Exponential Notation: \( x^n \)
- the expression above is read “\( x \) to the power of \( n \)”, where \( x \) is the base and \( n \) is the exponent
- when the exponent \( n \) is a positive integer, such as 1, 2, 3, 4, ..., exponential notation represents the product of repeated factors (the base times itself some number of times)
  - \( a^2 = a \cdot a \)
    - the exponent of 2 indicates there are 2 factors of \( a \)
  - \( b^5 = b \cdot b \cdot b \cdot b \cdot b \)
    - the exponent of 5 indicates there are 5 factors of \( b \)
  - \( x^n = x \cdot x \cdot ... \cdot x \)
    - the exponent of \( n \) indicates there are \( n \) factors of \( x \)
- if no exponent is denoted, it is understood to be a power of 1
  - \( x = x^1 \)
- if no coefficient is denoted, it is also understood to be 1
  - \( x = 1x^1 \)

Product Rule for Exponents:
- when common bases are multiplied, the exponents are added (order doesn’t matter)
- bases **MUST** be the same
  - \( x^2x^3 = \)
  - \( x^4y^3x^5y = \)
Quotient Rule for Exponents:
- when common bases are divided, the exponents are subtracted (order matters; exponent in numerator minus exponent in denominator)
- bases **MUST** be the same
  \[ \frac{x^3}{x^2} = \]  
  \[ \frac{x^6y^2}{x^4y} = \]

Power Rule for Exponents:
- when a base is raised to a power and then raised to another power, the exponents are multiplied
  \[ (x^2)^3 = \]  
  \[ ((x^3)^4)^5 = \]

Keep in mind that the Product, Quotient, and Power Rules for Exponents are just shortcuts. You can still go the long way on these problems and simplify by writing out all the factors and combining or canceling them.
Example 1: Simplify each expression COMPLETELY.

a. \(x^2y^3z^4x^8y^5\)

b. \(\frac{x^5y^6}{yz^3}\)

c. \((x^5)^3((y^2)^4)^6\)

d. \(\left(\frac{2x^3y}{-3xyz^6}\right)\left(\frac{6x^7y^8z^9}{5x^2y^3z}\right)\left(\frac{z^5}{2x^2y^2}\right)\)

There are a couple of different options for simplifying an expression such as \(\left(\frac{2x^3y}{-3xyz^6}\right)\left(\frac{6x^7y^8z^9}{5x^2y^3z}\right)\left(\frac{z^5}{2x^2y^2}\right)\):

<table>
<thead>
<tr>
<th>One option is to simplify each expression completely, then combine</th>
<th>Another option is to combine each expression first, then simplify the one expression that results</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\left(\frac{2x^3y}{-3xyz^6}\right)\left(\frac{6x^7y^8z^9}{5x^2y^3z}\right)\left(\frac{z^5}{2x^2y^2}\right))</td>
<td>(\left(\frac{2x^3y}{-3xyz^6}\right)\left(\frac{6x^7y^8z^9}{5x^2y^3z}\right)\left(\frac{z^5}{2x^2y^2}\right))</td>
</tr>
<tr>
<td>(-12x^{10}y^9z^{44})</td>
<td>(\frac{2}{5}x^5y^3z^{37})</td>
</tr>
<tr>
<td>(-30x^5y^6z^7)</td>
<td></td>
</tr>
</tbody>
</table>

Think about these different ways to simplify expressions when completing the homework. Try each method to see one you prefer, and then try to use that one method as much as possible for consistency from problem to problem.

Also, think about how to use each rule individually, but then be sure you understand how to use them together as well. Just like in Example 1 part d., there will be problems that require you to use more than one rule to simplify an expression completely.
Example 2: Simplify each expression **COMPLETELY**.

a. \((x^5)^2)(2xy)\)

b. \(((x^4)^3)^2 \left(\frac{y^5z^3}{x^7}\right)\)

c. \((-4xy^2z^3)(5x^4y^7)\left(\frac{3x^6z^8}{-10y^5z}\right)((y^2)^3)^4\)

\[
\begin{align*}
(-4xy^2z^3)(5x^4y^7)\left(\frac{3x^6z^8}{-10y^5z}\right)(y^2)^3 &
\Rightarrow
\end{align*}
\]

\[
\begin{align*}
& (-4xy^2z^3)(5x^4y^7)\left(\frac{3x^6z^8}{-10y^5z}\right)(y^2) &
\Rightarrow
\end{align*}
\]

\[
\begin{align*}
& -4xy^2z^3 \cdot 5x^4y^7 \cdot 3x^6z^8 \cdot y^2 &
\Rightarrow
\end{align*}
\]

\[
\begin{align*}
& -10y^5 &
\Rightarrow
\end{align*}
\]

\[
\begin{align*}
& -60x^{11}y^{33}z^{10} &
\Rightarrow
\end{align*}
\]

\[
\begin{align*}
& -10y^5 &
\Rightarrow
\end{align*}
\]

\[
6x^{11}y^{28}z^{10}
\]
d. \(\left(\frac{x^3y^5z^7}{xy^2}\right)\left(-\frac{2y^7}{x^6z^5}\right)\left(\frac{8x^6z^8}{xy^3z^7}\right)\left((1^2)^3\right)^\pi\)

Once again, being able to combine topics within each lesson, from lesson to lesson, and from week to week will be imperative to succeed in this course. Understanding each topic individually is the first step, but you’ll need to be able to synthesize topics and lessons in order to be successful in this, or any other math course.
Quiz #1 will take place at the end of class on Friday (August 25th), after we cover Lesson 2. Be sure to print out the notes for Lesson 2 (both sets), bring them to class on Friday, and follow along as I go through the examples. You will get to use your lecture notes while completing the quiz.

Also, be sure to review the Quiz Rules as well as the Sample Quiz and Sample Quiz Solution from the website where I keep my notes.

Answers to Examples:

1a. $x^{10}y^8z^4$; 1b. $\frac{x^5y^5}{z^3}$; 1c. $x^{15}y^{48}$; 1d. $-\frac{2}{5}x^5y^3z^{37}$;
2a. $2x^{11}y$; 2b. $x^{17}y^5z$; 2c. $6x^{11}y^{28}z^{10}$; 2d. $-16xy^7z^3$;