Shared Work Problems:
- when two or more people (or objects) are doing a job together
- to solve shared work problems, write an equation in terms of the rate of each person or object (how much of the job they do per unit of time)
  - person A does a job in 2 hours, so their rate is $\frac{1}{2}$ the job per hour
  - person B does the same job in 3 hours, so their rate is $\frac{1}{3}$ of the job per hour
  - persons A and B do the job together in $x$ hours, so their rate together is $\frac{1}{x}$ of the job per hour
    \[
    \frac{1}{2} + \frac{1}{3} = \frac{1}{x}
    \]
- make sure the units of time are consistent
  - if one person does a job in 45 minutes, and another person does the same job in 2 hours, convert both rates to either minutes or hours
    - in terms of minutes, the equation would be $\frac{1}{45} + \frac{1}{120} = \frac{1}{x}$
- make sure your answer makes sense
  - if one person does a job in 45 minutes, and another person does the same job in 2 hours, it shouldn’t take both people working together more than 45 minutes to do the job
    - the equation $\frac{1}{45} + \frac{1}{120} = \frac{1}{x}$ results in $x = \frac{360}{11}$, which is approximately 33 minutes, so each person saves time by working together
Example 1: It takes a boy 90 minutes to mow the lawn, but his sister can mow it in an hour. How long (in minutes) would it take them to mow the lawn if they worked together using two lawn mowers?

Example 2: Two pipes can be used to fill a swimming pool. When the first pipe is closed, the second pipe can fill the pool in 9 hours. When the second pipe is closed, the first pipe can fill the pool in 7 hours. How long (in hours) will it take to fill the pool if both pipes are open? Round your answer to two decimal places.
Example 3: On average, Mary can do her homework assignments in 100 minutes. It takes Frank about 2 hours to complete a given assignment. How long (in minutes) will it take the two of them working together to complete an assignment (round to the one decimal place, if necessary)?

Mary completes her homework assignment in 100 minutes, so working at a constant rate she completes \( \frac{1}{100} \) of the assignment per minute.

Frank completes his homework assignment in 2 hours, which is 120 minutes. Again, assuming Frank works at a constant rate he completes \( \frac{1}{120} \) of the assignment per minute.

Since we don’t know how long it will take the two of them working together, I will use a variable to express that quantity (\( x \) minutes). And again, assuming they work at a constant rate they will complete \( \frac{1}{x} \) of the assignment per minute.

\[
\frac{1}{100} + \frac{1}{120} = \frac{1}{x}
\]

\[
100 \cdot 120 \cdot x \left( \frac{1}{100} + \frac{1}{120} \right) = \frac{1}{x} \cdot 100 \cdot 120 \cdot x
\]

\[
\frac{100 \cdot 120 \cdot x}{100} + \frac{100 \cdot 120 \cdot x}{120} = \frac{100 \cdot 120 \cdot x}{x}
\]

\[
\frac{120x + 100x}{100} = \frac{100 \cdot 120 \cdot x}{x}
\]

\[
120x + 100x = 12,000
\]

\[
220x = 12,000
\]

\[
x = \frac{54.54}{1}
\]

So working together it would take Mary and Frank about \( 54.5 \) minutes to complete the assignment.
Example 4: Working together, Bill and Tom painted a fence in 4 hours. If Tom has painted the same fence before by himself in 7 hours, how long (in hours) would it take Bill on his own? Round your answer to one decimal place.

Answers to Exercises:
1. 36 minutes; 2. 3.94 hours; 3. 54.5 minutes; 4. 9.3 hours