Quiz #8 Solution

1. Solve the following equations. If there is no solution, write NO SOLUTION. If there are infinitely many solutions, list the restrictions using the notation $\mathbb{R} - \{\} \ (all \ real \ numbers \ except)$. 

\[
\frac{5}{2x + 3} + \frac{4}{2x - 3} = \frac{14x + 3}{4x^2 - 9}
\]

\[
(2x + 3)(2x - 3) \left( \frac{5}{2x + 3} + \frac{4}{2x - 3} \right) = (14x + 3) (2x + 3)(2x - 3)
\]

\[
\frac{5(2x + 3)(2x - 3)}{2x + 3} + \frac{4(2x + 3)(2x - 3)}{2x - 3} = \frac{(14x + 3)(2x + 3)(2x - 3)}{4x^2 - 9}
\]

\[
5(2x - 3) + 4(2x + 3) = 14x + 3
\]

\[
10x - 15 + 8x + 12 = 14x + 3
\]

\[
18x - 3 = 14x + 3
\]

\[
4x = 6
\]

\[
x = \frac{6}{4}
\]

\[
x = \frac{3}{2}
\]

$x = \frac{3}{2}$ is NOT a valid answer because it results in a denominator of zero in the second rational expression and the third rational expression of the original problem. Since we had one solution, but it was not valid, we now have no solutions.

NO SOLUTION
2. Working together, Bill and Tom painted a fence in 4 hours. If Tom has painted the same fence before by himself in 7 hours, how long (in hours) would it take Bill on his own? Round your answer to one decimal place.

If Tom has painted the fence on his own in 7 hours, his rate is $\frac{1}{7}$ of the job per hour. If Bill has never painted the fence on his own, we don’t know how long it will take him, so we can say it would take him $x$ hours. Therefore his rate would be $\frac{1}{x}$ of the job per hour. Working together, Bill and Tom can paint the fence in 4 hours, so their rate is $\frac{1}{4}$ of the job per hour. Since Tom’s rate plus Bill’s rate equals their rate together, we have the following equation:

$$\text{Tom’s rate} + \text{Bill’s rate} = \text{rate together}$$

$$\frac{1}{7} + \frac{1}{x} = \frac{1}{4}$$

To eliminate the fractions, we can multiply both sides of the equation by 7, $x$, and 4. This is the same as multiplying by the least common multiply, which is 28$x$.

$$28x \left( \frac{1}{7} + \frac{1}{x} \right) = \left( \frac{1}{4} \right) 28x$$

$$4x + 28 = 7x$$

$$28 = 3x$$

$$x = \frac{28}{3}$$

Since the directions say to round answers to one decimal place, we can express the answer as 9.3 hours. So if Bill painted the fence on his own, we would expect it to take him about **9.3 hours**.