When solving equations by factoring, we showed that an equation such as $x^2 - 25 = 0$ could be solved by factoring the binomial on the left hand side of the equation, and using Zero Factor Theorem.

$$x^2 - 25 = 0$$

$$(x - 5)(x + 5) = 0$$

$$x - 5 = 0 \quad ; \quad x + 5 = 0$$

$$\boldsymbol{x = 5 \quad ; \quad x = -5}$$

There is another way to solve this type of equation for x that does not involve factoring; instead we could isolate the perfect square x^2, then take the square root of both sides of the equation to solve for x. This is known as **Extracting Square Roots**.

$$x^2 - 25 = 0$$

Solving Quadratic Equations by Extracting Square Roots:

- a quadratic equation of the form $ax^2 + c = 0$ can be solved by isolating the perfect square containing the variable x, and taking the square root of both sides of the equation

$$ax^2 + c = 0 \quad \rightarrow \quad ax^2 = -c \quad \rightarrow \quad x^2 = -\frac{c}{a} \quad \rightarrow \quad x = \pm\sqrt{-\frac{c}{a}}$$

- keep in mind that any of the following quadratic equations can be solved by extracting square roots:

$$(5x)^2 - 20 = 0 \qquad (x+1)^2 - 9 = 0 \qquad 3(x-4)^2 - 16 = 0$$

as long as we can isolate the perfect square containing the variable x and take the square root of both sides of the equation, we can use this method to solve quadratic equations

- as shown on the previous page, extracting square roots produces the same answer as if we had solved by factoring

$49 - (2x)^2 = 0$	$49 - (2x)^2 = 0$
$49 = (2x)^2$	$(7 - 2x)(7 + 2x) = 0$
$\pm\sqrt{49} = \sqrt{(2x)^2}$	$7 - 2x = 0 \;\; ; \;\; 7 + 2x = 0$
$\pm 7 = 2x$	$7 = 2x \;\; ; \;\; 7 = -2x$
$\pm\frac{7}{2} = \boldsymbol{x}$	$\frac{7}{2} = \boldsymbol{x} \;\; ; \;\; -\frac{7}{2} = \boldsymbol{x}$

Anytime you're solving quadratic equations in LON-CAPA, you're welcome to use whichever method you'd like, assuming it is a viable option. Keep in mind that in Lessons 13 and 14, we'll see quadratic equations which are not factorable and are not set-up for extracting square roots, so the methods we're covering today will not be usable. Also keep in mind that when you encounter quadratic equations like the ones shown above, or the ones we're about to see in Example 1, it will be much quicker and easier to solve by extracting square roots than to use the options we'll learn in Lessons 13 and 14. So it's best to know each method so you can use the most efficient method for each equation.

Example 1: Solve the following equations for x and enter exact answers only (no decimal approximations). If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

a. $6(5+x)^2 - 42 = 0$

b. $\frac{(2x+3)^2}{3} - 13 = 0$

In each of the previous problems, the perfect squares containing the variable x:

$$\boldsymbol{x^2}$$

$$\boldsymbol{(2x)^2}$$

$$\boldsymbol{(5+x)^2}$$

$$\boldsymbol{(2x+3)^2}$$

were isolated first **BEFORE** taking the square root of both sides. Keep in this mind when solving similar problems.

Also, keep in mind that whatever you take the square root of must be nonnegative (zero or positive). You cannot take the square root of a negative number and get a real number back, because a real number times itself is never negative. So an equation such as $x^2 = -25$ would have no real solutions, because there is no real number that can be squared to produce -25.

Example 2: Solve the equation $x^2 + 4 = 0$ for x and enter exact answers only (no decimal approximations). If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

Since the square root of a negative number does not exist with real numbers, there are no real solutions. So there is **NO SOLUTION**.

Example 3: Solve the following equations for x and enter exact answers only (no decimal approximations). If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

a. $-4x^2 + 9 = 0$

b. $(x - 3)^2 = 16$

$$x - 3 = \pm\sqrt{16}$$

$$x - 3 = \pm 4$$

$$x = 3 \pm 4$$

$$\boldsymbol{x = -1, 7}$$

c. $51 - 3(2x - 3)^2 = 0$

d. $9(x - 1)^2 + 7 = 0$

e. $\left(\frac{1}{x}\right)^2 - \frac{1}{9} = 0$

$\frac{1}{x^2} = \frac{1}{9}$

$9x^2 \cdot \left(\frac{1}{x^2}\right) = \left(\frac{1}{9}\right) \cdot 9x^2$

$9 = x^2$

$\pm\sqrt{9} = x$

$\pm 3 = x$

$\boldsymbol{x = -3, 3}$

f. $\frac{x+2}{x-2} + \frac{1}{x^2-4} = 0$

$(x + 2)(x - 2)\left(\frac{x+2}{x-2} + \frac{1}{x^2-4}\right) = (0)(x + 2)(x - 2)$

$(x + 2)^2 + 1 = 0$

$(x + 2)^2 = -1$

$x + 2 = \pm\sqrt{-1}$

$\sqrt{-1}$ does not exist with real numbers. Therefore this equation has no solution.

NO SOLUTION

Keep in mind the difference between equations such as $x = \sqrt{25}$ and $x^2 = 25$. The equation $x = \sqrt{25}$ has only one solution ($x = 5$), while the quadratic equation $x^2 = 25$ has two solutions ($x = -5$ and $x = 5$). If you are unsure why the quadratic equation $x^2 = 25$ has two real solutions instead of just one, try solving it by factoring.

Answers to Examples:

1a. $x = -5 + \sqrt{7}, -5 - \sqrt{7}$; *1b.* $x = \frac{-3-\sqrt{39}}{2}, \frac{-3+\sqrt{39}}{2}$;

2. $NO\ SOLUTION$; *3a.* $x = -\frac{3}{2}, \frac{3}{2}$; *3b.* $x = -1, 7$;

3c. $x = \frac{3}{2} \pm \frac{\sqrt{17}}{2}$; *3d.* $NO\ SOLUTION$; *3e.* $x = -3, 3$;

3f. $NO\ SOLUTION$;