**Slope:**
- a ratio (or fraction) that compares the vertical change ($\Delta y$) of a graph to the horizontal change ($\Delta x$) when moving from one point to another.
- denoted symbolically by $m$
  - $m = \frac{\Delta y}{\Delta x}$ or $\frac{\uparrow \Delta}{\leftrightarrow \Delta}$ (the vertical change over the horizontal change)
- the slope of a line indicates whether that line is rising or falling as you move from left to right.
  - a positive slope indicates that a line is rising as it moves from left to right.
  - a negative slope indicates that a line is falling as it moves from left to right.

As the $x$-values are increasing, the $y$-values are doing the same and also increasing. Since $y$ and $x$ are both increasing, $\Delta y$ and $\Delta x$ are both positive, so the slope is positive.

\[ m = \frac{+}{+} = + \]

- a negative slope indicates that a line is falling as it moves from left to right.

As the $x$-values are increasing, the $y$-values are doing the opposite and decreasing. Since $y$ is decreasing and $x$ is increasing, $\Delta y$ is negative and $\Delta x$ is positive, so the slope is negative.

\[ m = \frac{-}{+} = - \]
Lesson 16
Finding slope algebraically

To find the slope of the line containing the points $A(-3,4)$ and $B(2,-2)$, find the $\Delta y$ and the $\Delta x$ when moving from one point to another. Going from point $A$ to point $B$, we have the following:

$$m = \frac{\Delta y}{\Delta x}$$

Going from $4$ to $-2$, the $y$ value decreases by $6$ ($\Delta y = -6$).

Going from $-3$ to $2$, the $x$ value increases by $5$ ($\Delta x = 5$).

$$m = \frac{-6}{5}$$

The slope of a line indicates whether that line is rising or falling as you move from left to right, but when finding the slope you can use points going from left to right or right to left. The slope of the line containing the points $A(-3,4)$ and $B(2,-2)$ is the same whether you move from point $A$ to point $B$ or from point $B$ to point $A$.

$$m = \frac{\Delta y}{\Delta x}$$

Going from $-2$ to $4$, the $y$ value increases by $6$ ($\Delta y = 6$).

Going from $2$ to $-3$, the $x$ value decreases by $5$ ($\Delta x = -5$).

$$m = \frac{6}{-5}$$

Regardless of which order we use (going from point $A$ to point $B$ or going from point $B$ to point $A$), the slope is the same. Also, using the formulas $m = \frac{y_2-y_1}{x_2-x_1}$ or $m = \frac{y_1-y_2}{x_1-x_2}$, will produce the same slope as well.
**Example 1:** Find the slope of the line that passes through points $A$ and $B$.

a. $A(-5, -4)$ and $B(2, 6)$

b. $A(-1, 4)$ and $B(7, 0)$

c. $A\left(-\frac{1}{2}, 5\right)$ and $B(0, -9)$

d. $A\left(\frac{3}{7}, -\frac{4}{5}\right)$ and $B\left(\frac{3}{5}, \frac{5}{6}\right)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{5 - (-4)}{6 - 3}$

$m = \frac{\frac{5}{6} - \frac{4}{5}}{3}$

$m = \frac{\frac{49}{30}}{\frac{35}{36}}$

$m = \frac{343}{36}$
**Special Cases of Slope**

- **Horizontal lines** (like the $x$-axis in a rectangular coordinate system) have a slope of 0 because there is no vertical change ($\Delta y = 0$)

  \[
  m = \frac{\Delta y}{\Delta x} \\
  = \frac{0}{\Delta x} \\
  = 0
  \]

  [Diagram showing a horizontal line with a slope of 0.]

- **Vertical lines** (like the $y$-axis in a rectangular coordinate system) have an undefined slope because there is no horizontal change ($\Delta x = 0$)

  \[
  m = \frac{\Delta y}{\Delta x} \\
  = \frac{\Delta y}{0}
  \]

  **UNDEFINED**

  [Diagram showing a vertical line with an undefined slope.]

**Example 2:** Find the slope of the line that passes through points $A$ and $B$, and enter exact answers only (no approximations). If the slope of line is undefined, write UNDEFINED.

  a. $A(-4, -1)$ and $B(3, -1)$

  b. $A(3, -4)$ and $B(3, 5)$
Answers to Examples:

1a. \( m = \frac{10}{7} \); 1b. \( m = -\frac{1}{2} \); 1c. \( m = -28 \); 1d. \( m = \frac{343}{36} \);
2a. \( m = 0 \); 2b. UNDEFINED ;