ASSIGNMENT I

1. Find a continuous r.v. that does not have a density function.

2. Prove that if $X_n \longrightarrow X$ a.s., then $X_n \xrightarrow{p} X$.

3. Suppose X is a non-negative valued random variable such that the moment generating function

$$M_X(\theta) = \mathbb{E}\left[e^{\theta X}\right]$$

exists for all $\theta \in \mathbb{R}$

(a) Use the Markov inequality to derive the Chernoff bound:

for non-negative X

$$P(X \geq t) \leq M(\theta)\, e^{-t\theta} \qquad \text{——— (1)}$$

(b) Specialize (1) for $X \sim \text{Poisson}(\lambda)$.

Find the value of θ that gives the best bound in (1).

4. (a) Give an example of a Markov process that is not a martingale.

(b) Give an example of martingale that is not a Markov process.

5. Suppose $\{X_n, n \geq 1\}$ is a real-valued random sequence and

$$X_n \xrightarrow{d} \sigma$$

where $|\sigma| < \infty$ is a constant.

Show that

$$X_n \xrightarrow{P} \sigma .$$

6. (a) Give an example of a covariance stationary process that is not Markov.

(b) Give an example of a Markov process that is not covariance stationary.

7. Give an example of a sequence $\{Z_n, n \geq 1\}$ that converges a.s. but does not converge in L_2.