are plotted. For the first point, note that as $\bar{v}$, $\bar{c}$, and c_H are all increasing, their derivatives are all non-negative; $\bar{v}$ is concave, hence its derivative is non-increasing; the derivatives of $\bar{c}$ and c_H are non-decreasing. As $c_H'(e) \geq c_L'(e)$, $\bar{c}'(e) + \Psi_p c_H'(e) \geq (1+\Psi_p)\bar{c}'(e)$; therefore this curve must intersect $\bar{v}'(e)$ at a lower value of e than in Case I. For the second point: the retailer's utility is shaded with vertical lines for Case II and with horizontal lines for Case I. It is easy to see that this area w.r.t Case II is contained within the area w.r.t. Case I. An example for the third point is given in Figure 3, using linear retailer value functions and quadratic costs. □

D Missing proofs from Section 5 (Case III)

D.1 Proof of Lemma 4

Proof of Lemma 4. Assume towards a contradiction that the retailer can enter into a contract C with the promise not to use prediction and the new contract stipulates an effort e^p, wage w^p, and bonus b^p. As the market is a continuum of identical suppliers and the number of suppliers is larger than the number of retailers, the retailer will always be able to secure the same contract, with a different supplier once the contract is broken. First, note that when a contract begins in the next period or if the contract is ongoing and the next state is v_H, the retailer's utility is the same for the next state regardless of whether she uses prediction. Assume now that the next state is v_L. If the retailer does not use predictions or predicts v_L and does not fire the supplier, her utility for the next period will be $v_L(e^p) - w^p - b^p$. If she predicts that the next state will be v_L and fires the supplier, her utility for the next period is $\bar{v}(e^p) - w^p - b^p$, which is strictly higher if $\bar{v} > v_L$, which we assume to hold. Therefore, her utility for the next state is strictly higher when using prediction and firing when predicting the state will be v_L than either not using prediction or not firing when the state is predicted to be v_L. □

D.2 Proof of Lemma 5

To prove Lemma 5, we use the following proposition.

Proposition 3. *Denote* $p^\dagger = 1 - \beta_v(1-p)$, $\Psi_{p^\dagger} = \frac{1-\delta(1-p^\dagger)}{\alpha\delta(1-p^\dagger)}$ *and* $\Omega_{p^\dagger} = \frac{\delta p^\dagger R}{(1-\delta)(S-R)+\delta p^\dagger S}$. *If only the retailer can predict her state for the next time period, and she fires the supplier whenever she predicts* v_L*, the following hold.*

(i) The rematch probability for unmatched suppliers is given by $q^\dagger := \frac{p^\dagger R}{S-R}$.

(ii) Let $\gamma_x := \frac{1-\delta}{\delta x}+1$. *The suppliers' expected utility from being matched and unmatched in a symmetric market equilibrium contract with effort, bonus, and wage given by* $\{e^*, w^*, b^*\}$ *is*

$$E^* = \frac{\gamma_{q^\dagger}}{\delta p^\dagger} \cdot \frac{w^* - \bar{c}(e^*) + b^*}{\gamma_{p^\dagger}\gamma_{q^\dagger} - 1} \ , \quad U^* = \frac{1}{\delta p^\dagger} \cdot \frac{w^* - \bar{c}(e^*) + b^*}{\gamma_{p^\dagger}\gamma_{q^\dagger} - 1} \ ,$$

(iii) For any contract $\{e, w, b\}$*, and assumed symmetric contract* $\{e^*, w^*, b^*\}$ *in the rest of the market, the suppliers' no-shirking constraint is*

$$w \geq \bar{c}(e) - b + \Psi_{p^\dagger}\left(\bar{c}(e) - \alpha b\right) + \Omega_{p^\dagger}(w^* + b^* - \bar{c}(e^*)) \ .$$

(iv) Let E^{III} *be the expected payoff of matched suppliers in contract* $\{e, w, b\}$. *The contract should be chosen such that the suppliers ex-ante prefer the contract to being unemployed (i.e.* $E^{III} \geq U^*$*). For any contract* $\{e, w, b\}$*, and assumed symmetric contract* $\{e^*, w^*, b^*\}$*, this is given by*

$$w + b - \bar{c}(e) \geq \Omega_{p^\dagger}\left(w^* + b^* - \bar{c}(e^*)\right) \ .$$

In addition, the suppliers should choose to participate in the market (i.e. $U^* \geq 0$). For this, the symmetric market equilibrium $\{e^*, w^*, b^*\}$ should satisfy

$$w^* + b^* - \bar{c}(e^*) \geq 0 \ .$$

(v) The retailer chooses the bonus such that

$$b \leq \delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)),$$

where e is the effort stipulated by the contract.

(vi) For any contract $\{e, w, b\}$, the retailers' expected utility is given by

$$v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - w - b.$$

Proof of Proposition 3. We begin with the suppliers' incentives. Let E^{III} and U^{III} be the expected payoff for suppliers of matched and unmatched suppliers, respectively, at the beginning of the period. Note that as the supplier and retailer states are independent, the supplier has the same expected payoff E^{III} whether continuing in an existing match or recently matched. In addition, U^{III} is a function only of the contract offered elsewhere in the market.

If the supplier does not shirk, the contract will continue only if the next state is predicted to be high (w.p. β_v), and the contract is not terminated due to exogenous reasons (w.p. $1-p$). Set $p^\dagger = 1 - \beta_v(1-p)$. Note this is exactly the probability that the contract breaks in this scenario. Therefore, the supplier's expected payoff from exerting the prescribed effort e is

$$u(e) := w + b - \bar{c}(e) + \delta\left(p^\dagger U^{III} + (1-p^\dagger)E^{III}\right) \ . \tag{20}$$

By Observation 5, a shirking supplier exerts no effort, and therefore has an expected payoff of

$$u(0) := w + \alpha\delta U^{III} + (1-\alpha)\left(b + \delta\left(p^\dagger U^{III} + (1-p^\dagger)E^{III}\right)\right) \ .$$

Thus, the supplier's incentive compatibility constraint (for exerting effort e) is

$$\alpha\left(b + \delta(1-p^\dagger)\left(E^{III} - U^{III}\right)\right) \geq \bar{c}(e) \ . \tag{21}$$

Part (i): Assuming the suppliers' IC constraints (21) are satisfied in the offered contract, retailers will have vacancies during matching from two sources: low future state predictions with rate $(1-\beta_v)R$, and if not, exogenous reasons creating vacancies at a rate $\beta_v pR$. That is, at equilibrium, $p^\dagger R$ vacancies are created, leading to the probability of rematch of unmatched suppliers being $q^\dagger = \frac{p^\dagger R}{S-R}$.

Part (ii): Denote the contract offered by the retailer by (e, w, b), and the assumed symmetric contract offered by other retailers in the market by (e^*, w^*, b^*). We now determine the best-response (e, w, b) to (e^*, w^*, b^*).

Let E^{III} be the expected utility of suppliers matched to contract (e, w, b), and let U^* and E^* be the expected utility of unmatched and matched suppliers, respectively, to the assumed symmetric market contracts (e^*, w^*, b^*).

Assume the wage, effort, and bonus in both contracts are such that the corresponding incentive constraints (21) are satisfied, and thus the expected rematch probability is given by $q^\dagger$. Then a matched supplier will not shirk in either new or existing contracts.

Given the assumed symmetric market contract (e^*, w^*, b^*), and that the probability of matching to contract (e, w, b) is negligible, the expected utility from being unmatched is

$$U^* = \delta(q^\dagger E^* + (1 - q^\dagger)U^*) \ , \tag{22}$$

where the supplier's expected utility E^* from being matched is given by (20) evaluated at (e^*, w^*, b^*). Let $\gamma_x := \frac{1-\delta}{\delta x} + 1$. Then we get $E^* = \gamma_{q^\dagger} U^*$, and

$$U^* = \frac{1}{\delta p^\dagger} \cdot \frac{w^* + b^* - \bar{c}(e^*)}{\gamma_{p^\dagger}\gamma_{q^\dagger} - 1} \ . \tag{23}$$

From (16), the following holds for contract (e, w, b):

$$E^{III} = w + b - \bar{c}(e) + \delta(p^\dagger U^* + (1 - p^\dagger)E^{III}) \ .$$

Therefore,

$$E^{III} = \frac{w + b - \bar{c}(e) + \delta p^\dagger U^*}{\delta p^\dagger \gamma_{p^\dagger}} \ . \tag{24}$$

Evaluating (24) at the equilibrium $(e, w, b) = (e^*, w^*, b^*)$ leads to the expression of E^{III} in part (ii) of the statement of the proposition.

Part (iii): For contract (e, w, b), the incentive constraint (21) is

$$\alpha\left(b + \delta(1 - p^\dagger)\left(E^{III} - U^*\right)\right) \geq \bar{c}(e) \ . \tag{25}$$

Substituting for U^* from (23) and E^{III} from (24) in the incentive constraint (25) gives the supplier's no-shirking constraint (NSC)

$$w \geq \bar{c}(e) - b + \left(\frac{1-\delta(1-p^\dagger)}{\delta\alpha(1-p^\dagger)}\right)(\bar{c}(e) - \alpha b) + \frac{\delta p^\dagger R}{(1-\delta)(S-R)+\delta p^\dagger S}(w^* + b^* - \bar{c}(e^*)) \ . \tag{26}$$

Part (iv): Next, we note that the retailers need to ensure that the supplier's participation constraints are satisfied in that they expect to gain from participation in their contract as opposed to quitting and joining the unmatched pool, i.e., that $E^{III} \geq U^*$ for the contract (e, w, b) they choose to offer, given the contract (e^*, w^*, b^*) elsewhere in the market. From (24), this is equivalent to having $w + b - \bar{c}(e) - (\gamma_{p^\dagger} - 1)\delta p^\dagger U^* \geq 0$, where U^* is given in (23). The participation constraint therefore reduces to $w + b - \bar{c}(e) \geq \frac{\gamma_{p^\dagger} - 1}{\gamma_{p^\dagger}\gamma_{q^\dagger} - 1}(w^* + b^* - \bar{c}(e^*))$. The retailer further knows that for contract (e^*, w^*, b^*), $U^* \geq 0$, which implies the parameters are chosen so that $w^* + b^* - \bar{c}(e^*) \geq 0$.

Part (v): We next consider the retailers' incentives, starting with incentives to pay the bonuses. Recall that we assume $S \geq 2R$. Let Π^{cont} be the retailer's expected profit at the beginning of the period when continuing her current match, and Π^{new} be her expected profit when starting a new match.

Denote the utility from the current period by ξ: if the contract is ongoing, $\xi = v_H(e)$, otherwise $\xi = \bar{v}(e)$. Then from paying the bonus, the retailer gets

$$\xi - w - b + \delta\left((1-p^\dagger)\Pi^{\mathsf{cont}} + p^\dagger\Pi^{\mathsf{new}}\right) \ . \tag{27}$$

On the other hand, from withholding the bonus, the retailer gets

$$\xi - w + \delta\Pi^{\mathsf{new}}$$

Therefore (regardless of the value of ξ) the retailer will pay the bonus if and only if

$$b \leq \delta(1-p^\dagger)(\Pi^{\mathsf{cont}} - \Pi^{\mathsf{new}}) \ . \tag{28}$$

Now, assume the bonus is selected such that (28) is satisfied. Then Π^{cont} and Π^{new} are equivalent to (27) with $\xi = v_H(e)$ and $\xi = \beta_v v_H(e) + (1-\beta_v)v_L(e)$ respectively. Then $\Pi^{\mathsf{cont}} - \Pi^{\mathsf{new}} = (1-\beta_v)(v_H(e) - v_L(e))$, and (28) reduces to,

$$b \leq \delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)) \ . \tag{29}$$

Part (vi): The retailer's goal is to maximize her expected profit. To determine the retailer's expected profit, note that the retailer's contracts break with probability $p^\dagger$, either due to low state predictions or exogenous reasons. Therefore, the retailer's expected profit is given by

$$(1-p^\dagger)\Pi^{\mathsf{cont}} + p^\dagger \Pi^{\mathsf{new}} = \frac{1}{1-\delta}\left(v_H(e) - w - b - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right)\right)$$

□

Proof of Lemma 5. We now turn to the optimal choice of the wage, bonus, and effort. Putting all the constraints from Proposition 3 together, the contracts' wage, bonus, and effort, are determined by

$$\begin{aligned}
\max_{e,w,b} \quad & v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - w - b \\
\text{s.t.} \quad & w \geq \bar{c}(e) - b + \Psi_{p^\dagger}\left(\bar{c}(e) - \alpha b\right) + \Omega_{p^\dagger}(w^* + b^* - \bar{c}(e^*)) && (30)\\
& w \geq \bar{c}(e) - b + \Omega_{p^\dagger}\left(w^* + b^* - \bar{c}(e^*)\right) && (31)\\
& b \leq \delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e))
\end{aligned}$$

where $\Psi_{p^\dagger} := \frac{1-\delta(1-p^\dagger)}{\alpha\delta(1-p^\dagger)}$ and $\Omega_{p^\dagger} := \frac{\delta p^\dagger R}{(1-\delta)(S-R)+\delta p^\dagger S}$.

For any fixed outside contract (e^*, w^*, b^*), the optimal effort is determined by the maximal solution of two programs, depending on which of the constraints (30) or (31) ends up binding. We can reduce these two programs to the following:

$$\begin{aligned}
\max_{e,b} \quad & v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - \bar{c}(e) - \Psi_{p^\dagger}\left(\bar{c}(e) - \alpha b\right) \\
\text{s.t.} \quad & b \leq \delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)) \\
& b \leq \frac{\bar{c}(e)}{\alpha}\ ,
\end{aligned}$$

and,

$$\begin{aligned}
\max_{e,b} \quad & v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - \bar{c}(e) \\
\text{s.t.} \quad & b \leq \delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)) \\
& b \geq \frac{\bar{c}(e)}{\alpha}\ .
\end{aligned}$$

In the first program, the retailer would wish to maximize b; hence she would set it to the minimum of $\delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e))$ and $\frac{\bar{c}(e)}{\alpha}$. In the second program, the objective function does not depend on the value of b, so we can arbitrarily set it so that one constraint is binding.

The bonus and effort are therefore determined by

$$\begin{aligned}
\max_{e,b} \quad & v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - \bar{c}(e) - \Psi_{p^\dagger}\left(\bar{c}(e) - \alpha b\right) \\
\text{s.t.} \quad & b = \min\left\{\delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)), \frac{\bar{c}(e)}{\alpha}\right\}
\end{aligned}$$

The wage is set according to either inequality (30) or (31), depending on the value of b, leading to

$$w = \bar{c}(e) - b + \Phi_{p^\dagger} \max\{\bar{c}(e) - \alpha b, 0\}$$

where $\Phi_{p^\dagger} := \frac{\Psi_{p^\dagger}}{1-\Omega_{p^\dagger}} = \frac{1}{\alpha\delta(1-p^\dagger)}\left(1 - \delta + \delta p^\dagger \frac{S}{S-R}\right)$. □

D.3 Proof of Theorem 2

Proof of Theorem 2. Part (i): Fix all of the parameters except β_v. Denote by $e^\emptyset$ the minimal effort stipulated by Program 1 over all choices of β_v; $e^\emptyset$ provides a lower bound on the effort in the no prediction case. Let $\Lambda = \max_{e \geq e^\emptyset} \frac{v_H(e)}{\bar{c}(e)}$. Find β_v^* such that

1. $\beta_v^* v_H(e) \leq \frac{\bar{c}(e)}{2}$ for all $e \geq e^\emptyset$. To see why such β_v^* exists, note that $\frac{v_H}{\bar{c}}$ is maximized at $e^\emptyset$, and is non-increasing for $e > e^\emptyset$ (the latter can be verified given that v_H is concave, $\bar{c}$ is convex, and $e^\emptyset > 0$). The condition also holds for any $\beta_v \leq \beta_v^*$.

2. $\Phi_{p^\dagger} \geq 2\Lambda$. Recall that $p^\dagger = 1 - \beta_v(1-p)$ and $\Phi_{p^\dagger} := \frac{1}{\alpha\delta(1-p^\dagger)}\left(1 - \delta + \frac{\delta p^\dagger S}{S-R}\right)$. Such β_v^* exists because $\Phi_{p^\dagger}$ is increasing in $p^\dagger$, and $p^\dagger$ is decreasing in β_v. The condition also holds for any $\beta_v \leq \beta_v^*$.

Set β_v^* to be the minimum of these two values.

Recall that by the condition on the optimal bonus in (3b) we can obtain the following upper bound on the bonus:

$$\begin{aligned}
b &\leq \delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)) \\
&= \delta(1-\beta_v)\beta_v(1-p)(v_H(e) - v_L(e)) \leq \beta_v(v_H(e) - v_L(e)).
\end{aligned}$$

Note also that condition 1 above implies that $\beta_v^*\left(v_H\left(e^\emptyset\right) - v_L\left(e^\emptyset\right)\right) \leq \frac{\bar{c}(e^\emptyset)}{2}$. Therefore $\hat{b}\left(e^\emptyset\right)$, the bonus offered by the Case III contract if the stipulated effort is $e^\emptyset$, satisfies $\hat{b}\left(e^\emptyset\right) \leq \frac{\bar{c}(e^\emptyset)}{2}$.

By (3c), we have $w = \bar{c}(e) - b + \Phi_{p^\dagger}\max\{\bar{c}(e) - \alpha b, 0\}$. As a result, using $\hat{b}\left(e^\emptyset\right) \leq \frac{\bar{c}(e^\emptyset)}{2}$ and $\alpha \leq 1$, we get $\hat{w}(e^\emptyset) \geq \frac{\Phi_{p^\dagger}\bar{c}(e)}{2}$, where $\hat{w}(e^\emptyset)$ is the wage offered in Case III for effort $e^\emptyset$. This, together with condition 2 above, implies that $\hat{w}(e^\emptyset) \geq v_H(e^\emptyset)$.

Now, using Part *(vi)* of Proposition 3, the retailers' expected utility for effort $e^{\emptyset}$ is

$$v_H(e^{\emptyset}) - p^{\dagger}(1-\beta_v^*)\left(v_H(e^{\emptyset}) - v_L(e^{\emptyset})\right) - w(e^{\emptyset}) - b(e^{\emptyset}) \le v_H(e^{\emptyset}) - w(e^{\emptyset}).$$

This means that the retailer's utility for $e^{\emptyset}$ is strictly negative. As $v_H - v_L$ is non-increasing, $\hat{b}\,(e) \le \hat{b}\left(e^{\emptyset}\right)$ for $e \ge e^{\emptyset}$, hence $\hat{w}(e) \ge \hat{w}(e^{\emptyset})$, and so the retailer's utility is negative for all $e \ge e^{\emptyset}$. The optimal effort of Case III (when $\beta_v \le \beta_v^*$) must therefore be lower than $e^{\emptyset}$.

Part (ii): Note that the retailer's utility in Case I, $\bar{v} - w$, is non-decreasing in β_v; hence, the solution of Program 1 at $\beta_v = 0$ is a lower bound on the utility of the retailer in Case I. Let e° be the effort stipulated by this contract. The retailer's utility in Case I is therefore at least $v_L(e^{\circ}) - (1+\Phi_p)\bar{c}(e^{\circ})$ (for any value of β_v). Set $e^{\star}$ to be such that $v_H(e^{\star}) = v_L(e^{\circ}) - (1+\Phi_p)\bar{c}(e^{\circ})$. Such a value must exist as $v_H(0) = 0$ and the revenue of Case I is non-zero. Note that this means that if we can guarantee that the effort exerted by the contract in Case III is at most $e^{\star}$, then the revenue in Case III must be at most the revenue in Case I, as the revenue of Case III is upper bounded by $v_H(e^{\star})$. The proof now proceeds similarly to Case I, but with $e^{\star}$ replacing $e^{\emptyset}$.

Part (iii): Fix all of the parameters except β_v. Denote by $e^{\emptyset}$ the minimal effort stipulated by Program 1 over all choices of β_v; $e^{\emptyset}$ provides a lower bound on the effort in the no prediction case.

Find the minimal $\beta_v^{\dagger}$ such that $\delta(1-\beta_v)(1-p^{\dagger})(v_H(e) - v_L(e)) \le \frac{\bar{c}(e)}{\alpha}$ for all $e > e^{\emptyset}$, $\beta_v \ge \beta_v^{\dagger}$. Such a $\beta_v^{\dagger}$ must exist because $v_H(e) - v_L(e)$ is decreasing in e, $\bar{c}(e)$ is increasing in e and $\delta(1-\beta_v)(1-p^{\dagger})(v_H(e) - v_L(e))$ is decreasing in β_v for $\beta_v > 0.5$, with $\delta(1-\beta_v)(1-p^{\dagger})(v_H(e) - v_L(e)) = 0$ for $\beta_v = 1$.

For any $\dot{\beta}_v \ge \beta_v^{\dagger}$, let e^I be the optimal effort in Case I for $\dot{\beta}_v$. By the first order optimality condition, it must hold that (rewriting the objective function (1a)),

$$\dot{\beta}_v {v_H}'(e^I) - (1-\dot{\beta}_v)({v_H}'(e^I) - {v_L}'(e^I)) - (1+\Psi_p)\bar{c}'(e^I) = 0.$$

As (1a) is concave, it must hold that for every $\dot{e} \ge e^I$,

$$\dot{\beta}_v {v_H}'(\dot{e}) - (1-\dot{\beta}_v)({v_H}'(\dot{e}) - {v_L}'(\dot{e})) - (1+\Psi_p)\bar{c}'(\dot{e}) \le 0.$$

Set $\dot{p} = 1 - \dot{\beta}_v(1-p)$. The derivative w.r.t. e of the objective function at $\dot{\beta}_v$ is

$$\begin{aligned}
& v_H'(e) - \dot{p}(1-\dot{\beta}_v)\left(v_H{}'(e) - v_L{}'(e)\right) - \vec{c}'(e) - \Psi_{\dot{p}}\left(\vec{c}'(e) - \alpha b'\right) \\
= \;& v_H'(e) - (1-\dot{\beta}_v)\left(\dot{p} - \delta\alpha\Psi_{p^\dagger}(1-\dot{p})\right)\left(v_H{}'(e) - v_L{}'(e)\right) - (1+\Psi_{\dot{p}})\vec{c}'(e) \qquad (32)\\
\leq \;& v_H{}'(\dot{e}) - (1-\dot{\beta}_v)(v_H{}'(\dot{e}) - v_L{}'(\dot{e})) - (1+\Psi_p)\vec{c}'(\dot{e}) \qquad (33)\\
\leq \;& 0,
\end{aligned}$$

where (32) is due to the choice of $\beta_v^\dagger$ (guaranteeing that the bonus is $\delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e))$), and (33) is because $\Psi_{p^\dagger} \geq \Psi_p$, $v_H{}'(e) - v_L{}'(e) \leq 0$ and $\dot{p} - \delta\alpha\Psi_{p^\dagger}(1-\dot{p}) < 1$.

As $v_H - v_L$ is concave, the objective function (3a) is concave, hence if the derivative at $\dot{e}$ is negative, this implies the maximum is attained for some effort $e < \dot{e}$. □

D.4 Proof of Theorem 3

Proof of Theorem 3. First note that

$$\beta_v v_H(e) + (1-\beta_v)v_L(e) = \beta_v \bar{v_H}(e) + (1-\beta_v)\bar{v_L}(e) \qquad (34)$$

for all e if and only if

$$\beta_v v_H{}'(e) + (1-\beta_v)v_L{}'(e) = \beta_v \bar{v_H}{}'(e) + (1-\beta_v)\bar{v_L}{}'(e) \qquad (35)$$

for all e. This is because $v_H(0) = \bar{v_H}(0)$, $v_L(0) = \bar{v_L}(0)$, hence, (34) is equivalent to the definite integral of (35).

The retailer's optimization problem (as in Lemma 5) can be written as follows:

$$\begin{aligned}
\max_e \quad & \min\left\{v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - \bar{c}(e),\right. \qquad (36a)\\
& \left. v_H(e) + (1-\beta_v)(1-\delta)(1-p^\dagger)\left(v_H(e) - v_L(e)\right) - (1+\Psi_{p^\dagger})\bar{c}(e)\right\} \qquad (36b)
\end{aligned}$$

As v_H and v_L are concave and $\bar{c}$ is concave, and from the assumption that $v_H - v_L$ is concave, the objective function is the minimum of two concave functions and hence also concave.

We verify that for both expressions in (36), the derivative w.r.t. to any effort is greater for $\bar{S}$ than for S: for (36b), this is immediate from the fact that $\bar{v_H}{}'(e) > v_H{}'(e)$ and $\bar{v_L}{}'(e) < v_L{}'(e)$. For (36a), it is easy to verify that

$$\begin{aligned}
& \bar{v_H}{}'(e) - p^\dagger(1-\beta_v)\left(\bar{v_H}{}'(e) - \bar{v_L}{}'(e)\right) - \left(v_H{}'(e) - p^\dagger(1-\beta_v)\left(v_H{}'(e) - v_L{}'(e)\right)\right)\\
& = (1-p^\dagger)(\bar{v_H}{}'(e) - v_H{}'(e)),
\end{aligned}$$

using the equality in the theorem statement. As the derivative of the retailer payoff is greater in $\bar{S}$ than in S for all values of e, and the function being maximized is concave, the optimal effort must be higher in $\bar{S}$.

Regarding the bonus, first note that the conditions $v_H(0) = \bar{v_H}(0)$, $v_L(0) = \bar{v_L}(0)$, for every $e > 0$, $\bar{v_H}'(e) > v_H'(e)$ and $\bar{v_L}'(e) < v_L'(e)$, imply that for every $e > 0$, $\bar{v_H}(e) > v_H(e)$ and $\bar{v_L}(e) < v_L(e)$, and therefore, $\bar{v_H}(e) - \bar{v_L}(e) > v_H(e) - v_L(e)$ for all $e > 0$. The bonus is $b = \min\left\{\delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)), \frac{\bar{c}(e)}{\alpha}\right\}$; as both of the expressions in the brackets are larger in $\bar{S}$ than in S for every $e > 0$, their minimum is as well. As the effort is increasing in e, and the effort in $\bar{S}$ is greater than in S by part 1, this implies that the bonus in $\bar{S}$ is greater.

For the third part, recall from Proposition 3 that the retailer's utility is $v_H(e) - p^\dagger(1 - \beta_v)\left(v_H(e) - v_L(e)\right) - w - b$, which is equivalent to

$$v_H(e) - p^\dagger(1-\beta_v)\left(v_H(e) - v_L(e)\right) - \bar{c}(e) - \Phi_{p^\dagger}\max\{\bar{c}(e) - \alpha\delta(1-\beta_v)(1-p^\dagger)(v_H(e) - v_L(e)), 0\}.$$

Similar to the proof of the first part, we can show that the derivative of the utility is larger in $\bar{S}$ than in S for every value of e, and hence the retailer's utility is greater in $\bar{S}$ than S for the effort level exerted in S. As the retailer is a utility maximizer, her utility must be at least this value. □

E Missing proofs from Section 6 (Case IV)

E.1 Proof of Lemma 7

Lemma 7 follows directly from the following proposition, in similar fashion to the Proof of Lemma 5.

Proposition 4. *Denote $p^\dagger = 1 - \beta_v(1-p)$, $\Psi_{p^\dagger} = \frac{1-\delta(1-p^\dagger)}{\alpha\delta(1-p^\dagger)}$ and $\Omega_{p^\dagger} = \frac{\delta p^\dagger R}{(1-\delta)(S-R)+\delta p^\dagger S}$. If both retailers and suppliers can predict their state for the next time period, and the retailer fires the supplier whenever she predicts v_L, the following hold.*

(i) The rematch probability for unmatched suppliers is given by $q^\dagger := \frac{p^\dagger R}{S-R}$.

(ii) Let $\gamma_x := \frac{1-\delta}{\delta x} + 1$. The suppliers' expected utility from being matched and unmatched in a symmetric market equilibrium contract with effort, bonus, and wage given by $\{e^*, w^*, b^*\}$ is

$$E^* = \frac{\gamma_{q^\dagger}}{\delta p^\dagger} \cdot \frac{w^* + b^* - \bar{c}(e^*)}{\gamma_{p^\dagger}\gamma_{q^\dagger} - 1}, \quad U^* = \frac{1}{\delta p^\dagger} \cdot \frac{w^* + b^* - \bar{c}(e^*)}{\gamma_{p^\dagger}\gamma_{q^\dagger} - 1},$$

(iii) For any contract $\{e, w, b\}$, and assumed symmetric contract $\{e^*, w^*, b^*\}$ in the rest of the market, the suppliers' no-shirking constraint is

$$w \geq \bar{c}(e) - b + \Psi_{p^\dagger}\left(c_H(e) - \alpha b\right) + \Omega_{p^\dagger}(w^* + b^* - \bar{c}(e^*)) \ .$$

(iv) Let E^{IV} be the expected payoff of matched suppliers in contract $\{e, w, b\}$. The contract should be chosen such that the suppliers ex-ante prefer the contract over unemployment (i.e. $E^{IV} \geq U^*$). For any contract $\{e, w, b\}$, and assumed symmetric contract $\{e^*, w^*, b^*\}$, this is given by

$$w + b - \bar{c}(e) \geq \Omega_{p^\dagger}\left(w^* + b^* - \bar{c}(e^*)\right) \ .$$

In addition, the suppliers should choose to participate in the market (i.e. $U^* \geq 0$). For this, the symmetric market equilibrium $\{e^*, w^*, b^*\}$ should satisfy

$$w^* + b^* - \bar{c}(e^*) \geq 0 \ .$$

(v) The retailer chooses the bonus such that

$$b \leq \delta(1 - \beta_v)\left(1 - p^\dagger\right)\left(v_H(e) - v_L(e)\right),$$

where e is the effort stipulated by the contract.

(vi) For any contract $\{e, w, b\}$, the retailers' expected utility is given by

$$v_H(e) - p^\dagger(1 - \beta_v)\left(v_H(e) - v_L(e)\right) - w - b.$$

Proof. The proofs for Parts *(i)*, *(v)*, and *(vi)* are identical to Proposition 3, as the suppliers' cost (predictions) do not change those arguments. The proofs for Parts *(ii)*-*(iv)* are as follows.

Let E^{IV} and U^{IV} be the expected payoff for matched and unmatched suppliers, respectively, at the beginning of the period. Note that E^{IV} depends on the contract offered by the supplier's current retailer, while U^{IV} is only a function of the contract offered elsewhere in the market.

We analyze the incentives of a supplier predicting c_H. If the supplier exerts effort e, in the next period, the contract will continue only if the next state is predicted to be high (w.p. β_v), and the contract is not terminated due to exogenous reasons (w.p. $1 - p$). Thus

$p^\dagger = 1 - \beta_v(1-p)$ is the probability the contract breaks. Therefore,

$$u(e, c_H) := w + b - c_H(e) + \delta\left(p^\dagger U^{IV} + (1-p^\dagger)E^{IV}\right) \; . \tag{37}$$

By Observation 5, a shirking supplier exerts no effort, and expects a payoff of

$$u(0, c_H) := w + \alpha\delta U^{IV} + (1-\alpha)\left(b + \delta\left(p^\dagger U^{IV} + (1-p^\dagger)E^{IV}\right)\right) \; .$$

Thus, the supplier's incentive compatibility (IC) constraint is

$$\alpha\left(b + \delta(1-p^\dagger)\left(E^{IV} - U^{IV}\right)\right) \geq c_H(e) \; . \tag{38}$$

Note that as $c_H(e) \geq c_L(e)$, this IC constraint also implies that the supplier will also exert effort when c_L is predicted, and also in new contracts where the state is unknown.

Denote the contract offered by the retailer by $\{e, w, b\}$, and the assumed symmetric contract offered by other retailers in the market by $\{e^*, w^*, b^*\}$. We now determine the best-response $\{e, w, b\}$ to $\{e^*, w^*, b^*\}$.

Part (ii): Let E^{IV} be the expected utility of suppliers matched to contract $\{e, w, b\}$, and let U^* and E^* be the expected utility of unmatched and matched suppliers, respectively, to $\{e^*, w^*, b^*\}$.

Assuming the suppliers' incentive constraint (38) is satisfied, and given the rematch probability $q^\dagger$, similar to Part (ii) of Proposition 3, we have $E^* = \gamma_{q^\dagger} U^*$, where, $\gamma_x := \frac{1-\delta}{\delta x} + 1$. We therefore get

$$U^* = \frac{1}{\delta p^\dagger} \cdot \frac{w^* + b^* - \bar{c}(e^*)}{\gamma_{p^\dagger}\gamma_q^\dagger - 1} \; . \tag{39}$$

Also, similar to Part (ii) of Proposition 3, we have

$$E^{IV} = \frac{w + b - \bar{c}(e) + \delta p^\dagger U^*}{\delta p^\dagger \gamma_{p^\dagger}} \; . \tag{40}$$

Part (iii): Substituting the expression for $q^\dagger$, as well as U^{IV} from (39) and E^{IV} from (40), in the IC constraint (38), we get the supplier's non-shirking constraint (NSC)

$$w \geq \bar{c}(e) - b + \left(\frac{1-\delta(1-p^\dagger)}{\delta\alpha(1-p^\dagger)}\right)(c_H(e) - \alpha b) + \frac{\delta p^\dagger R}{(1-\delta)(S-R) + \delta p^\dagger S}(w^* + b^* - \bar{c}(e^*)) \; . \tag{41}$$

Part (iv): Similar to the previous cases, the retailer needs to ensure that all suppliers' participation constraints are satisfied at any time step. To ensure that all suppliers (whether matched or unmatched, and ex-ante with regard to their prediction) prefer being in this market to opting to their respective outside options, we need $E^{IV} \geq U^*$ and $U^* \geq 0$. The required conditions can be obtained by the expressions in Part *(ii)*, and are identical to Part *(iv)* of Proposition 3. □

E.2 Proof of Theorem 4

Proof of Theorem 4. We begin by rewriting the retailers' objective function for determining the effort, (3a) and (4a), by inserting the constraints for the bonus in these objective functions, and taking the min out of the expression. In particular, denote

$$\begin{aligned}
\mu &:= (1 - p^\dagger (1 - \beta_v))v_H(e), \\
\nu &:= p^\dagger (1 - \beta_v)\, v_L(e), \\
\xi &:= (1 - \beta_v)(1 - \delta)(1 - p^\dagger)(v_H(e) - v_L(e)).
\end{aligned}$$

Then, finding the effort of Program 3, by combining (3a) and (3b) reduces to:

$$\max_e \min \left\{ \mu + \nu - \bar{c}(e), v_H(e) + \xi - \bar{c}(e) - \Psi_{p^\dagger} \bar{c}(e) \right\} \tag{42}$$

Similarly, finding the effort of Program 4, by combining (4a) and (4b) reduces to:

$$\max_e \min \left\{ \mu + \nu - \bar{c}(e), v_H(e) + \xi - \bar{c}(e) - \Psi_{p^\dagger} c_H(e) \right\} \tag{43}$$

By the theorem's statement, we assume ξ is concave. Then, as v_H and v_L are concave and $\bar{c}$ is convex, the objective function in both (42) and (43) is the minimum of two concave functions and hence also concave. Then, as by the assumption in the theorem statement, $c'_H(e) \geq \bar{c}'(e)$ for every e, we have that the derivative of (43) is pointwise always at most the derivative of (42), and hence, it must equal zero for a lower value of e. This proves the first part of the theorem.

The second point is immediate from noting that the retailer's utility for every value of e is at least as high in (42) as in (43). An example for the third point is given in Figure 6(d). □

F Missing proofs from Section 7 (Case V)

F.1 Proof of Lemma 8

To prove Lemma 8, we require the following proposition.

Proposition 5. *Denote $p^{\ddagger} = \beta_c + (1-\beta_c)p$. If both the supplier and retailer can predict the supplier's cost for one step ahead, and the retailer fires suppliers with high cost, the following hold.*

(i) The retailer will choose $b = 0$.

(ii) The rematch probability for unmatched suppliers is given by $q^{\ddagger} := \frac{p^{\ddagger}R}{S-R}$.

(iii) Let $\gamma_x := \frac{1-\delta}{\delta x} + 1$. The suppliers' expected utility at the beginning of a period when being matched and unmatched, under a symmetric market contract with effort and wage $\{e^*, w^*\}$, are

$$E^* = \frac{\gamma_{q^{\ddagger}}}{\delta p^{\ddagger}} \cdot \frac{w^* - c_L(e^*)}{\gamma_{p^{\ddagger}}\gamma_{q^{\ddagger}} - 1} - \frac{\beta_c\left(c_H(e^*) - c_L(e^*)\right)}{\gamma_{p^{\ddagger}}\gamma_{q^{\ddagger}} - 1} .$$

$$U^* = \frac{1}{\delta p^{\ddagger}} \cdot \frac{w^* - c_L(e^*)}{\gamma_{p^{\ddagger}}\gamma_{q^{\ddagger}} - 1} - \frac{\gamma_{p^{\ddagger}}\beta_c\left(c_H(e^*) - c_L(e^*)\right)}{\gamma_{p^{\ddagger}}\gamma_{q^{\ddagger}} - 1} .$$

(iv) For any given choice of effort and wage $\{e, w\}$ and assumed symmetric contract $\{e^*, w^*\}$ elsewhere in the market, the suppliers' no shirking constraint (NSC) is

$$w \geq c_L(e) + \Psi_p^{\ddagger}\bar{c}(e) + \Omega_p^{\ddagger}\left((w^* - c_L(e^*)) - \beta_c\left(1 - \delta(1-p^{\ddagger})\right)\left(c_H(e^*) - c_L(e^*)\right)\right) ,$$

where $\Psi_p^{\ddagger} := \frac{1-\delta(1-p^{\ddagger})}{\alpha\delta(1-p^{\ddagger})}$ and $\Omega_p^{\ddagger} := \frac{\delta p^{\ddagger}R}{(1-\delta)(S-R)+\delta p^{\ddagger}S}$.

(v) Let E^V be the expected payoff of matched suppliers in contract $\{e, w\}$. Then, given that the (NSC) of Part (iv) is satisfied, the suppliers' participation constraints will be satisfied as well; that is, the suppliers choose to participate in the market (i.e. $U^* \geq 0$), and they prefer entering the contract than being unemployed (i.e. $E^V \geq U^*$).

(vi) The retailers' expected utility from contract $\{e, w\}$ is $\bar{v}(e) - w$.

Proof. The proof of *(i)* is similar to the proof of Part *(i)* of Propositions 1 and 2 and is omitted.

Parts (ii)-(iv): We next evaluate the suppliers' incentives. Let E^V, $\bar{E}^V$, and U^V denote the expected payoffs of suppliers continuing in existing matches, recently matched suppliers, and unmatched suppliers, respectively, at the beginning of a given period.

First, note that assuming the supplier has exerted effort e, or if he has not been caught shirking, match break probabilities for the next period are $p^{\ddagger} := \beta_c + (1-\beta_c)p$; this is because matches break either when the next cost state is high, or due to exogenous reasons.

We now analyze what happens when the supplier is in a new match, with expected cost $\bar{c}$ for the next period. From exerting effort, the supplier expects a payoff of

$$u(e,\bar{c}) := w - \bar{c}(e) + \delta\left(p^{\ddagger}U^{V} + (1-p^{\ddagger})E^{V}\right) \ . \tag{44}$$

By Observation 5, a shirking supplier will exert no effort, and expects a payoff of

$$u(0,\bar{c}) := w + \alpha\delta U^{V} + (1-\alpha)\delta\left(p^{\ddagger}U^{V} + (1-p^{\ddagger})E^{V}\right) \ .$$

Thus, the incentive compatibility (IC) constraint of the supplier, $u(e,\bar{c}) \geq u(0,\bar{c})$, becomes

$$\alpha\delta(1-p^{\ddagger})\left(E^{V} - U^{V}\right) \geq \bar{c}(e) \ . \tag{45}$$

Note that as $c_H(e) \geq c_L(e)$, this IC implies the supplier is also incentivized to exert effort when the state is c_L.

Part (ii): In terms of the probability of rematch of unmatched suppliers, $q^{\ddagger}$, given all IC constraints are satisfied, retailers will have vacancies during matching from two sources: high future cost state predictions with rate $\beta_c R$, and if not, exogenous reasons creating vacancies at a rate $(1-\beta_c)pR$. That is, at equilibrium, $p^{\ddagger}R$ vacancies are created, leading to $q^{\ddagger} = \frac{p^{\ddagger}R}{S-R}$.

Part (iii): Denote the contract offered by the retailer by (e,w), and the assumed symmetric contract offered by other retailers in the market by (e^*,w^*). Recall that E^V is the expected utility of matched suppliers in this ongoing contact (e,w). Let U^*, $\bar{E}^*$, and E^* be the expected utility of unmatched, recently matched, and matched suppliers, respectively, in the assumed symmetric market contracts (e^*,w^*).

Given the assumed symmetric contract (e^*,w^*) elsewhere in the market, and that the probability of matching to contract (e,w) is negligible, similar to Part (ii) of Proposition 1, we get $\bar{E}^* = \gamma_{q^{\ddagger}}U^*$, where $\gamma_x := \frac{1-\delta}{\delta x} + 1$.

We also know that $E^* = u(e^*,c_L)$ and $\bar{E}^* = u(e^*,\bar{c})$. Therefore,

$$E^* - \bar{E}^* = u(e^*,c_L) - u(e^*,\bar{c}) = \bar{c}(e^*) - c_L(e^*) = \beta_c(c_H(e^*) - c_L(e^*)) \ .$$

We therefore get $E^* - \gamma_{q^{\ddagger}}U^* = \beta_c(c_H(e^*) - c_L(e^*))$.

In addition, from (44),

$$E^* = u(e^*,c_L) = w^* - c_L(e^*) + \delta\left(p^{\ddagger}U^* + (1-p^{\ddagger})E^*\right) \ .$$

From this, we get $\gamma_{p^\ddagger} E^* = \frac{w^* - c_L(e^*)}{\delta p^\ddagger} + U^*$. Therefore,

$$E^* = \frac{\gamma_{q^\ddagger}}{\delta p^\ddagger} \cdot \frac{w^* - c_L(e^*)}{\gamma_{p^\ddagger}\gamma_{q^\ddagger} - 1} - \frac{\beta_c \left(c_H(e^*) - c_L(e^*)\right)}{\gamma_{p^\ddagger}\gamma_{q^\ddagger} - 1} \; .$$

$$U^* = \frac{1}{\delta p^\ddagger} \cdot \frac{w^* - c_L(e^*)}{\gamma_{p^\ddagger}\gamma_{q^\ddagger} - 1} - \frac{\gamma_{p^\ddagger}\beta_c \left(c_H(e^*) - c_L(e^*)\right)}{\gamma_{p^\ddagger}\gamma_{q^\ddagger} - 1} \; . \tag{46}$$

Further, for contract (e, w), the expected utility is $u(e, c_L)$, defined similarly to (44), i.e.,

$$E^V = w - c_L(e) + \delta(p^\ddagger U^* + (1 - p^\ddagger)E^V)$$

Therefore,

$$E^V = \frac{w - c_L(e) + \delta p^\ddagger U^*}{\delta p^\ddagger \gamma_p^\ddagger} \tag{47}$$

Part (iv): Substituting $q^\ddagger = \frac{p^\ddagger R}{S-R}$, E^V from (47) and U^V from (46) (using $U^V = U^*$), into the incentive compatibility constraint (45), we get the supplier's non-shirking constraint (NSC):

$$w \geq c_L(e) + \left(\frac{1 - \delta(1 - p^\ddagger)}{\delta\alpha(1 - p^\ddagger)}\right) \bar{c}(e)$$
$$+ \frac{\delta p^\ddagger R}{(1-\delta)(S-R) + \delta p^\ddagger S} \left((w^* - c_L(e^*)) - \beta_c \left(1 - \delta(1 - p^\ddagger)\right)(c_H(e^*) - c_L(e^*))\right) \; . \tag{48}$$

Part (v): Similar to the previous cases, the retailer needs to ensure that all suppliers' participation constraints are satisfied at any time step. To ensure that all suppliers (whether matched or unmatched, and ex-ante with regard to their prediction) prefer being in this market to opting out, we need $E^V \geq U^*$, $\bar{E}^V \geq U^*$, and $U^* \geq 0$. First, note that by (44), $E^V - \bar{E}^V = \bar{c}(e) - c_L(e)$. Further, when (45) holds, $E^V - U^* \geq \frac{\bar{c}(e)}{\alpha\delta(1-p^\ddagger)}$. Therefore, combining the two, when the (NSC) is satisfied, we have

$$\bar{E}^V - U^* \geq \frac{\bar{c}(e)}{\alpha\delta(1 - p^\ddagger)} - \bar{c}(e) + c_L(e) \; .$$

Since $\alpha\delta(1 - p^\ddagger) < 1$, this implies that $\bar{E}^V \geq U^*$, and consequently that $E^V \geq U^*$.

By Equation (46), to guarantee $U^* \geq 0$, the symmetric market equilibrium contract should satisfy $w^* - c_L(e^*) - \beta_c \left(1 - \delta(1 - p^\ddagger)\right)(c_H(e^*) - c_L(e^*)) \geq 0$.

We now note that the (NSC) constraint (48) needs to be satisfied for the symmetric equilibrium contract $\{e^*, w^*\}$ as well. This is given by

$$w^* \geq c_L(e^*) + \frac{\Psi_{p^\ddagger}}{1-\Omega_p^\ddagger}\bar{c}(e^*) - \frac{\Omega_{p^\ddagger}}{1-\Omega_p^\ddagger}\beta_c\left(1-\delta(1-p^\ddagger)\right)\left(c_H(e^*) - c_L(e^*)\right) \ . \tag{49}$$

It is straightforward to verify that

$$\frac{\Psi_{p^\ddagger}}{1-\Omega_p^\ddagger}\bar{c}(e^*) \geq \frac{1}{1-\Omega_p^\ddagger}\beta_c\left(1-\delta(1-p^\ddagger)\right)\left(c_H(e^*) - c_L(e^*)\right).$$

Therefore, Inequality (49) implies Inequality (46), hence, as the NSC is satisfied, $U^* \geq 0$ will hold as well. □

Proof of Lemma 8. We turn to the choice of the wage and effort. The retailer's goal is to maximize her expected profit, while satisfying the supplier's no shirking constraint (48). Given Proposition 5, the optimal contract will be the solution to

$$\begin{aligned} \max_{e,w} \quad & \bar{v}(e) - w \\ \text{s.t.} \quad & w \geq c_L(e) + \Psi_p^\ddagger \bar{c}(e) + \Omega_p^\ddagger \left((w^* - c_L(e^*)) - \beta_c\left(1-\delta(1-p^\ddagger)\right)\left(c_H(e^*) - c_L(e^*)\right)\right) \ , \end{aligned}$$

where $\Psi_p^\ddagger := \frac{1-\delta(1-p^\ddagger)}{\alpha\delta(1-p^\ddagger)}$ and $\Omega_p^\ddagger := \frac{\delta p^\ddagger R}{(1-\delta)(S-R)+\delta p^\ddagger S}$. Noting that the revenue is maximized when the inequality is tight, we can rewrite this as

$$\begin{aligned} \max_{e} \quad & \bar{v}(e) - w \\ \text{s.t.} \quad & w = c_L(e) + \Psi_p^\ddagger \bar{c}(e) + \Omega_p^\ddagger \left((w^* - c_L(e^*)) - \beta_c\left(1-\delta(1-p^\ddagger)\right)\left(c_H(e^*) - c_L(e^*)\right)\right) \ , \end{aligned} \tag{50a}$$

where $\{e^*, w^*\}$ is the assumed contract offered elsewhere in the market. Note that as e^* and w^* are determined exogenously, the last term in (50a) is a constant, hence the program can be rewritten as follows.

$$\begin{aligned} e &:= \arg\max_{e'} \quad \bar{v}(e') - c_L(e) - \Psi_p^\ddagger \bar{c}(e) \\ w &= c_L(e) + \Psi_p^\ddagger \bar{c}(e) + \Omega_p^\ddagger \left((w^* - c_L(e^*)) - \beta_c\left(1-\delta(1-p^\ddagger)\right)\left(c_H(e^*) - c_L(e^*)\right)\right) \ . \end{aligned}$$

At equilibrium $w = w^*$, $e = e^*$, giving Program 5 and completing the proof. □

F.2 Proof of Theorem 5

Observation 6. *Let* $\Phi^* := \frac{1}{\alpha\delta(1-p)}\left(1-\delta+\delta\frac{S}{S-R}\right)$. *Then* $\Phi_{p^\ddagger} - \Phi_p = \frac{\beta_c}{1-\beta_c}\Phi^*$.

Proof. Denote $X := \frac{S}{S-R}$. We have

$$\begin{aligned}
\Phi_{p^\ddagger} - \Phi_p &= \frac{1-\delta+\delta p^\ddagger X}{\alpha\delta(1-p^\ddagger)} - \frac{1-\delta+\delta pX}{\alpha\delta(1-p)} \\
&= \frac{1}{\alpha\delta(1-p)}\left(\frac{\left(1-\delta+\delta p^\ddagger X\right)-(1-\beta_c)\left(1-\delta+\delta pX\right)}{(1-\beta_c)}\right) \\
&= \frac{1}{\alpha\delta(1-p)}\left(\frac{\beta_c(1-\delta+\delta X)}{1-\beta_c}\right) = \frac{\beta_c}{1-\beta_c}\Phi^* \ .
\end{aligned}$$

□

Proof of Theorem 5. Part (i): Let e^* be the optimal effort decided by the contract of Case I (Lemma 1). We show that the exists a $\beta_c^* > 0$ such that for all $\beta_c \leq \beta_c^*$, the contracted wage for exerting effort e^* can be lower in Contract C^P than in Case I, by comparing Constraint (1b) of Case I and Constraint (5b) of Contract C^P. Specifically, we show that

$$c_L(e) + \Phi_{p^\ddagger}\bar{c}(e) - \beta_c\frac{\delta p^\ddagger R}{S-R}\left(c_H(e)-c_L(e)\right) \leq \bar{c}(e) + \Phi_p\bar{c}(e)$$

For the first point, it suffices to show that $c_L(e) + \Phi_{p^\ddagger}\bar{c}(e) \leq \bar{c}(e) + \Phi_p\bar{c}(e)$. Rearranging and substituting $\Phi_{p^\ddagger} - \Phi_p = \frac{\beta_c}{1-\beta_c}\Phi^*$ (Observation 6),

$$\begin{aligned}
& c_L(e) + \Phi_{p^\ddagger}\bar{c}(e) \leq \bar{c}(e) + \Phi_p\bar{c}(e) \\
\iff & c_L(e) + \frac{\beta_c}{1-\beta_c}\Phi^*\bar{c}(e) \leq \bar{c}(e) \\
\iff & \Phi^*\bar{c}(e) \leq (1-\beta_c)(c_H(e)-c_L(e)) \\
\iff & \beta_c(\Phi^*+1)(c_H(e)-c_L(e)) \leq (c_H(e)-c_L(e)) - \Phi^* c_L(e) \\
\iff & \beta_c \leq \frac{c_H(e)-(1+\Phi^*)c_L(e)}{(1+\Phi^*)(c_H(e)-c_L(e))}
\end{aligned}$$

Note that the requirement on $c_H(e) \geq (1+\Phi^*)c_L(e)$ guarantees that the right hand side of the last inequality is positive.

Therefore, setting $\beta_c^* = \frac{c_H(e)-(1+\Phi^*)c_L(e)}{(1+\Phi^*)(c_H(e)-c_L(e))}$, the wage required to incentivize the supplier to exert e^* is lower in contract C^P for $\beta_c < \beta_c^*$ and hence in Case V. The retailer's utility is at least as high as her utility for $e = e^*$.

Part (ii): We now show that for sufficiently large β_c, for every $e > 0$,

$$c_L(e) + \Phi_{p^\ddagger}\bar{c}(e) - \beta_c\delta\frac{p^\ddagger R}{S-R}\left(c_H(e)-c_L(e)\right) \geq \bar{c}(e) + \Phi_p c_H(e) \ . \tag{51}$$

This would mean that in order to incentivize any given effort, the incentive constraint of C^P imposes a higher wage than that of contract C^N, so that for sufficiently large β_c, the supplier would rather offer contract C^N over C^P.

Denote $\eta = c_H(e)+\Phi_p c_H(e)-c_L(e)+\delta\frac{p^\ddagger R}{S-R}\left(c_H(e)-c_L(e)\right)$. We do not compute its value, but note that it is independent of β_c. Therefore, there exists a sufficiently large $\beta_c^\dagger > 0$ such that for every $\beta_c > \beta_c^\dagger$ it holds that $\frac{1-\delta}{\alpha\delta(1-\beta_c)}c_L(e) > \eta$. Then

$$
\begin{aligned}
\Phi_{p^\ddagger}\bar{c}(e) =& \frac{1}{\alpha\delta(1-p^\ddagger)}\left(1-\delta+\delta p^\ddagger\frac{S}{S-R}\right)\bar{c}(e) \\
=& \frac{1}{\alpha\delta(1-\beta_c)(1-p)}\left(1-\delta+\delta p^\ddagger\frac{S}{S-R}\right)\bar{c}(e) \\
\geq& \frac{1-\delta}{\alpha\delta(1-\beta_c)}c_L(e) \\
>& \,\eta = c_H(e)+\Phi_p c_H(e)-c_L(e)+\delta\frac{p^\ddagger R}{S-R}\left(c_H(e)-c_L(e)\right). \\
&\geq \bar{c}(e)+\Phi_p c_H(e)-c_L(e)+\beta_c\delta\frac{p^\ddagger R}{S-R}\left(c_H(e)-c_L(e)\right).
\end{aligned}
$$

where the first inequality holds if $p < 1$, and as $\bar{c}(e) \geq c_L(e)$, and the last inequality is due to $c_H(e) \geq \bar{c}(e)$ and $\beta_c \leq 1$. Rearranging the outer inequality gives Inequality (51). Therefore, for sufficiently high β_c, the retailer offers contract C^N in Case V, which by Theorem 1, results in lower payoff for the retailer than Case I. □