To determine the value of a function for a particular input, we replace the original input of the function (usually \(x\)) with the new input. For instance, to determine the value of the function \(f(x) = \frac{x+2}{1-x}\) when \(x = -1\), we would replace \(x\) (the original input) with \(-1\) (the new input). This means not only replacing the \(x\) in parentheses with \(-1\), but also replacing any \(x\)’s in the expression that defines the function with \(-1\)’s.

\[
f(x) = \frac{x + 2}{1 - x}
\]

\[
f(-1) = \frac{-1 + 2}{1 - (-1)}
\]

\[
f(-1) = \frac{1}{1 + 1}
\]

\[
f(-1) = \frac{1}{2}
\]

This shows that for the function \(f(x) = \frac{x+2}{1-x}\), when \(-1\) is the input \((x = -1)\), \(\frac{1}{2}\) is the output \((f(-1) = \frac{1}{2})\). The same idea is true when finding \(f(0)\); simply replace all the \(x\)’s in the function with \(0\)’s.

\[
f(x) = \frac{x + 2}{1 - x}
\]

\[
f(0) = \frac{0 + 2}{1 - 0}
\]

\[
f(0) = \frac{2}{1}
\]

\[
f(0) = 2
\]
One function value we would not be able to find would be $f(1)$. Keep in mind that any input of a function must be part of the domain of that function, and in this case, 1 is not part of the domain of $f$. We could not find the value of the function $f$ when $x$ is replaced with 1 because 1 would result in a denominator of zero.

$$f(x) = \frac{x + 2}{1 - x}$$

$$f(1) = \frac{1 + 2}{1 - 1}$$

$$f(1) = \frac{3}{0}$$

$f(1)$ is **UNDEFINED**

Division by zero is not defined for any function, so that is why we say $f(1)$ is undefined. This is one reason it is important to be able to find the domain of a function; so that we know what we can (and what we cannot) plug into the function for $x$.

**When working with a function that contains a square root, ALWAYS state that the radicand must be greater than or equal to zero.**

**When working with a function that contains a fraction, ALWAYS state that the denominator can never be equal to zero.**
**Example 1:** Given \( g(x) = 2 - 5\sqrt{x - 3} \), find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

a. Find the domain of \( g \) and list in interval notation

b. Find \( g(4) \); *When \( x \) is replaced with 4, the output is ?*

c. Find \( g(3) \); *When \( x \) is replaced with 3, the output is ?*

d. Find \( g(2) \); *When \( x \) is replaced with 2, the output is ?*

Keep in mind that when finding the domain of the function \( g(x) = 2 - 5\sqrt{x - 3} \), we only focused on the radicand \( x - 3 \) being non-negative \( (x - 3 \geq 0) \). However when finding the function values, we used the entire expression \(-5\sqrt{x - 3}\):

\[
\begin{align*}
g(4) &= 2 - 5\sqrt{4 - 3} \\
g(3) &= 2 - 5\sqrt{3 - 3}
\end{align*}
\]
Example 2: Given \( h(x) = \sqrt{1 - \frac{x}{3}} \), find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

\[
1 - \frac{x}{3} \geq 0 \\
3 - x \geq 0 \\
3 \geq x
\]

Domain of \( h \): \((-\infty, 3]\)

a. \( h(-9) \) 
   \[
h(-9) = \sqrt{1 - \frac{-9}{3}} = \sqrt{1 + 3} = \sqrt{4} = 2
\]

b. \( h(0) \) 
   \[
h(0) = \sqrt{1 - \frac{0}{3}} = \sqrt{1} = 1
\]

Since 6 is not part of the domain of the function, \( h(6) \) is not defined.

Example 3: Given \( j(x) = \frac{x^2 - 4}{x^2 + 4} \), find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

Domain of \( j \):

a. \( j(-2) \) 

b. \( j(0) \) 
   \[
j(0) = \frac{0^2 - 4}{0^2 + 4} = \frac{-4}{4} = -1
\]

Since \( x = 0 \) is not part of the domain of the function, \( j(0) \) is not defined.

   c. \( j(2) \) 
   \[
j(2) = \frac{2^2 - 4}{2^2 + 4} = \frac{0}{8} = 0
\]
Example 4: Given \( k(x) = \frac{1}{x\sqrt{1+x}} \), find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

**Domain of \( k \):**

\[
x\sqrt{1+x} \neq 0 \quad \text{AND} \quad 1 + x \geq 0 \]

\[
(x\sqrt{1+x})^2 \neq (0)^2 \quad \text{AND} \quad x \geq -1
\]

\[
(x\sqrt{1+x})(x\sqrt{1+x}) \neq 0 \quad \text{AND} \quad -1 \leq x
\]

\[
x^2(1+x) \neq 0 \quad \text{AND} \quad -1 \leq x
\]

\[
x^2 \neq 0 \quad \text{AND} \quad 1 + x \neq 0 \quad \text{AND} \quad -1 \leq x
\]

\[
x \neq 0 \quad \text{AND} \quad x \neq -1 \quad \text{AND} \quad -1 \leq x
\]

\[
x \neq 0 \quad \text{AND} \quad -1 < x
\]

\[-1 < x \quad x \neq 0\]

\[\text{(-1, 0) } \cup \text{ (0, } \infty)\]

b. \( k(-1) \)

b. \( k(0) \)

c. \( k(3) \)
Example 5: Given \( m(x) = \frac{\sqrt{x}}{2-\sqrt{x}+1} \), find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

Domain of \( m \):

\[ x = 0 \]

a. \( m(2) \)  

b. \( m(3) \)  

c. \( m(4) \)
Example 6: Given \( f(x) = \frac{x^2+4}{(x+2)\sqrt{x-2}} \), find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

**Domain of \( n \):**

\[
(x + 2)\sqrt{x - 2} \neq 0 \quad \text{AND} \quad x - 2 \geq 0
\]

\[
\left( (x + 2)\sqrt{x - 2} \right)^2 \neq (0)^2 \quad \text{AND} \quad x \geq 2
\]

\[
(x + 2)^2 (\sqrt{x - 2})^2 \neq 0 \quad \text{AND} \quad 2 \leq x
\]

\[
(x + 2)^2 (x - 2) \neq 0 \quad \text{AND} \quad 2 \leq x
\]

\[
(x + 2)^2 \neq 0 \quad \text{AND} \quad x - 2 \neq 0 \quad \text{AND} \quad 2 \leq x
\]

\[
x + 2 \neq 0 \quad \text{AND} \quad x \neq 2 \quad \text{AND} \quad 2 \leq x
\]

\[
x \neq -2 \quad \text{AND} \quad 2 < x
\]

\[
x = 2
\]

\[
2 < x
\]

\[
(2, \infty)
\]

a. \( n(2) \)  

b. \( n(3) \)  

c. \( n(6) \)
Answers to Exercises:
1a. Domain of h: [3, ∞); 1b. −3; 1c. 2; 1d. UNDEFINED;
2. Domain of g: (−∞, 3]; 2a. 2; 2b. 1; 2c. UNDEFINED;
3. Domain of j: (−∞, ∞); 3a. 0; 3b. −1; 3c. 0;
4. Domain of k: (−1, 0) ∪ (0, ∞); 4a. UNDEFINED;
4b. UNDEFINED; 4c. \(\frac{1}{6}\);
5. Domain of m: [0, 3) ∪ (3, ∞); 5a. \(\frac{\sqrt{2}}{2-\sqrt{3}}\); 5b. UNDEFINED;
5c. \(\frac{2}{2-\sqrt{5}}\);
6. Domain of n: (2, ∞); 6a. UNDEFINED; 6b. \(\frac{13}{5}\); 6c. \(\frac{5}{2}\);