Example 2: Divide the following rational expressions and simplify your answers completely.

a. \[ \frac{x^2-x-6}{x^2+6x+9} \div \frac{x^2-4}{x+3} \]

b. \[ \frac{9x^2-1}{3x^2+5x-2} \div \frac{2x-1}{6x^2+17x+10} \]

On both part a. of Example 2, and part b., I will start by converting division to multiplication by taking the reciprocal of the divisor (the fraction to the right of the division sign).

\[
\frac{x^2-x-6}{x^2+6x+9} \cdot \frac{x+3}{x^2-4} \quad \frac{9x^2-1}{3x^2+5x-2} \cdot \frac{6x^2+17x+10}{2x-1}
\]

Now that division has been changed to multiplication in each example, I will multiply the numerators together and the denominators together. However I’ll simply write out the products in factored form, so I can factor further and then cancel common factors.

\[
\frac{(x^2-x-6)(x+3)}{(x^2+6x+9)(x^2-4)} \quad \frac{(9x^2-1)(6x^2+17x+10)}{(3x^2+5x-2)(2x-1)}
\]

\[
\frac{(x-3)(x+2)(x+3)}{(x+3)(x+3)(x+2)(x-2)} \quad \frac{(3x+1)(3x-1)(6x+5)(x+2)}{(3x-1)(x+2)(2x-1)}
\]

Once each fraction has been reduced, I simply leave my answers in factored form.

\[
\frac{x-3}{(x+3)(x-2)} \quad \frac{(3x+1)(6x+5)}{2x-1}
\]
c. \( \frac{x^3-25x}{4x^2} \cdot \frac{2x^2-2}{x^2-6x+5} \div \frac{x^2+5x}{7x+7} \)

d. \( \frac{x^2+5x+6}{x^2-2x-3} \div \frac{x+3}{x^2+7x+6} \cdot \frac{x^2+x-12}{x^2+x-30} \)

Keep in mind that order of operation does not state that multiplication trumps division, or that division trumps multiplication. When those are the only two operations we have in an expression, we simply work from left to right. So for each problem (part c. and part d.), I’ll combine the first two rational expressions before doing anything with the third.

\[
\left( \frac{(x)(x^2-25)(2)(x^2-1)}{(4x^2)(x-1)(x-5)} \right) \div \frac{x^2+5x}{7x+7}
\]

\[
\left( \frac{(x)(x-5)(x+5)(2)(x-1)(x+1)}{(2)(2)(x)(x-1)(x-5)} \right) \div \frac{x^2+5x}{7x+7}
\]

\[
\left( \frac{(x+5)(x+1)}{(2)(x)} \right) \div \frac{x^2+5x}{7x+7}
\]

\[
\left( \frac{(x+5)(x+1)}{(2)(x)} \right) \cdot \frac{7x+7}{x^2+5x}
\]

\[
\frac{(x+5)(x+1)(7)(x+1)}{(2)(x)(x)(x+5)}
\]

\[
\frac{7(x+1)^2}{2x^2}
\]