Exponential Notation:

- the expression above is read “$x$ to the power of $n$”, where $x$ is the base and $n$ is the exponent
- if no exponent is denoted, it is understood to be a power of 1
  - $x = x^1$
- if no coefficient is denoted, it is also understood to be 1
  - $x = 1 \cdot x^1$
- when the exponent $n$ is a positive integer, such as $1, 2, 3, 4, \ldots$, exponential notation represents the product of repeated factors (the base times itself some number of times)
  - $a^2 = a \cdot a$
    - the exponent of 2 indicates there are 2 factors of $a$
  - $b^5 = b \cdot b \cdot b \cdot b \cdot b$
    - the exponent of 5 indicates there are 5 factors of $b$
  - $x^n = x \cdot x \cdot \ldots \cdot x$
    - the exponent of $n$ indicates there are $n$ factors of $x$

Product Rule for Exponents:

- when common bases are multiplied, the exponents are added (order doesn’t matter)
- bases **MUST** be the same
  - $x^2x^3 = x^{2+3} = x^5$
  - $x^4y^3x^5y = x^{4+5}y^{3+1} = x^9y^4$
**Quotient Rule for Exponents:**
- when common bases are divided, the exponents are subtracted (order matters; exponent in numerator minus exponent in denominator)
- bases **MUST** be the same
  \[ \frac{x^3}{x^2} = \quad \frac{x^6y^2}{x^4y} = \]

**Power Rule for Exponents:**
- when a base is raised to a power and then raised to another power, the exponents are multiplied
  \[ (x^2)^3 = \quad ((x^3)^4)^5 = \]

*Keep in mind that the Product, Quotient, and Power Rules for Exponents are just shortcuts. You can still go the long way on these problems and simplify by writing out all the factors and combining or canceling them.*
Example 1: Simplify each expression **COMPLETELY**.

a. $x^2 y^3 z^4 x^8 y^5$

b. $\frac{x^5 y^6}{yz^3}$

c. $(x^5)^3 ((y^2)^4)^6$

d. $\left( \frac{2x^3 y}{-3xyz^6} \right) \left( \frac{6x^7 y^8 z^9}{5x^2 y^3 z} \right) \left( \frac{z^5}{2x^2 y^2} \right)$

There are a couple of different options for simplifying an expression such as $\left( \frac{2x^3 y}{-3xyz^6} \right) \left( \frac{6x^7 y^8 z^9}{5x^2 y^3 z} \right) \left( \frac{z^5}{2x^2 y^2} \right)$:

<table>
<thead>
<tr>
<th>One option is to simplify each expression completely, then combine</th>
<th>Another option is to combine each expression first, then simplify the one expression that results</th>
</tr>
</thead>
<tbody>
<tr>
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Think about these different ways to simplify expressions when completing the homework. Try each method to see one you prefer, and then try to use that one method as much as possible for consistency from problem to problem.

Also, think about how to use each rule individually, but then be sure you understand how to use them together as well. Just like in Example 1 part d., there will be problems that require you to use more than one rule to simplify an expression completely.
**Example 2:** Simplify each expression **COMPLETELY**.

a. \((x^5)^2)(2xy)\)

b. \(((x^4)^3)^2 \left(\frac{y^5z}{x^7}\right)\)

c. \((-4xy^2z^3)(5x^4y^7) \left(\frac{3x^6z^8}{-10y^5z}\right) ((y^2)^3)^4\)

\[
\begin{align*}
(-4xy^2z^3)(5x^4y^7) & \left(\frac{3x^6z^8}{-10y^5z}\right) (y^2)^{24} \\
& = \frac{-4xy^2z^3 \cdot 5x^4y^7 \cdot 3x^6z^8 \cdot y^{24}}{-10y^5} \\
& = \frac{-60x^{11}y^{33}z^{10}}{-10y^5} \\
& = 6x^{11}y^{28}z^{10}
\end{align*}
\]
d. \( \left( \frac{x^3 y^5 z^7}{xy^2} \right) \left( -\frac{2y^7}{x^6 z^5} \right) \left( \frac{8x^6 z^8}{xy^3 z^7} \right) ((1^2)^3)^\pi \)

\[
\left( x^2 y^3 z^7 \right) \left( -\frac{2y^7}{x^6 z^5} \right) \left( \frac{8x^5 z^3}{y^3} \right) (1)^6 \pi
\]

\[
\left( \frac{x^2 y^3 z^7}{1} \right) \left( -\frac{2y^7}{x^6 z^5} \right) \left( \frac{8x^5 z^3}{y^3} \right) 1
\]

\[
- \frac{(x^2 y^3 z^7)(2y^7)(8x^5 z^3)}{(x^6 z^5)(y^3)}
\]

\[
- \frac{16x^7 y^{10} z^8}{x^6 y^3 z^5}
\]

\[
-16xy^7 z^3
\]

Answers to Examples:

1a. \( x^{10}y^8z^4 \); 1b. \( \frac{x^5 y^5}{z^3} \); 1c. \( x^{15}y^{48} \); 1d. \( -\frac{2}{5}x^5 y^3 z^{37} \);

2a. \( 2x^{11}y \); 2b. \( x^{17}y^5z \); 2c. \( 6x^{11}y^{28}z^{10} \); 2d. \( -16xy^7z^3 \);