**Example 1:** The two lines given below are perpendicular. What do you notice about the slopes of those two lines?

1. To find the slope of $l_1$ (the blue line) or $l_2$ (the red line), simply pick any two points that $l_1$ passes through, and then find the vertical change divided by the horizontal change ($m = \frac{\Delta y}{\Delta x}$).

2. Two points that $l_1$ passes through: $(0, -2), (3, 2)$
   - Slope of $l_1$: $m = \frac{\Delta y}{\Delta x} = \frac{4}{3}$

3. Two points that $l_2$ passes through: $(-4, 4), (0, 1)$
   - Slope of $l_2$: $m = \frac{\Delta y}{\Delta x} = \frac{-3}{4}$

$l_1$ and $l_2$ have slopes that are the negative reciprocal of each other ($l_1 \ m = \frac{4}{3}, \ l_2 \ m = \frac{-3}{4}$). This is true not just for these two lines which are perpendicular, but for all perpendicular lines.

**Perpendicular lines:**
- two lines that intersect at a right angle ($90^\circ$)
- the slopes of the lines are the negative reciprocal of each other (opposite signs, flipped over)
  - if two lines are perpendicular, and the slope of the first line is $m$, the slope of the second line is $-\frac{1}{m}$
**Example 2:** Find the slope of a line that is perpendicular to the line \( y = \frac{2}{3}x - 4 \). Enter exact answers only (no approximations).

When a linear equation is expressed in slope-intercept form, like \( y = \frac{2}{3}x - 4 \), the slope is simply the coefficient of \( x \). So in this case the slope is \( \frac{2}{3} \).

In order to be perpendicular to the line \( y = \frac{2}{3}x - 4 \), any other line must have a slope that is the negative reciprocal of that. So perpendicular lines must have a slope of \( -\frac{3}{2} \).

**Example 3:** Find the slope of a line that is perpendicular to the line \( 9x - 4y = 5 \). Enter exact answers only (no approximations).
**Example 4:** Find the equation of the line passing through the point $A(2, -3)$ and is perpendicular to a line with a slope of $m = -\frac{8}{7}$. Enter exact answers only (no approximations), and write the equation in slope-intercept form (if possible).

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{7}{8}(x - 2)$$

$$y + 3 = \frac{7}{8}x - \frac{7}{4}$$

$$y = \frac{7}{8}x - \frac{19}{4}$$

On this problem we need to find the equation of a line that is perpendicular to some other line with a slope of $-\frac{8}{7}$. If the line we want to be perpendicular to has a slope of $-\frac{8}{7}$, then the slope of our line needs to $\frac{7}{8}$. So to find the equation that passes through the point $A(2, -3)$ and has a slope of $\frac{7}{8}$, I replace $m$ with $\frac{7}{8}$ and $(x_1, y_1)$ with the point $(2, -3)$ and I plugged this information into point-slope form.

**Example 5:** Find the equation of the line that passes through the point $A(4, 5)$ and is perpendicular to the line $y = -\frac{3}{2}x + \frac{7}{2}$. Enter exact answers only (no approximations), and write the equation in slope-intercept form (if possible).
**Example 6:** Find the equation of the line that passes through the point \(A(7, -3)\) and is perpendicular to the line \(2x - 5y = 8\). Enter exact answers only (no approximations), and write the equation in slope-intercept form (if possible).

**Example 7:** Find the equation of the line with a \(y\)-intercept of 2, that is perpendicular to the \(y\)-axis. Enter exact answers only (no approximations), and write the equation in slope-intercept form (if possible).

Keep in mind that horizontal lines have a slope of zero and are of the form \(y = \#\), while vertical lines have an undefined slope and are of the form \(x = \#\).
Example 8: Find the equation of the line with an \( x \)-intercept of 5, that is perpendicular to the line \( 5x + 9y = \frac{1}{8} \). Enter exact answers only (no approximations), and write the equation in slope-intercept form (if possible).

On this problem I’ll start by converting the linear equation \( 5x + 9y = \frac{1}{8} \) from general form to slope-intercept form so I can identify its slope.

\[
5x + 9y = \frac{1}{8}
\]

\[
9y = -5x + \frac{1}{8}
\]

\[
y = -\frac{5}{9}x + \frac{1}{72}
\]

Now that I know the slope of the given line is \(-\frac{5}{9}\), I know that in order for another line to be perpendicular to the given line, its slope must be the negative reciprocal of that \( \left( \frac{9}{5} \right) \). So I now have the slope of the line I’m trying to find \( (m = \frac{9}{5}) \). And since the line I’m trying to find has an \( x \)-intercept of 5, I also have a point that I can use \((5, 0)\). So now I’ll plug this information into point-slope form:

\[
y - y_1 = m(x - x_1)
\]

\[
y - 0 = \frac{9}{5}(x - 5)
\]

\[
y = \frac{9}{5}x - 9
\]
Example 9: Find the equation of the line that crosses the y-axis at $-\frac{1}{4}$ and is perpendicular to the line $y = -\frac{7}{4}x + 5$. Enter exact answers only (no approximations), and write the equation in slope-intercept form (if possible).

Answers to Exercises:

1. negative reciprocal
2. $m = -\frac{3}{2}$
3. $m = -\frac{4}{9}$
4. $y = \frac{7}{8}x - \frac{19}{4}$
5. $y = \frac{2}{3}x + \frac{7}{3}$
6. $y = -\frac{5}{2}x + \frac{29}{2}$
7. $y = 2$
8. $y = \frac{9}{5}x - 9$
9. $y = \frac{4}{7}x - \frac{1}{4}$