When solving application problems, it is helpful to have a procedure that you follow in order to solve the problem. The following are the steps that I will use when solving Applications of Quadratic Equations:

Steps for Solving Quadratic Story Problems:

1. draw a picture
2. define unknown variables
3. set-up equations
4. solve

Once again when solving application problems I will include questions in my notes to help set-up equations. Keep in mind that the questions below in red will not appear in the homework, or on quizzes and exams; these are simply questions that you should be asking yourself when you see problems like these in order to help get equations that you can solve.

Also keep in mind the various methods that we've covered thus far for solving quadratic equations:

- solve by factoring (only works when polynomials are factorable)
  - write the equation as a polynomial set equal to zero, factor, use Zero Factor Theorem
- solve by extracting square roots (only works with perfect squares)
  - isolate the perfect square and take the square root of both sides of the equation
- solve by completing the square (works for all quadratic equations)
  - divide by the leading coefficient, isolate the constant, add half the square of the coefficient of x to both sides of the equation, factor, solve as a special quadratic equation

<u>Example 1:</u> A $12\ x\ 15$ inch picture is mounted on a wall and surrounded by a border of uniform width. The total area of the picture with the border around it is $304\ \text{in}^2$. What is the width of just the border?

Draw a diagram of the picture with the border around it (a rectangle inside of another rectangle), and list the dimensions of the inner rectangle and the dimensions of the outer rectangle.

What is the width of the border? (if you don't know, assign a variable to represent this value)

Write an equation for the total area of the picture with the border around it, and use that equation to solve for your variable.

$$total\ width\ *\ total\ length\ =\ total\ area$$

ac	***b***	**Think about the signs of the product and the sum.**
-124	27	
$-1, 124$	123	
$-2, 62$	60	
$\mathbf{-4, 31}$	$\mathbf{27}$	

$$(12 + 2x)(15 + 2x) = 304$$

$$180 + 24x + 30x + 4x^2 = 304$$

$$4x^2 + 54x + 180 = 304$$

$$4x^2 + 54x - 124 = 0$$

$$2x^2 + 27x - 62 = 0$$

$$2x^2 - 4x + 31x - 62 = 0$$

$$2x(x - 2) + 31(x - 2) = 0$$

$$(x - 2)(2x + 31) = 0$$

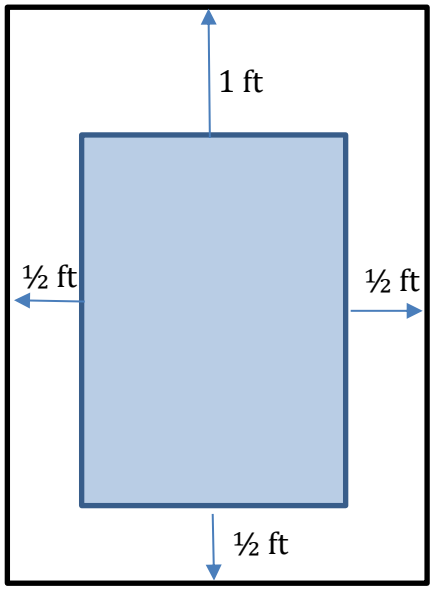
$$x - 2 = 0 \quad ; \quad 2x + 31 = 0$$

$$x = 2 \quad ; \quad x = -\frac{31}{2}$$

Since x represents the width of the border surrounding the picture, x cannot be negative. So I will disregard the negative answer and focus only on the positive answer of $\boldsymbol{x = 2}$. Therefore the width of the border is $\mathbf{2}$ inches.

<u>Example 2:</u> The blueprints for a new bathroom show a raised, rectangular bathtub with an area of 9 ft^2. The blueprints show that the raised tub will be surrounded by tile on all four sides to provide a ledge; half a foot of tile on each side of the rectanglar tub as well as at the bottom of the rectangle, and 1 foot of tile at the top of the rectangle (see picture below). The total length of the raised tub including the tile will be twice the total width. Find the dimensions of the bathtub only.

Draw a diagram of the bathtub and the surrounding tile (a rectangle inside of another rectangle), and list the dimensions of the inner rectangle and the dimensions of the outer rectangle.



Write an equation for the area of the bathtub, and use that equation to solve for your variable.

$$length\ of\ tub\ *\ width\ of\ tub\ =\ area\ of\ tub$$

$$\left(2x - \frac{3}{2}\right)(x - 1) = 9$$

$$2x^2 - 2x - \frac{3}{2}x + \frac{3}{2} = 9$$

$$2\left(2x^2 - 2x - \frac{3}{2}x + \frac{3}{2}\right) = (9)2$$

$$4x^2 - 4x - 3x + 3 = 18$$

$$4x^2 - 7x + 3 = 18$$

$$4x^2 - 7x - 15 = 0$$

$$4x^2 + 5x - 12x - 15 = 0$$

$$x(4x + 5) - 3(4x + 5) = 0$$

$$(4x + 5)(x - 3) = 0$$

$$4x + 5 = 0 \quad ; \quad x - 3 = 0$$

$$x = -\frac{5}{4} \quad ; \quad x = 3$$

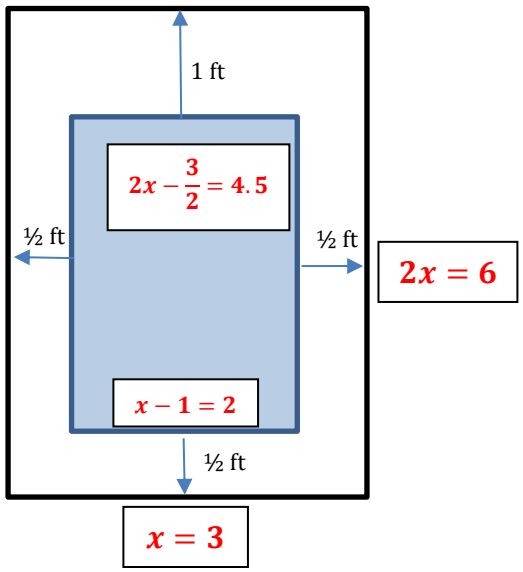
ac	***b***	**Think about the signs of the product and the sum.**
-60	-7	
$1, -60$	-59	
$2, -30$	-28	
$3, -20$	-17	
$4, -15$	-11	
$\mathbf{5, -12}$	$\mathbf{-7}$	

Once again, our variable is representing a dimension (the width of the tile border surrounding a bathtub), so it cannot be negative. Therefore once again I will disregard the negative answer, so the total width of the bathtub surrounded by tile is 3 feet.

$x = 3$ means that the total width of the bathtub plus the tile surrounding it is 3 feet; it also means the total length of the bathtub surrounded by tile is 6 feet. The remaining dimensions can be found by replacing x with 3 and $2x$ with 6 in the diagram from the previous page.

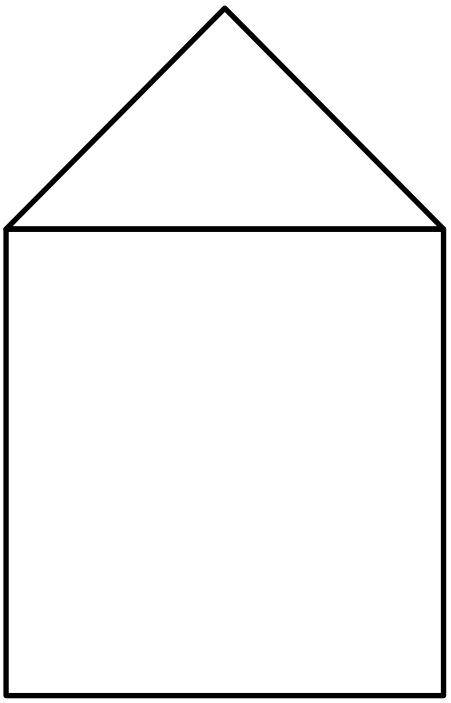
The dimensions of the bathtub only will be **2 ft.** by **4.5 ft.**

Be sure to refer back to your diagram when solving a similar problem on the homework.



<u>Example 3:</u> A gardener plans to enclose a rectangular region using fencing on three sides and part of a shed on the fourth side. The side parallel to the shed needs to be twice the length of an adjacent side. If the area of the region is $6050\ ft^2$, how many feet of fencing should be purchased?

Draw a diagram of the region, and list the length of each side.



Write an equation for the area of the rectangular region.

$$length\ of\ the\ region\ *\ width\ of\ the\ region\ =\ area$$

$$(x)(2x) = 6050$$

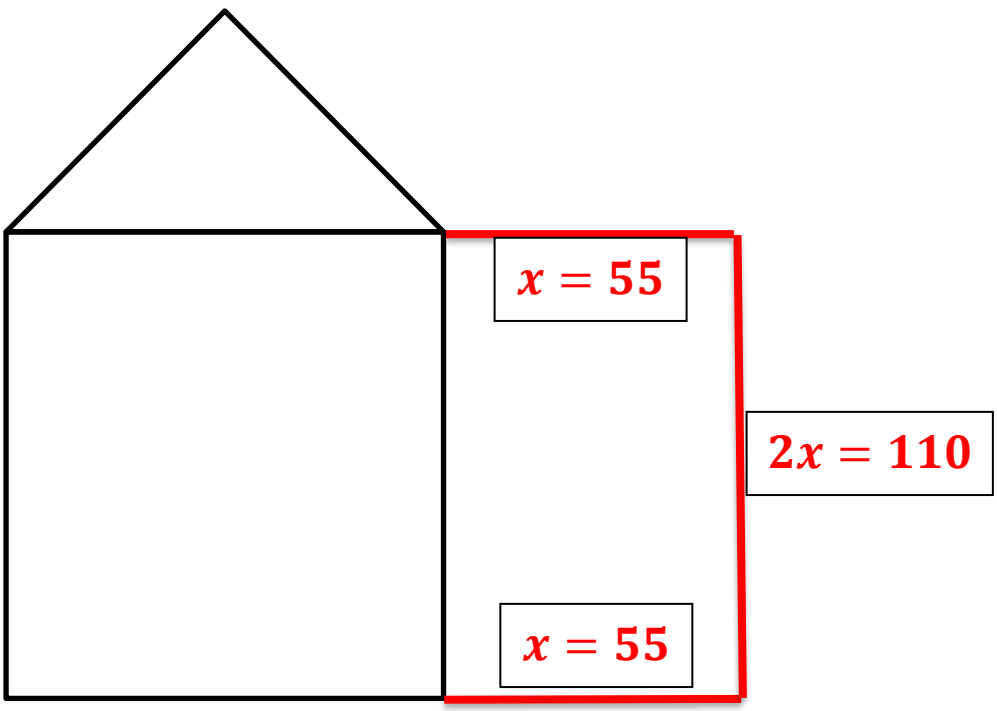
$$2x^2 = 6050$$

$$x^2 = 3025$$

$$\sqrt{x^2} = \pm\sqrt{3025}$$

$$x = \pm 55$$

Since x represents the length of a section of fence, x cannot be negative. So I will disregard the negative answer and focus only on the positive answer of $x = 55$. Referring back to our diagram, we have the following:



So we need to purchase **220 feet** of fencing to enclose a rectangular region with an area of 6050 ft^2.

Answers to Exercises:

1. 2 inches ; *2.* $2\,ft \times 4.5ft$; *3.* $220\,feet$;