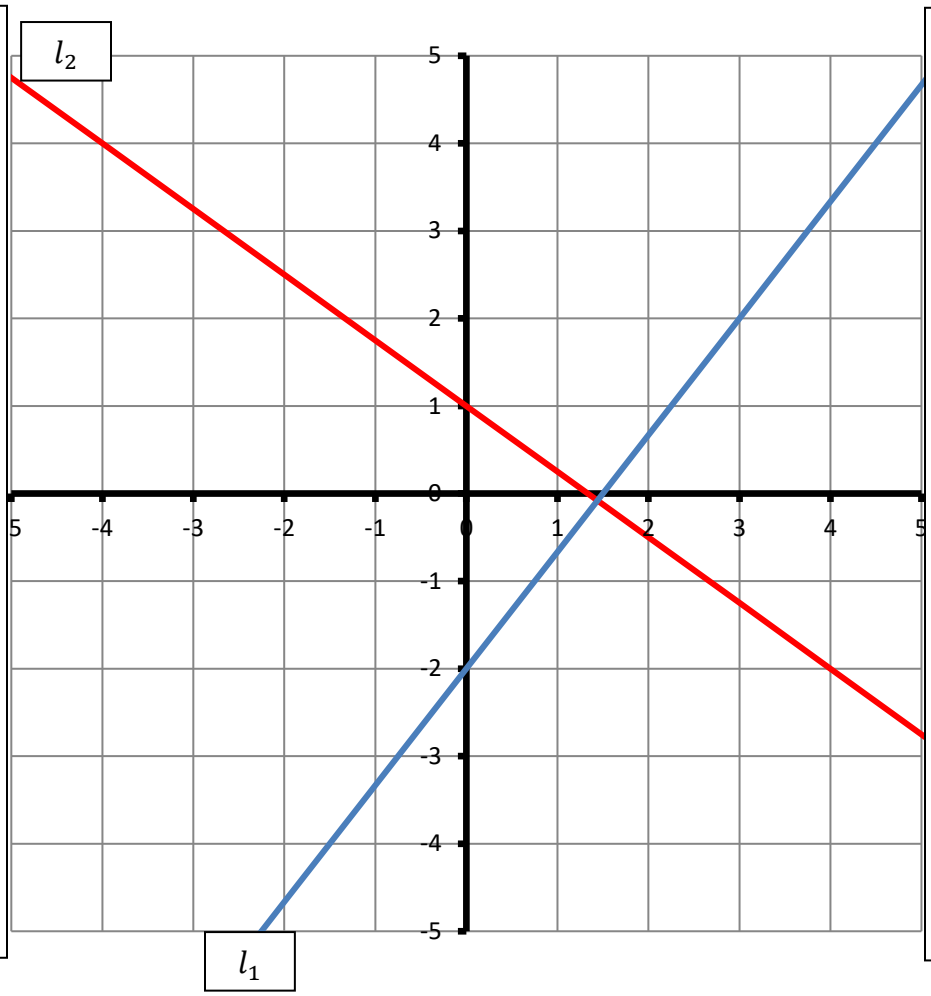
Example 1: The two lines given below are perpendicular. What do you notice about the slopes of those two lines?

To find the slope of l_1 (the blue line) or l_2 (the red line), simply pick any two points that l_1 passes through, and then find the vertical change divided by the horizontal change $\left(m = \frac{\Delta y}{\Delta x}\right)$



Two points that l_1 passes through: $(0, -2), (3,2)$

Slope of l_1: $m = \frac{\Delta y}{\Delta x} = \frac{4}{3}$

Two points that l_2 passes through: $(-4,4), (0, 1)$

Slope of l_2: $m = \frac{\Delta y}{\Delta x} = \frac{-3}{4}$

l_1 and l_2 have slopes that are the negative reciprocal of each other $\left(l_1\ m = \frac{4}{3},\ l_2\ m = -\frac{3}{4}\right)$. This is true not just for these two lines which are perpendicular, but for all perpendicular lines.

Perpendicular lines:

- two lines that intersect at a right angle (90°)
- the slopes of the lines are the negative reciprocal of each other (opposite signs, flipped over)
  - if two lines are perpendicular, and the slope of the first line is m, the slope of the second line is $-\frac{1}{m}$

<u>Example 2:</u> Find the slope of a line that is perpendicular to the line $y = \frac{2}{3}x - 4$. Enter exact answers only (no approximations).

When a linear equation is expressed in slope-intercept form, like $y = \frac{2}{3}x - 4$, the slope is simply the coefficient of x. So in this case the slope is $\frac{2}{3}$.

In order to be perpendicular to the line $y = \frac{2}{3}x - 4$, any other line must have a slope that is the negative reciprocal of that. So perpendicular lines must have a slope of $\mathbf{-\frac{3}{2}}$.

<u>Example 3:</u> Find the slope of a line that is perpendicular to the line $9x - 4y = 5$. Enter exact answers only (no approximations).

Example 4: Find the equation of the line passing through the point $A(2,-3)$ and is perpendicular to a line with a slope of $m=-\frac{8}{7}$. Enter exact answers only (no approximations), and write the equation in slope-intercept form ($y=mx+b$), if possible.

$$y-y_1=m(x-x_1)$$

$$y-(-3)=\frac{7}{8}(x-2)$$

$$y+3=\frac{7}{8}x-\frac{7}{4}$$

$$\boldsymbol{y=\frac{7}{8}x-\frac{19}{4}}$$

On this problem we need to find the equation of a line that is perpendicular to some other line with a slope of $-\frac{8}{7}$. If the line we want to be perpendicular to has a slope of $-\frac{8}{7}$, then the slope of our line needs to $\frac{7}{8}$. So to find the equation that passes through the point $A(2,-3)$ and has a slope of $\frac{7}{8}$, I replace m with $\frac{7}{8}$ and (x_1,y_1) with the point $(2,-3)$ and I plugged this information into point-slope form.

Example 5: Find the equation of the line that passes through the point $A(4,5)$ and is perpendicular to the line $y=-\frac{3}{2}x+\frac{7}{2}$. Enter exact answers only (no approximations), and write the equation in slope-intercept form ($y=mx+b$), if possible.

$$y-y_1=m(x-x_1)$$

$$y-5=\frac{2}{3}(x-4)$$

$$y-5=\frac{2}{3}x-\frac{8}{3}$$

$$\boldsymbol{y=\frac{2}{3}x+\frac{7}{3}}$$

On this problem we need to find the equation of a line that is perpendicular to a given line with a slope of $-\frac{3}{2}$. If the line we want to be perpendicular to has a slope of $-\frac{3}{2}$, then the slope of our line needs to $\frac{2}{3}$. So to find the equation that passes through the point $A(4,5)$ and has a slope of $\frac{2}{3}$, I replace m with $\frac{2}{3}$ and (x_1,y_1) with the point $(4,5)$ and I plugged this information into point-slope form.

Example 6: Find the equation of the line that passes through the point $A(7,-3)$ and is perpendicular to the line $2x-5y=8$. Enter exact answers only (no approximations), and write the equation in slope-intercept form $(y=mx+b)$, if possible.

Start by converting the equation $2x-5y=8$ to slope-intercept form (basically just get y by itself).

$2x-5y=8$

$-5y=-2x+8$

$y=\frac{2}{5}x-\frac{8}{5}$

Then find the equation of the line that parallel to the given line (remember that parallel lines have the same slope).

$y-y_1=m(x-x_1)$

$y- \quad = \quad (x- \quad)$

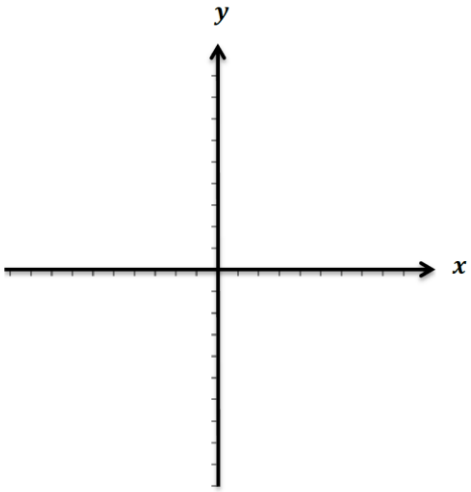
Example 7: Find the equation of the line with a y-intercept of 2, that is perpendicular to the y-axis. Enter exact answers only (no approximations), and write the equation in slope-intercept form $(y=mx+b)$, if possible.

The y-axis is a vertical line. A line perpendicular to the y-axis would be a horizontal line. That means its slope would be zero.

$m=0 \quad ; \quad (0,2)$

$y=0x+2$

$\boldsymbol{y=2}$



Keep in mind that horizontal lines are **ALWAYS** of the form $y=\#$, while vertical lines are **ALWAYS** of the form $x=\#$.

Example 8: Find the equation of the line with an x-intercept of 5, that is perpendicular to the line $5x + 9y = \frac{1}{8}$. Enter exact answers only (no approximations), and write the equation in slope-intercept form ($y = mx + b$), if possible.

On this problem I'll start by converting the linear equation $5x + 9y = \frac{1}{8}$ from general form to slope-intercept form so I can identify its slope.

$$5x + 9y = \frac{1}{8}$$

$$8(5x + 9y) = \left(\frac{1}{8}\right)8$$

$$40x + 72y = 1$$

$$72y = -40x + 1$$

$$y = \frac{-40}{72}x + \frac{1}{72}$$

$$y = -\frac{5}{9}x + \frac{1}{72}$$

The slope of the given line is $-\frac{5}{9}$. I know that in order for another line to be perpendicular to the given line, its slope must be the negative reciprocal of that $\left(\frac{9}{5}\right)$. I now have the slope of the line I'm trying to find $\left(m = \frac{9}{5}\right)$; since the line I'm trying to find has an x-intercept of 5, I also have a point that I can use, $(5, 0)$:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{9}{5}(x - 5)$$

$$\boldsymbol{y = \frac{9}{5}x - 9}$$

<u>Example 9:</u> Find the equation of the line that crosses the y-axis at $-\frac{1}{4}$ and is perpendicular to the line $y = -\frac{7}{4}x + 5$. Enter exact answers only (no approximations), and write the equation in slope-intercept form ($y = mx + b$), if possible.

Answers to Exercises:

1. negative reciprocal ; *2.* $m = -\frac{3}{2}$; *3.* $m = -\frac{4}{9}$; *4.* $y = \frac{7}{8}x - \frac{19}{4}$;
5. $y = \frac{2}{3}x + \frac{7}{3}$; *6.* $y = -\frac{5}{2}x + \frac{29}{2}$; *7.* $y = 2$; *8.* $y = \frac{9}{5}x - 9$;
9. $y = \frac{4}{7}x - \frac{1}{4}$;