

Assignment VII, STAT 532 (Pasupathy), Fall 2022

1. A certain Markov chain in genetics has states  $0, 1, \dots, 2d$  and transition function

$$P(x, y) = \binom{2d}{y} \left(\frac{x}{2d}\right)^y \left(1 - \frac{x}{2d}\right)^{2d-y}.$$

Find  $P(T_0 < \infty \mid X_0 = x)$ .

2. Consider a branching chain with  $f(x) = p(1-p)^x, x \geq 0$ , where  $0 < p < 1$ . Show that  $\rho = 1$  if  $p \geq 1/2$  and that  $\rho = p/(1-p)$  if  $p < 1/2$ .
3. Consider a branching chain such that  $f(1) < 1$ . Show that every state other than 0 is transient.
4. Let  $X_n, n \geq 0$  be the Ehrenfest chain with  $d = 4$  and  $X_0 = 0$ .
- (a) Find the approximate distribution of  $X_n$  for  $n$  large and even.
- (b) Find the approximate distribution of  $X_n$  for  $n$  large and odd.
5. Consider a Markov chain on  $\{0, 1, 2\}$  having the transition matrix

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1/2 & 1/2 & 0 \end{bmatrix}.$$

- (a) Show that the chain is irreducible.
- (b) Find the period.
- (c) Find the stationary distribution.
6. Consider a Markov chain on  $\{0, 1, 2, 3, 4\}$  having the transition matrix

$$P = \begin{bmatrix} 0 & 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 3/4 \\ 0 & 0 & 0 & 1/4 & 3/4 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

- (a) Show that the chain is irreducible.
- (b) Find the period.
- (c) Find the stationary distribution.