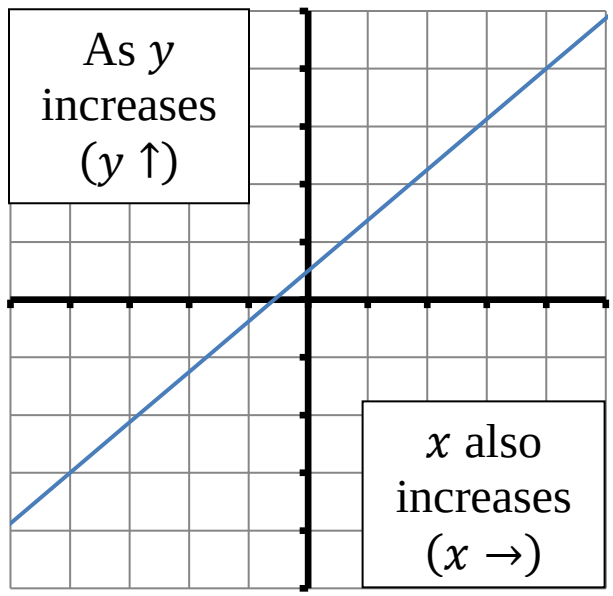
Slope:

- a ratio (or fraction) that compares the vertical change (Δy) of a graph to the horizontal change (Δx) when moving from one point to another
- denoted symbolically by m
  - $m = \frac{\Delta y}{\Delta x}$ or $\frac{\updownarrow\Delta}{\leftrightarrow\Delta}$ (the vertical change over the horizontal change)
- the slope of a line indicates whether that line is rising or falling as you move from left to right
  - a positive slope indicates that a line is rising as it moves from left to right

As the x-values are increasing, the y-values are doing the same and also increasing. Since y and x are both increasing, Δy and Δx are both positive, so the slope is positive.

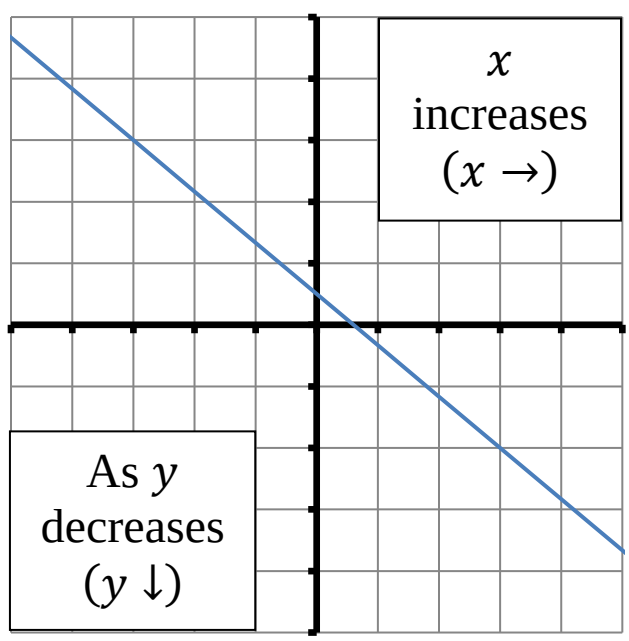
$$\boldsymbol{m} = \frac{+}{+} = +$$



  - a negative slope indicates that a line is falling as it moves from left to right

As the x-values are increasing, the y-values are doing the opposite and decreasing. Since y is decreasing and x is increasing, Δy is negative and Δx is positive, so the slope is negative.

$$\boldsymbol{m} = \frac{-}{+} = -$$



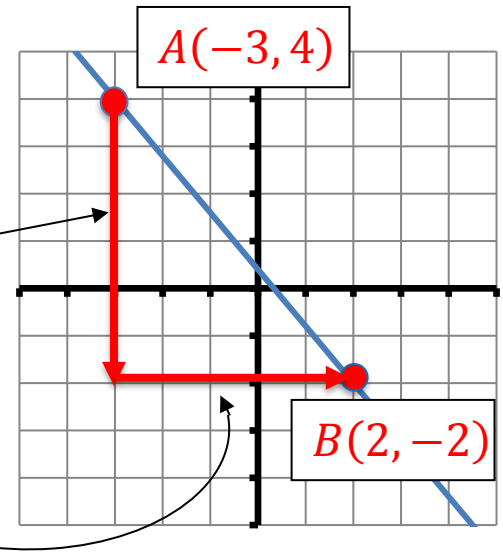
To find the slope of the line containing the points $A(-3,4)$ and $B(2,-2)$, find the Δy and the Δx when moving from one point to another. Going from point A to point B, we have the following:

$$m = \frac{\Delta y}{\Delta x}$$

Going from 4 to -2, the y value decreases by 6 ($\Delta y = -6$).

Going from -3 to 2, the x value increases by 5 ($\Delta x = 5$).

$$\boldsymbol{m = \frac{-6}{5}}$$



The slope of a line indicates whether that line is rising or falling as you move from left to right, but when finding the slope you can use points going from left to right or right to left. The slope of the line containing the points $A(-3,4)$ and $B(2,-2)$ is the same whether you move from point A to point B or from point B to point A.

$$m = \frac{\Delta y}{\Delta x}$$

Going from -2 to 4, the y value increases by 6 ($\Delta y = 6$).

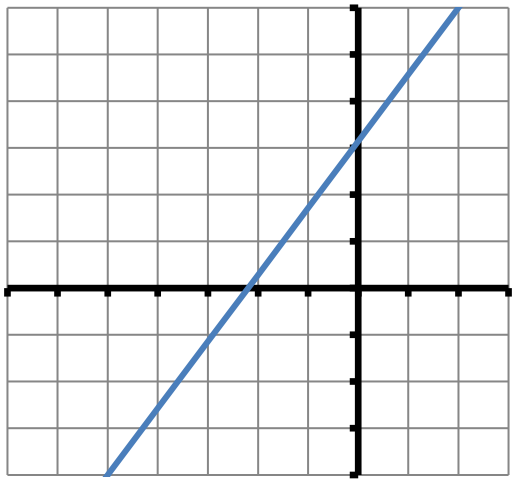
Going from 2 to -3, the x value decreases by 5 ($\Delta x = -5$).

$$\boldsymbol{m = \frac{6}{-5}}$$

Regardless of which order we use (going from point A to point B or going from point B to point A), the slope is the same. Also, using the formulas $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{y_1 - y_2}{x_1 - x_2}$, will produce the same slope as well.

<u>Example 1:</u> Find the slope of the line that passes through points A and B.

a. $A(-5,-4)$ and $B(2,6)$



b. $A(-1,4)$ and $B(7,0)$

c. $A\left(-\frac{1}{2},5\right)$ and $B(0,-9)$

d. $A\left(\frac{3}{7},-\frac{4}{5}\right)$ and $B\left(\frac{3}{5},\frac{5}{6}\right)$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{\frac{5}{6}-\left(-\frac{4}{5}\right)}{\frac{3}{5}-\frac{3}{7}}$$

$$m=\frac{\frac{49}{30}}{\frac{6}{35}}$$

$$\boldsymbol{m=\frac{343}{36}}$$

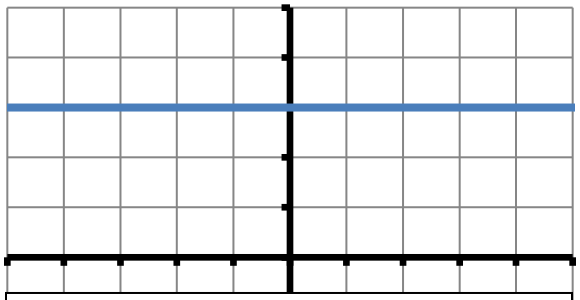
Special Cases of Slope

- horizontal lines (like the x-axis in a rectangular coordinate system) have a slope of 0 because there is no vertical change ($\Delta y = 0$)

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{0}{\Delta x}$$

$$= \mathbf{0}$$



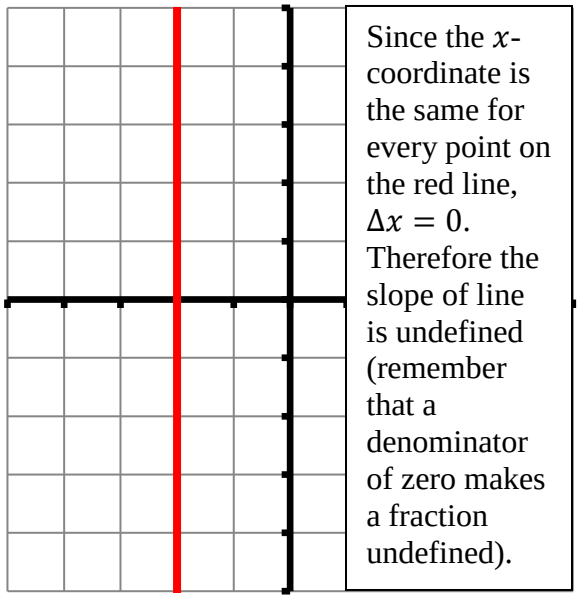
Since the y-coordinate is the same for every point on the blue line, $\Delta y = 0$. Therefore the slope of line is zero (remember that a numerator of zero makes a fraction equal to zero).

- vertical lines (like the y-axis in a rectangular coordinate system) have an undefined slope because there is no horizontal change ($\Delta x = 0$)

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{\Delta y}{0}$$

UNDEFINED



Since the x-coordinate is the same for every point on the red line, $\Delta x = 0$. Therefore the slope of line is undefined (remember that a denominator of zero makes a fraction undefined).

Example 2: Find the slope of the line that passes through points A and B, and enter exact answers only (no approximations). If the slope of line is undefined, write UNDEFINED.

a. $A(-4, -1)$ and $B(3, -1)$

b. $A(3, -4)$ and $B(3, 5)$

Answers to Examples:

1a. $m = \frac{10}{7}$ *; 1b.* $m = -\frac{1}{2}$ *; 1c.* $m = -28$ *; 1d.* $m = \frac{343}{36}$ *;*

2a. $m = 0$ *; 2b.* UNDEFINED *;*