An **angle** is defined as the set of points determined by two rays, or half-lines, $l_1$ and $l_2$ having the same end point $O$. An angle can also be considered as two finite line segments with a common point.

We call $l_1$ the **initial side**, $l_2$ the **terminal side**, and $O$ the **vertex** of angle $\angle AOB$. The direction and number of rotations of $l_1$ makes before stopping at $l_2$ is not restricted.

The **standard position** of an angle is obtained by placing the vertex at the origin and letting the initial side $l_1$ coincide with the positive x-axis.

If $l_1$ is rotated in a counterclockwise direction to the terminal position $l_2$, then the angle is considered **positive**.

If $l_1$ is rotated in a clockwise direction, the angle is **negative**.
A **straight angle** is an angle whose sides lie on the same straight line but extend in opposite directions from its vertex.

A **right angle** is half of a straight angle and has measure $90^\circ$.

An **acute angle** $\theta$; $0^\circ < \theta < 90^\circ$

An **obtuse angle** $\theta$; $90^\circ < \theta < 180^\circ$

**Complementary angles** $\alpha$, $\beta$; $\alpha + \beta = 90^\circ$

**Supplementary angles** $\alpha$, $\beta$; $\alpha + \beta = 180^\circ$

An angle is called a **quadrantal angle** if its terminal side lies on a coordinate axis.
Two units for measuring angles:

**Degrees**: A circle can be divided into 360 equal pieces. One complete revolution has measure 360 degrees, $360^\circ$

**Radians**: One complete revolution has measure $2\pi$ radians, $2\pi$.

One radian is the measure of the central angle of a circle **subtended** by an arc equal in length to the radius of the circle.

\[360^\circ = 2\pi \text{ radians and } 180^\circ = \pi \text{ radians}.\]

Radian measure do not use units.

When $\theta = 5$, it means $\theta = 5$ radians

When $\theta = 5^\circ$, it means $\theta = 5$ degrees.
Convert the following angles from degrees into radians.

\[ \theta = 70^\circ \quad \theta = 210^\circ \]

Convert the following angles from radians to degrees. Give the exact answer.

\[ \theta = \frac{2\pi}{3} \quad \theta = 1.58\pi \]
If two angles have the same initial and terminal sides, they are **coterminal angles**.

List some coterminal angles for…

\[ \theta = 55^\circ \quad \theta = -70^\circ \quad \theta = \frac{\pi}{8} \]
For smaller units than degrees we have two choices:

For an angle that is between $150^\circ$ and $151^\circ$…

**Degrees Decimal, DD**: example: $150.1234^\circ$

**Degrees, Minutes, Seconds, DMS**: example: $150^\circ 7' 24''$

- Divide one **degree** into 60 equal parts, **minutes** (')
- Divide one **minute** into 60 equal parts, **seconds** (")

$1^\circ = 60'$

$1' = 60''$

$1^\circ = ________''$
Convert the following angle to degrees, minutes, and seconds. Round to the nearest second.

$\theta = 102.1459^\circ$

Convert the following angles from degrees-minutes-seconds to decimals. Round your answer to four decimal places.

$\theta = 15^\circ 46' 32''$
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Convert the following angles from radians to degrees. For the decimal answers, round to four decimal places. For the DMS answers, round to the nearest second.

\[ \theta = \frac{-14\pi}{13} \]

Decimal: \( \theta = \)
DMS: \( \theta = \)

\( \theta = 5^\circ \)

Decimal: \( \theta = \)
DMS: \( \theta = \)
Complementary angles $\alpha, \beta; \alpha + \beta = 90^\circ$

Supplementary angles $\alpha, \beta; \alpha + \beta = 180^\circ$

Find an angle that is **Complement** and find the angle that is the **Supplement** to the given angles.

$\theta = 15.25^\circ$  \hspace{1cm} $\theta = 28^\circ 41' 29''$
**Definition of Reference Angle:** Let \( \theta \) be a nonquadrantal angle in standard position. The reference angle of \( \theta \) is the acute angle \( \theta_R \) that the terminal side of \( \theta \) makes with the \( x \)-axis.

If \( \theta \) is in QI, \( \theta_R = \theta \)
If \( \theta \) is in QII, \( \theta_R = 180^\circ - \theta \) or \( \pi - \theta \)
If \( \theta \) is in QIII, \( \theta_R = \theta - 180^\circ \) or \( \theta - \pi \)
If \( \theta \) is in QIV, \( \theta_R = 360^\circ - \theta \) or \( 2\pi - \theta \)

Find the reference angle \( \theta_R \).

\[
\begin{align*}
\theta &= 132^\circ & \theta &= 311^\circ \\
\theta &= 236^\circ & \theta &= -120^\circ 
\end{align*}
\]
Find the reference angle $\theta_R$.

$$\theta = \frac{13\pi}{4}$$

$$\theta = \frac{43\pi}{6}$$

$$\theta = \frac{7\pi}{3}$$

$$\theta = -\frac{5\pi}{6}$$
Match the following angles to the quadrant or axis on which the terminal side of the angle lies.

\[ \theta = 360^\circ \]

\[ \theta = \frac{\pi}{3} \]

\[ \theta = 750^\circ \]

\[ \theta = \frac{17\pi}{6} \]

\[ \theta = 215^\circ \]

\[ \theta = -2\pi \]

\[ \theta = -85^\circ \]

\[ \theta = \frac{-3\pi}{2} \]