Outline

• Recap from Wednesday
• BPM Mode Solver
• Vectorial BPM Applications:
  – Waveguide
  – Photonic Crystal Fiber
BPM Mode Solver

• Can extend BPM method to solve for modes, by propagating in the imaginary direction
• First, drop all derivatives in BPM equation:

\[ [K] \{ h_{t,l} \} = -\gamma^2 [M] \{ h_{t,l} \} \]

• Second, write down next step in z:

\[
\{ h_{t,l} \}_{k+1} = \frac{-2\gamma - 0.5\Delta z k_o^2 (n_{\text{eff},l}^2 - n_o^2)}{-2\gamma + 0.5\Delta z k_o^2 (n_{\text{eff},l}^2 - n_o^2)} \{ h_{t,l} \}_k
\]

• Third, substitute special value of \( \Delta z \):

\[
\Delta z \approx j \frac{4n_o}{(n_{\text{eff},l}^2 - n_o^2) k_o}
\]
BPM Mode Solver

• Since $\Delta z$ initially unknown, assume largest index possible, and decrease it as needed

• Will eventually converge to correct answer and effective refractive index:

$$n_{eff, \ell, k}^2 = \frac{\{h_t\}_k^* [K]_k \{h_t\}_k}{k_o^2 \{h_t\}_k^* [M]_k \{h_t\}_k}$$

• Can use Gram-Schmidt normalization procedure to find higher-order modes:

$$\{h_{t,1, new}\} = \{h_{t,1}\} - \sum_{\ell=1}^{i-1} \frac{\{h_{t,\ell}\}^* [M] \{h_{t,\ell}\}}{\{h_{t,\ell}\}^* [M] \{h_{t,\ell}\}} \{h_{t,\ell}\}$$
VBPM on a Waveguide: Problem Description

- Cross section defined above; $\lambda = 1.3 \, \mu m$
- Propagation along $z$ is semi-infinite
- Must grid space with first-order triangular elements in cross-sectional plane; choose PML to reduce reflections to $10^{-100}$
- Will vary $\Delta z$ for maximum effectiveness
VBPM on a Waveguide

• Fundamental mode is calculated accurately with 12,800 first-order triangular elements
• Propagation step size in $Z$, known as $\Delta Z$, should equal transverse dimensions for best accuracy
VBPM on a Waveguide: Longitudinal Imaginary Propagation

- With optimal step size, can solve the fundamental mode of both polarizations in a pretty modest number of steps!
VBPM on a Waveguide: Accuracy

- Accuracy of calculation of waveguide coupling length as a function of mesh divisions $N$
VBPM on a Waveguide

- Accuracy of coupling length as a function of $\Delta Z$ saturates below one wavelength
VBPM on a Photonic Crystal Fiber

- Originally conceived of by P.J. Russell
- Confines light to core without total internal reflection!
VBPM on a PhC Fiber

- Effective index vs. PhC period
VBPM on a PhC Fiber

- $H_y$ field distributions for the fundamental TE modes
Confinement loss decreases sharply as period $\Lambda$ increases.
VBPM on a PhC Fiber

- Variation of the effective mode area with PhC period $\Lambda$
VBPM on a PhC Fiber

- Effective index increases modestly with increasing period $\Lambda$, indicating increased mode confinement
VBPM on a PhC Fiber

- Calculated dispersion relation (effective index versus wavelength) for a PhC Fiber
VBPM on a PhC Fiber

- Obtained dispersion \( D = \frac{d^2 k}{d\omega^2} \) from earlier data
- Note modest changes in parameters flip sign of \( D \)
Next Class

• Is on Monday, Feb. 18
• Next time, we shall finish the applications of BPM, and possibly cover other FEM applications
• Recommended reading: Obayya, Chapter 3