A logarithm is the inverse of an exponential. 

\[ a^x = y \iff \log_a y = x \]

\(a\), the base and must be positive. 
\(y\), the argument and must be positive. 
\(x\), the exponent and can be positive or negative.

\[ 3^x = 81 \iff \log_3 81 = x \]

**The Natural Logarithm:**

A special logarithm occurs when the base of the logarithm is the natural number \(e\). When the base of a logarithm is \(e\), then it is called a natural logarithm, and is denoted \(\ln x\). \[ \log_e x = \ln x \]

\[ y = e^x \quad \text{and} \quad \ln x = y \]

…are inverse functions, and have all the properties of inverse functions.
General Properties of Logarithms:

\[ \ln(1) = ? \quad \ln(e) = ? \quad \ln(e^x) = ? \quad e^{\ln(x)} = ? \]

\[ \ln(x^m) = \quad \ln(xy) = \quad \ln\left(\frac{x}{y}\right) = \]
The Domain of $y = \ln x$ is $(0, \infty)$ since $x$ has to be positive. For the domain of $y = \ln(ax+b)$ simply solve $(ax+b) > 0$.

Find the Domain:

a) $\ln(5x)$  

b) $\ln(-8x)$  

c) $\ln(5x+3)$  

d) $\ln(15x^2-135)$  

e) $\ln(-2x^2-6x+36)$
Simplify the following expressions:

a) \( \ln(e) \)

b) \( \ln(e^{6x}) \)

c) \( \ln(1) \)

d) \( e^{\ln(15x)} \)

e) \( e^{3+\ln(4x)} \)

f) \( e^{3\ln(4x)} \)

g) \( e^{3-\ln(4x)} \)
Use a calculator to evaluate the following expressions to four decimal places, if it exist.

a) \( \ln(7) \)  

b) \( \ln(0.67) \)  

c) \( \ln(359.2) \)

d) \( \ln(-3.27) \)  

e) \( \ln(485,165,195) \)

Use a calculator to find the value of \( x \) to four decimal places. If it exist.

a) \( \ln(x) = 7.5 \)  

b) \( \ln(x) = 0.025 \)

c) \( \ln(5x) = 3.1 \)  

d) \( \ln(x-0.9) = -4 \)

e) \( \ln(3x) = -0.6547 \)
Solve the following equation for the **exact** value of x.

a) \( \ln(12x-4) = \ln(x+3) \)  
b) \( \ln(3x-4) = \ln(6x-6) \)

c) \( \ln(4x+5) = 17 \)  
d) \( \ln(x^2+4) = \ln(3x+8) \)

e) \( \ln(x^2) = 18 \)  
f) \( \ln(3x^2+1) = \ln(12x+14) \)