A logarithm is the inverse of an exponential.

$$a^x = y \Leftrightarrow \log_a y = x$$

a, the base and must be positive.

y, the argument and must be positive.

x, the exponent and can be positive or negative.

$$3^x = 81 \Leftrightarrow \log_3 81 = x$$

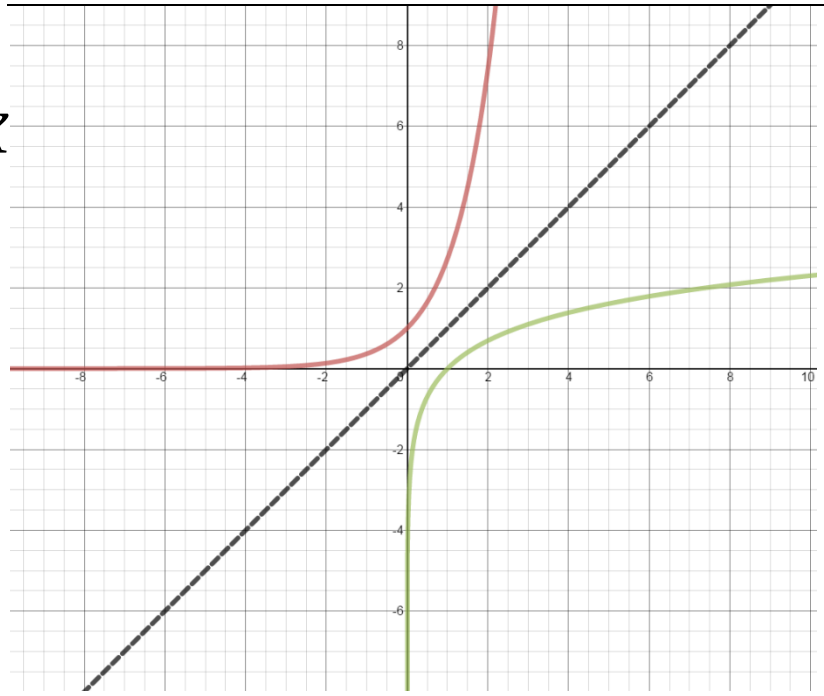
The Natural Logarithm:

A special logarithm occurs when the base of the logarithm is the natural number *e*. When the base of a logarithm is *e*, then it is called a natural logarithm, and is denoted $\ln x$. $\log_e x = \ln x$

$$y = e^x \qquad \textit{and} \qquad \ln x = y$$

…are inverse functions, and have all the properties of inverse functions.

$y=x$



General Properties of Logarithms:

$\ln(1)=?$ $\ln e=?$ $\ln e^{x}=?$ $e^{\ln x}=?$

$\ln x^{m}=$ $\ln(xy)=$ $\ln\left(\frac{x}{y}\right)=$

The Domain of $y=\ln x$ is $(0,\infty)$ since x has to be positive. For the domain of $y=\ln\left(ax+b\right)$ simply solve $\left(ax+b\right)>0$.

<u>Find the Domain:</u>

a) $\ln\left(5x\right)$

b) $\ln\left(-8x\right)$

c) $\ln\left(5x+3\right)$

d) $\ln\left(15x^2-135\right)$

e) $\ln\left(-2x^2-6x+36\right)$

Simplify the following expressions:

a) $\ln(e)$

b) $\ln\left(e^{6x}\right)$

c) $\ln(1)$

d) $e^{\ln(15x)}$

e) $e^{3+\ln(4x)}$

f) $e^{3\ln(4x)}$

g) $e^{3-\ln(4x)}$

Use a calculator to evaluate the following expressions to four decimal places, if it exist.

a) $\ln(7)$ b) $\ln(0.67)$ c) $\ln(359.2)$

d) $\ln(-3.27)$ e) $\ln(485{,}165{,}195)$

Use a calculator to find the value of x to four decimal places. If it exist.

a) $\ln(x)=7.5$ b) $\ln(x)=0.025$

c) $\ln(5x)=3.1$ d) $\ln(x-0.9)=-4$

e) $\ln(3x)=-0.6547$

Solve the following equation for the **exact** value of x.

a) $\ln(12x-4)=\ln(x+3)$

b) $\ln(3x-4)=\ln(6x-6)$

c) $\ln(4x+5)=17$

d) $\ln(x^2+4)=\ln(3x+8)$

e) $\ln(x^2)=18$

f) $\ln(3x^2+1)=\ln(12x+14)$