1. If the point $P$ is on the graph of a function $y = f(x)$, find the corresponding point on the graph of the transformed function given below. **Keep in mind order of operation.**

$$P(-7, 9); \quad y = 4f\left(-\frac{1}{3}x\right) - 1$$

When transforming the graph of a function, look at whether the changes are taking place inside the parentheses or outside the parentheses. In this case we have both. Inside the parentheses we have $-\frac{1}{3}x$, which means we will take the inputs of the function $f$ and **divide** them by $-\frac{1}{3}$ (or multiply them by $-3$). $-\frac{1}{3}x$ indicates that we need to use inputs that 3 times larger than our original inputs in order to produce the same outputs; we also need to use inputs with the opposite sign of the original inputs. Since $-7$ is the input in the ordered pair $(-7, 9)$, I will take $-7$ and divide it by $-\frac{1}{3}$. This will produce the same result as multiplying by $-3$.

$$\text{new input} = \frac{\text{old input}}{-\frac{1}{3}}$$

$$\text{new input} = \frac{-7}{-\frac{1}{3}}$$

$$\text{new input} = -7 \cdot -3$$

$$\text{new input} = 21$$
To find the new output, I will perform the operations that are outside the parentheses on the original output of 9. Outside the parentheses we have $4f\left(-\frac{1}{3}x\right) - 1$, which means we will take the output and multiply by 4 first, then subtract 1 from the product. Keep in mind that order of operation states that multiplication is completed before addition or subtraction.

\[
\text{new output} = 4(\text{old output}) - 1
\]

\[
\text{new output} = 4(9) - 1
\]

\[
\text{new output} = 36 - 1
\]

\[
\text{new output} = 35
\]

**So our new ordered pair will be (21, 35).**

2. Let $y = f(x)$ be a function with domain $D = [-10, 10]$ and range $R = [-5, 7]$. Find the domain $D$ and range $R$ for the following transformation of the function $f(x)$. **Keep in mind order of operation.**

\[
y = -\frac{3}{2}f(x + 4) - \frac{1}{2}
\]

When transforming the graph of a function, look at whether the changes are taking place inside the parentheses or outside the parentheses. In this case we have both. Inside the parentheses we have $x + 4$, which means we will take the inputs of the function $f$ and **SUBTRACT** 4 from each of them. Since the domain of a function represents the set of inputs, I will subtract 4 from each element of the domain.
new inputs = (old inputs) − 4

new Domain = [−10 − 4, 10 − 4]

new Domain = [−14,6]

Outside the parentheses we have \(-\frac{3}{2} \cdot f\), and then that product \(-\frac{1}{2}\), which means we will take the outputs of the function \(f\), multiply them by \(-\frac{3}{2}\), and then subtract \(\frac{1}{2}\) from them. Since the range of a function represents the set of outputs, I will multiply each element of the range by \(-\frac{3}{2}\), and then subtract \(\frac{1}{2}\).

\[
\text{new outputs} = -\frac{3}{2} (\text{old outputs}) - \frac{1}{2}
\]

new Range = \([-\frac{3}{2}(-5) - \frac{1}{2}, -\frac{3}{2}(7) - \frac{1}{2}]\)

new Range = \([\frac{15}{2} - \frac{1}{2}, -\frac{21}{2} - \frac{1}{2}]\)

new Range = [7,−11]

Remember that just like a number line, an interval must go from smallest to largest when going from left to right. So the new range should be [−11,7].

new Range = [−11,7]

So the new domain is [−14, 6] and the new range is [−11, 7].