ASSIGNMENT IX, STAT 532 (Pasupathy)

1. Let $Z_{11}, Z_{12}, Z_{21}, Z_{22}, \ldots, Z_{n1}, Z_{n2}$ be $2n$ independent normally distributed random variables each having mean zero and such that

$$\mathrm{Var}(Z_{k1}) = \mathrm{Var}(Z_{k2}) = \sigma_k^2, \quad k = 1, 2, \ldots, n.$$

Let $\lambda_i, i = 1, 2, \ldots, n$ be real constants and set

$$X(t) = \sum_{k=1}^{n} (Z_{k1} \cos \lambda_k t + Z_{k2} \sin \lambda_k t), \quad -\infty < t < \infty.$$

Show that $X(t), -\infty < t < \infty$ is a Guassian process.

2. Let $X(t), -\infty < t < \infty$ be a Gaussian process and let f and g be functions from $(-\infty, \infty)$ to $(-\infty, \infty)$. Show that $Y(t) = f(t)X(g(t)), -\infty < t < \infty$, is a Gaussian process and find its mean and covariance functions.

3. Let $X(t), -\infty < t < \infty$ be a Gaussian process having mean zero. Set $Y(t) = X^2(t), -\infty < t < \infty$.
   (a) Find the mean and covariance functions of the $Y(t)$ process.
   (b) Show that if $X(t), -\infty < t < \infty$ is covariance stationary, then so is $Y(t), -\infty < t < \infty$.

4. Let $W(t), -\infty < t < \infty$ denote the Wiener process with parameter σ^2. Find the mean and covariance functions of the following processes:
   (a) $X(t) = (W(t))^2, \quad t \geq 0;$
   (b) $X(t) = tW(1/t), \quad t > 0;$
   (c) $X(t) = c^{-1}W(c^2 t), \quad t \geq 0.$