**My Steps for Factoring:**

1. **ALWAYS check for a GCF first**
   
a. this should be done regardless of how you are factoring or what type of polynomial you have (binomial, trinomial …)

2. if the polynomial has two terms (binomial), check to see if both terms are perfect squares or perfect cubes
   
a. If the two terms are perfect squares, and they are being subtracted, use the difference of squares formula
      
i. \( x^2 - y^2 = (x + y)(x - y) \)
   
b. If the two terms are perfect cubes, and they are being added or subtracted, use the sum or difference of cubes formulas
      
i. \( x^3 - y^3 = (x - y)(x^2 + xy + y^2) \)
      
ii. \( x^3 + y^3 = (x + y)(x^2 - xy + y^2) \)
   
c. If none of the above apply to a binomial, it is not factorable

3. if the polynomial has three terms (trinomial), use the \( ac \)-method

4. if the polynomial has four terms, factor by grouping

**Regardless of how you factor, ALWAYS check to see if your factors are factorable and ALWAYS factor completely (see Example 1).**

**Example 1:** Factor the following polynomial completely.

\[
x^7 + 8x^4 - 16x^3 - 128
\]

\[
x^4(x^3 + 8) - 16(x^3 + 8)
\]

\[
(x^3 + 8)(x^4 - 16)
\]

\[
(x + 2)(x^2 - 2x + 4)(x^2 - 4)(x^2 + 4)
\]

\[
(x + 2)(x^2 - 2x + 4)(x + 2)(x - 2)(x^2 + 4)
\]

\[
(x^2 + 4)(x^2 - 2x + 4)(x + 2)^2(x - 2)
\]

Notice there are no common factors among the four terms, so we can’t factor out a GCF.

Since the polynomial has four terms, the next step is to factor it by grouping.

After factoring by grouping, I end up with two binomials which are both still factorable; one using the sum of cubes and the other using the difference of squares.

Finally, I am able to factor using the difference of squares once more, before all the factors are prime.
Again, regardless of how you factor, **ALWAYS** check to see if your factors are factorable and **ALWAYS** factor completely. It is very likely that you will use more than one of the Steps for Factoring when completing the problems in HW7 and when you see similar problems on Exam 2. On Example 1 from the previous page I used factor by grouping, sum of cubes, difference of squares, and difference of squares again to factor the polynomial $x^7 + 8x^4 - 16x^3 - 128$ completely.

**Example 2:** Factor each polynomial completely.

a. $-3x^3 + 48x$

The first thing I notice is that both terms have a common factor of $3x$. Since the leading coefficient is negative, I will factor out $-3x$ rather than $3x$ in order to get a positive leading coefficient:

$-3x(x^2 - 16)$

Now I’m left with a monomial $(-3x)$ times a binomial $(x^2 - 16)$, and since both terms in the binomial $(x^2$ and 16) are perfect squares, I can factor that binomial as a difference of squares:

$-3x(x + 4)(x - 4)$

Since all three factors ($3x$, $x - 4$, and $x + 4$) are prime, this expression is factored completely.

c. $45x^3 + 90x^2 - 5x - 10$

Since this polynomial has four terms, and all four terms have a common factor of 5, I’ll start by factoring out the GCF of 5, then I’ll factor by grouping:

$5(9x^3 + 18x^2 - x - 2)$

$5(9x^2(x + 2) - 1(x + 2))$

$5(x + 2)(9x^2 - 1)$

I now have three factors, one of which I can factor as a difference of squares $(9x^2 - 1)$:

$5(x + 2)((3x)^2 - (1)^2)$

$5(x + 2)(3x - 1)(3x + 1)$

b. $x^4 - 10x^2 + 24$

Since the three terms in this polynomial ($x^4$, $-10x^2$, and 24) have no common factors, and since this is a trinomial, I will go ahead and factor it using the $ac$-method. In order to do so, I’ll need to find two numbers whose product is $ac$ (24) and whose sum is $b$ ($-10$); those two numbers are $-4$ and $-6$, so I’ll replace $-10x^2$ with $-4x^2 - 6x^2$ and factor by grouping:

$x^4 - 4x^2 - 6x^2 + 24$

$x^2(x^2 - 4) - 6(x^2 - 4)$

$(x^2 - 4)(x^2 - 6)$

I now have two factors, both of which are binomials. Since the first binomial $(x^2 - 4)$ has two terms that are both perfect squares, I can factor it using the difference of squares:

$(x^2 - 2^2)(x^2 - 6)$

$(x - 2)(x + 2)(x^2 - 6)$

Since all three factors are prime, this expression is factored completely.

d. $2x^4 + 250x$

The first thing I notice is that both terms have a common factor of $2x$:

$2x(x^3 + 125)$

Now I have a binomial $(x^3 + 125)$ that is a sum of cubes, since both terms $(x^3$ and 125) are perfect cubes. To factor the binomial, I’ll use the Sum of Cubes formula, by leaving $x$ as $x$ but replacing $y$ with 5:

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 + 5^3 = (x + 5)(x^2 - x \cdot 5 + 5^2)$

$2x(x + 5)(x^2 - 5x + 25)$
Rational expression:
- a fraction with polynomials in the numerator and denominator
  (also called a quotient or ratio of polynomials)
  \(\frac{x+2}{x^2-5x-6}\) is an example of a rational expression; the numerator is a binomial and the denominator is a trinomial

Since a rational expression is simply a fraction, it can be simplified just like a fraction by factoring the numerator and denominator, and then canceling any common factors that they may have.

**Example 1:** Simplify the following fractional expressions completely.

a. \(\frac{6}{8}\)

\[
\begin{array}{c}
2 \div 2 \\
2 \div 2
\end{array}
\]

\[
\frac{3}{4}
\]

b. \(\frac{27x^5y}{48x^3y^2}\)

\[
\begin{array}{c}
2 \div 3 \\
2 \div 3
\end{array}
\]

\[
\begin{array}{c}
2 \div 4 \\
2 \div 4
\end{array}
\]

\[
\begin{array}{c}
3 \div 3 \\
4
\end{array}
\]

\[
\frac{9x^2}{16y}
\]

**Steps for Simplifying Rational Expressions:**
1. remove parentheses and combine like terms (if necessary)
2. factor all the polynomials
3. cancel common factors

A rational expression is simplified if its numerator and denominator have no common factors other than 1, just like the rational expression \(\frac{x-2}{x+2}\).

It is imperative that you understand how to factor polynomials prior to simplify rational expressions. Keep in mind that rational expressions are simply fractions, and just like any other type of fraction, they should be simplified completely by canceling common factors.
**Example 2:** Simplify the rational expressions completely.

a. \( \frac{8 + x^3}{x^4 - 16} \)

I’ll factor the numerator as a sum of cubes (since 8 and \( x^3 \) are both perfect cubes) and I’ll factor the denominator as a difference of squares:

\[
\frac{(2 + x)(4 - 2x + x^2)}{(x^2 - 4)(x^2 + 4)}
\]

Now I’m left with a binomial times a trinomial in the numerator (neither of which are factorable), and a binomial times a binomial in the denominator (one of which is factorable). So I’ll factor \( x^2 - 4 \) in the denominator, since it’s a difference of squares, but I’ll \( x^2 + 4 \), since it’s a sum of squares, which we can’t factor:

\[
\frac{(2 + x)(4 - 2x + x^2)}{(x - 2)(x + 2)(x^2 + 4)}
\]

Now that both the numerator and denominator are factored completely, I need to cancel common factors:

\[
\frac{x^2 - 2x + 4}{(x - 2)(x^2 + 4)}
\]

b. \( \frac{5 - 17x^2 + 12x^4}{8x^9 - 8x^8} \)

I’ll factor the numerator using the \( ac \)-method (since it’s a trinomial) and I’ll factor the denominator by removing a factor of \( 8x^8 \) form both terms (since that’s the GCF):

\[
\frac{12x^4 - 17x^2 + 5}{8x^8(x - 1)}
\]

\[
\frac{12x^4 - 12x^2 - 5x^2 + 5}{8x^8(x - 1)}
\]

\[
\frac{(12x^2 - 5)(x^2 - 1)}{8x^8(x - 1)}
\]

The denominator is factored completely, since \( 8x^8 \) and \( x - 1 \) are both prime, but the numerator can be factored further since \( x^2 - 1 \) is a difference of squares:

\[
\frac{(12x^2 - 5)(x^2 - 1)}{8x^8(x - 1)}
\]

\[
\frac{(12x^2 - 5)(x - 1)(x + 1)}{8x^8(x - 1)}
\]

Now that both the numerator and denominator are factored completely, I need to cancel common factors:

\[
\frac{(12x^2 - 5)(x - 1)(x + 1)}{8x^8(x - 1)}
\]

\[
\frac{(12x^2 - 5)(x + 1)}{8x^8}
\]

c. \( \frac{3x^5 - 26x^4 - 40x^3}{\pi x^6 - 100\pi x^4} \)

\[
\frac{x^3(3x^2 - 26x - 40)}{\pi x^4(x^2 - 100)}
\]

\[
\frac{x^3(3x^2 + 4x - 30x - 40)}{\pi x^4(x - 10)(x + 10)}
\]

\[
\frac{x^3(x - 10)(3x + 4)}{\pi x^4(x - 10)(x + 10)}
\]

\[
\frac{x^2(x - 10)(3x + 4)}{\pi x^4(x - 10)(x + 10)}
\]

\[
\frac{3x + 4}{\pi(x + 10)}
\]

d. \( \frac{3 + 13x - 10x^2}{25x^2 - 1} \)

\[
\frac{-1(10x^2 - 13x - 3)}{(5x)^2 - (1)^2}
\]

\[
\frac{-1(10x^2 + 2x - 15x - 3)}{(5x + 1)(5x - 1)}
\]

\[
\frac{-1(2x(5x + 1) - 3(5x + 1))}{(5x + 1)(5x - 1)}
\]

\[
\frac{-1(5x + 1)(2x - 3)}{(5x + 1)(5x - 1)}
\]

\[
\frac{3 - 2x}{5x - 1}
\]