ECE 595, Section 10
Numerical Simulations
Lecture 29: Eigenmode Layered Computations (CAMFR)

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Recap from Friday: S-Matrices

• S-matrix method in plane wave basis
  – Calculate field eigenvectors in plane-wave basis
  – Calculate interface s-matrices and layer s-matrices
  – Compose S-matrices iteratively

• S4 simulation tool:
  – User interface
  – Lua commands
  – Results for example problems: multilayer stack; 1D grating; 2D Tikhodeev example
CAMFR: Rationale

• Many problems consist of layers with varying widths

• Examples:
  – LED stack
  – Rod-hole photonic crystal

• Natural form of solutions is semi-analytic, in terms of eigenmodes

CAMFR: Basic Strategy

- Break up structure into layers
- Calculate eigenmodes in each layer (of four types)
- Apply Lorentz reciprocity to match BC’s
- Propagate within layers using S-matrix method
- Apply inputs to calculate physical outputs
CAMFR: Eigenmode Decomposition

- This stage resembles BPM
- Begin with the Helmholtz equation:
  \[ [\nabla_t^2 + \epsilon \mu \omega^2] \psi = \beta^2 \psi \]
- Where \( \psi \) represents \( E \)-field or \( H \)-field, and \( \beta \) is the eigenvalue (wavevector along \( z \))
- Write 3D solutions in this form for each layer:
  \[
  \begin{pmatrix}
  E(r) \\
  H(r)
  \end{pmatrix}
  = \sum_k \sum_k A_k e^{-j \beta_k z}
  \begin{pmatrix}
  E(r_t) \\
  H(r_t)
  \end{pmatrix}
  \]
CAMFR: Eigenmode Decomposition

Can express eigenvalues in terms of $\text{Re} \ n_{\text{eff}}$ and $\text{Im} \ n_{\text{eff}}$

Eigenmode Classification

**Guided mode**

- $\text{Im } \beta = 0$; discrete

**Complex mode**

- $\text{Im } \beta \neq 0$; $\text{Re } \beta \neq 0$; discrete complex-conjugate pairs

**Radiation mode**

- $\text{Re } \beta = 0$ or $\text{Im } \beta = 0$; continuous

**Leaky mode**

- $\text{Im } \beta \neq 0$; $\text{Re } \beta \neq 0$; discrete

Lorentz Reciprocity

• Evaluate Maxwell’s equations across boundary using this surface

Lorentz Reciprocity

• Starting with Maxwell’s equations:

\[ \nabla \times E_1 = -j\omega \mu H_1 \quad \nabla \times E_2 = -j\omega \mu H_2 \]
\[ \nabla \times H_1 = J_1 + j\omega \varepsilon E_1 \quad \nabla \times H_2 = J_2 + j\omega \varepsilon E_2 \]

• Can form the expression:

\[ \nabla \cdot (E_1 \times H_2 - E_2 \times H_1) = J_1 \cdot E_2 - J_2 \cdot E_1 \]

• Integrating over \( V \) and using Gauss’ theorem:

\[ \int \int_S (E_1 \times H_2 - E_2 \times H_1) \cdot dS = \int \int \int_V (J_1 \cdot E_2 - J_2 \cdot E_1) dV \]
Lorentz Reciprocity

- Lorentz Reciprocity theorem becomes:

\[
\int \int_S \frac{\partial}{\partial z} (E_1 \times H_2 - E_2 \times H_1) \cdot u_z dS = \int \int_S (J_1 \cdot E_2 - J_2 \cdot E_1) dS
\]

- For \( z \)-invariant media:

\[
\int \int_S (E_{m,t} \times H_{n,t}) \cdot u_z dS = 0
\]

Boundary Conditions

• Assuming:
  \[ E_{p,t}^I + \sum_j R_{j,p} E_{j,t}^I = \sum_j T_{j,p} E_{j,t}^{II} \]

• Defining overlap between modes to be:
  \[ \langle E_m, H_n \rangle \equiv \int \int_S (E_m \times H_n) \cdot u_z \, dS \]

• We obtain the transmission coefficient:
  \[ \sum_j \left[ \langle E_i^I, H_{j}^{II} \rangle + \langle E_{j}^{II}, H_i^I \rangle \right] T_{j,p} = 2\delta_{ip} \langle E_p^I, H_p^I \rangle \]

• And reflection coefficient:
  \[ R_{i,p} = \frac{1}{2 \langle E_i^I, H_i^I \rangle} \sum_j \left[ \langle E_{j}^{II}, H_i^I \rangle - \langle E_i^I, H_{j}^{II} \rangle \right] T_{j,p} \]
S-Matrix Solution

• Now we can employ the standard S-matrix scheme from Li ’96:

\[
T_{1,p+1} = t_{p,p+1} \cdot (I - R_{p,1} \cdot r_{p,p+1})^{-1} \cdot T_{1,p}
\]
\[
R_{p+1,1} = t_{p,p+1} \cdot (I - R_{p,1} \cdot r_{p,p+1})^{-1} \cdot R_{p,1} \cdot t_{p+1,p} + r_{p+1,p}
\]
\[
R_{1,p+1} = T_{p,1} \cdot (I - r_{p,p+1} \cdot R_{p,1})^{-1} \cdot r_{p,p+1} \cdot T_{1,p} + R_{1,p}
\]
\[
T_{p+1,1} = T_{p,1} \cdot (I - r_{p,p+1} \cdot R_{p,1})^{-1} \cdot t_{p+1,p}
\]

• We can compose the S-matrix starting from the identity matrix until we include all layers
Periodic Eigenproblems

• Periodic layered structures will:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \cdot \begin{bmatrix}
F \\
B
\end{bmatrix} = e^{-jkzp} \cdot \begin{bmatrix}
F \\
B
\end{bmatrix}
\]

• Since T-matrix is nearly singular, use SVD:

\[
A = U \cdot \Sigma \cdot V^H
\]

• Where \( U \) and \( V \) are unitary; \( \Sigma \) diagonal. Then:

\[
A^{-1} = V \cdot \text{diag} \left( \frac{1}{\sigma_i} \right) \cdot U^H
\]
CAMFR: 2D Photonic Crystals

CAMFR: 2D PhC Waveguide

3/25/2013 ECE 595, Prof. Bermel
Next Class

• Is on Wednesday, March 27
• Will discuss CAMFR interface: http://camfr.sourceforge.net