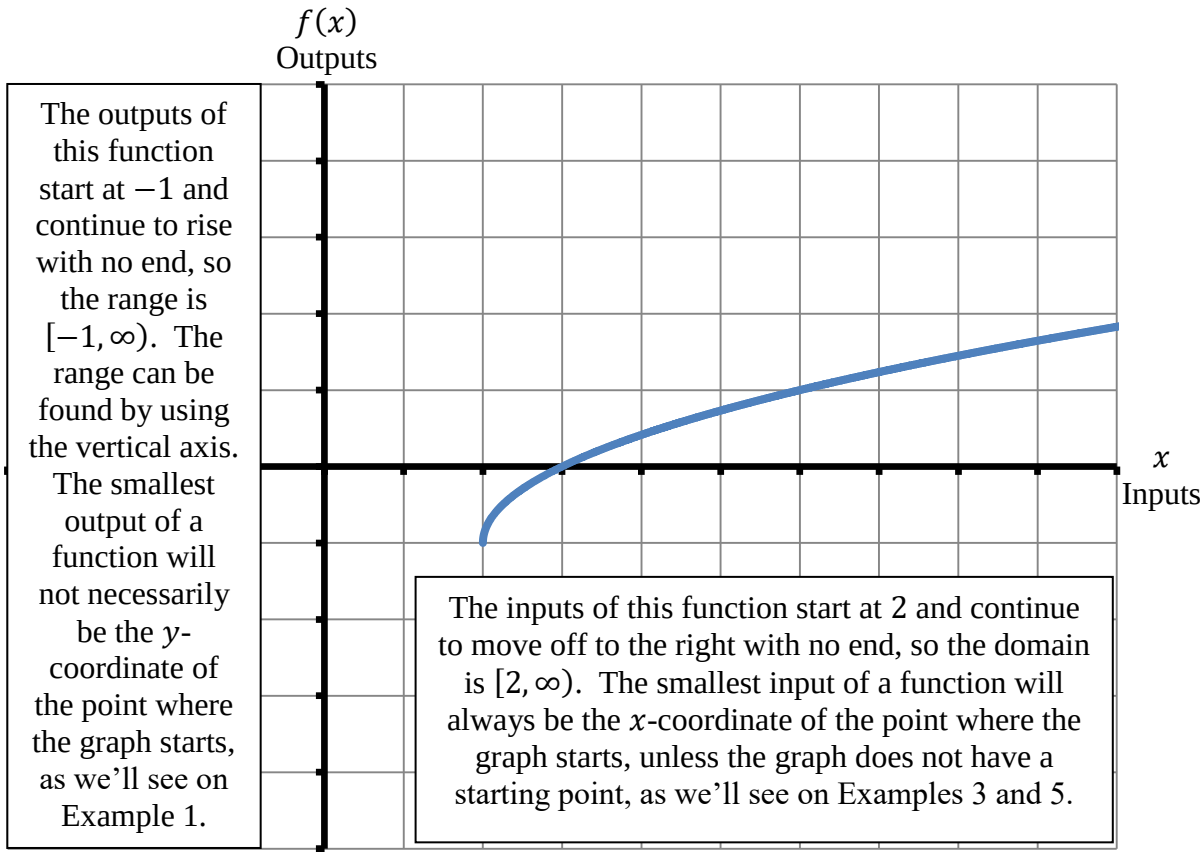
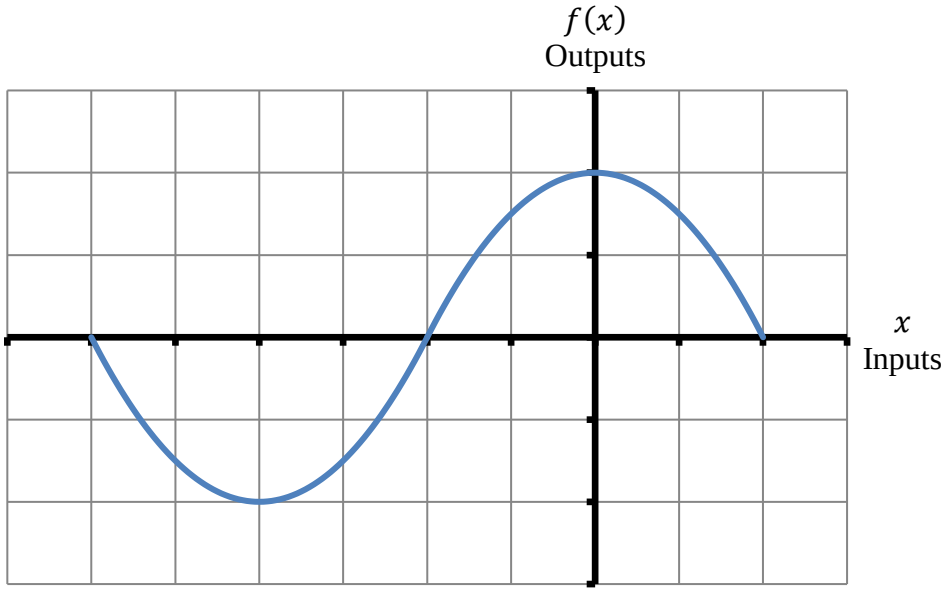
We've seen how to work with functions algebraically, by finding domains as well as function values. In this set of notes we'll be working with functions graphically, and we'll see how to find the domain and range of a function based on its graph, as well as how to find functions values graphically.

When working with the graph of a function, the inputs (the elements of the domain) are always the values on the horizontal axis (x-axis) and the outputs (the elements of the range) are always the values on the vertical axis (y-axis). This means ordered pairs are written $(\text{input}, \text{output})$. In the graph below, the ordered pair $(2, -1)$ means that when 2 goes into the function, -1 comes out, or $f(2) = -1$.



To identify different features about the graph of a function, we will use the axes and ordered pairs from the function. Keep in mind that the domain of a function is the set of inputs, and inputs lie on the horizontal axis. Also, keep in mind that the range of a function is the set of outputs, and outputs lie on the vertical axis.

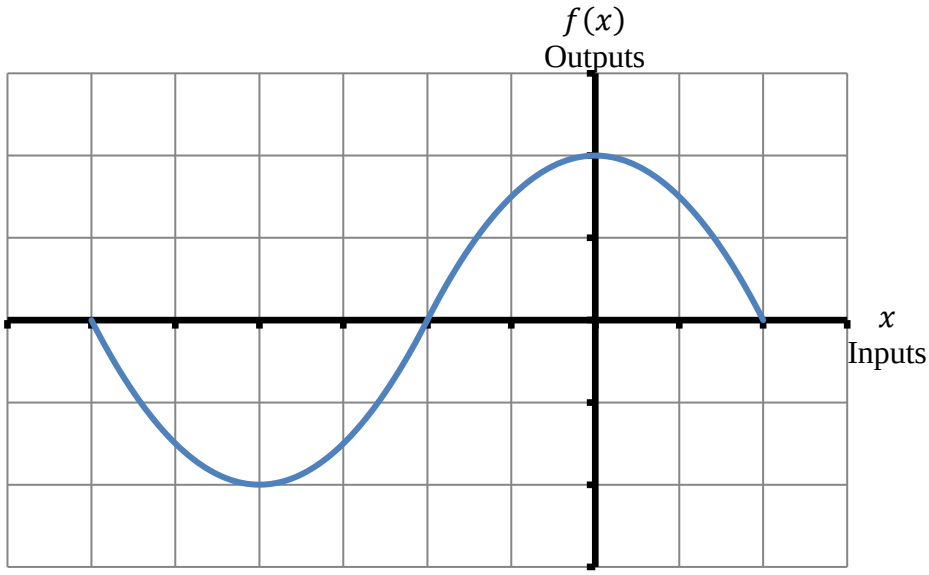
Example 1: Use the graph of the function f given below to find the following.



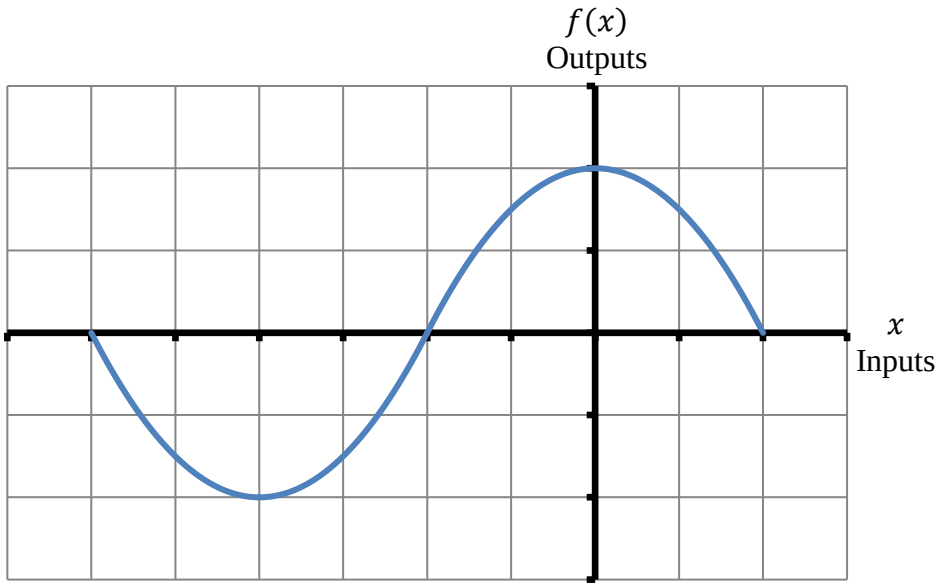
a. Find the domain of f and list your answer in interval notation

b. Find the range of f and list your answer in interval notation

c. Find $f(-4)$ d. Find $f(0)$ e. Find $f(2)$



On parts c., d., and e., we're using inputs of the function f to find outputs. To do so, it might be helpful to write ordered pairs, and to keep in mind that ordered pairs for a function are expressed $(\text{input}, \text{output})$.



f. Find all the x-values, such that $f(x) = -2$

g. Find all the x-values, such that $f(x) = 2$

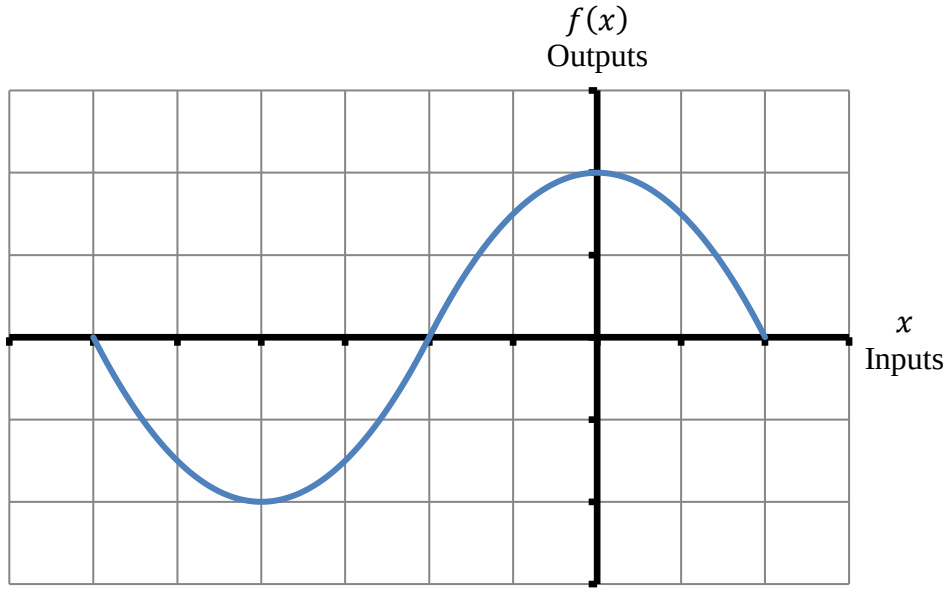
h. Find all the x-values, such that $f(x) = 0$

On parts f., g., and h., we're using outputs of the function f to find inputs. Once again, it might be helpful to write ordered pairs, and to keep in mind that ordered pairs for a function are expressed $(\text{input}, \text{output})$.

The x-values that make a function equal to zero are called the Zeros of the Function. These are simply the values on the horizontal axis (the x-axis) where the graph of the function touches or crosses the axis. The Zeros of a Function are just the x-values that make a function equal to zero.

The Zeros of a Function are important not only for identifying the x-intecepts of a function (which we'll see later), but also for determining the intervals where a function is positive or negative. These intervals are formed by the zeros of the function.

<u>Example 2:</u> Use the graph of the function f given below to find the following.



a. Find the zeros of the function ($f(x) = 0$ when $x =$?), and label them on the graph of $f(x)$

$$\boldsymbol{x = -6, -2, 2}$$

These are just the x-value that make the function zero.

b. Find all the x-values, such that $f(x) > 0$ and list your answers in interval notation.

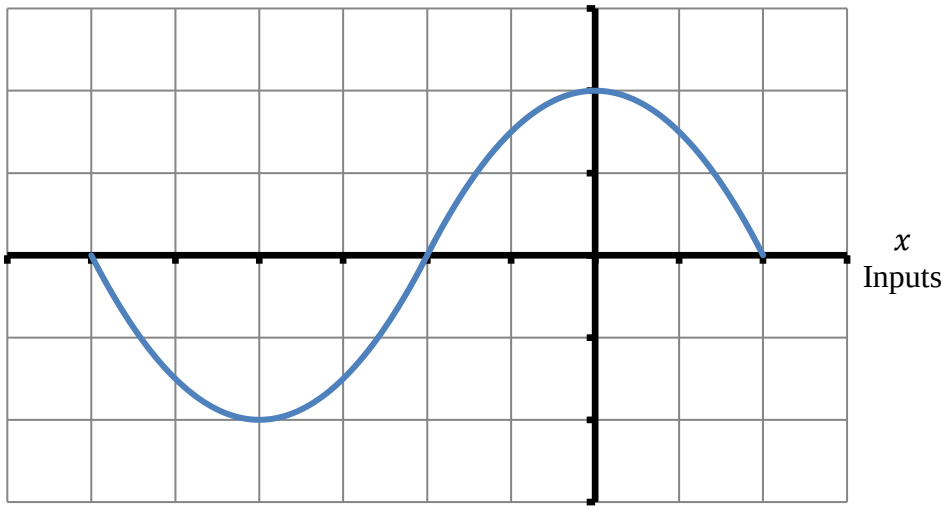
Where does the function lie above the x-axis?

c. Find all the x-values, such that $f(x) < 0$ and list your answers in interval notation

Where does the function lie below the x-axis?

The zeros of a function, along with the positive interval and the negative interval, make up the entire domain of a function. Basically every input of a function will produce a positive output, a negative output, or an output of zero. You can refer back to your domain, $[-6, 2]$ for the graph given above, to verify that every element of the domain is a zero, or is part of the positive interval or the negative interval.

<u>Example 2 Continued:</u> Use the graph of the function f given below to find the following.



d. Find the x-intercepts and the y-intercepts of the function, and list your answers as ordered pairs

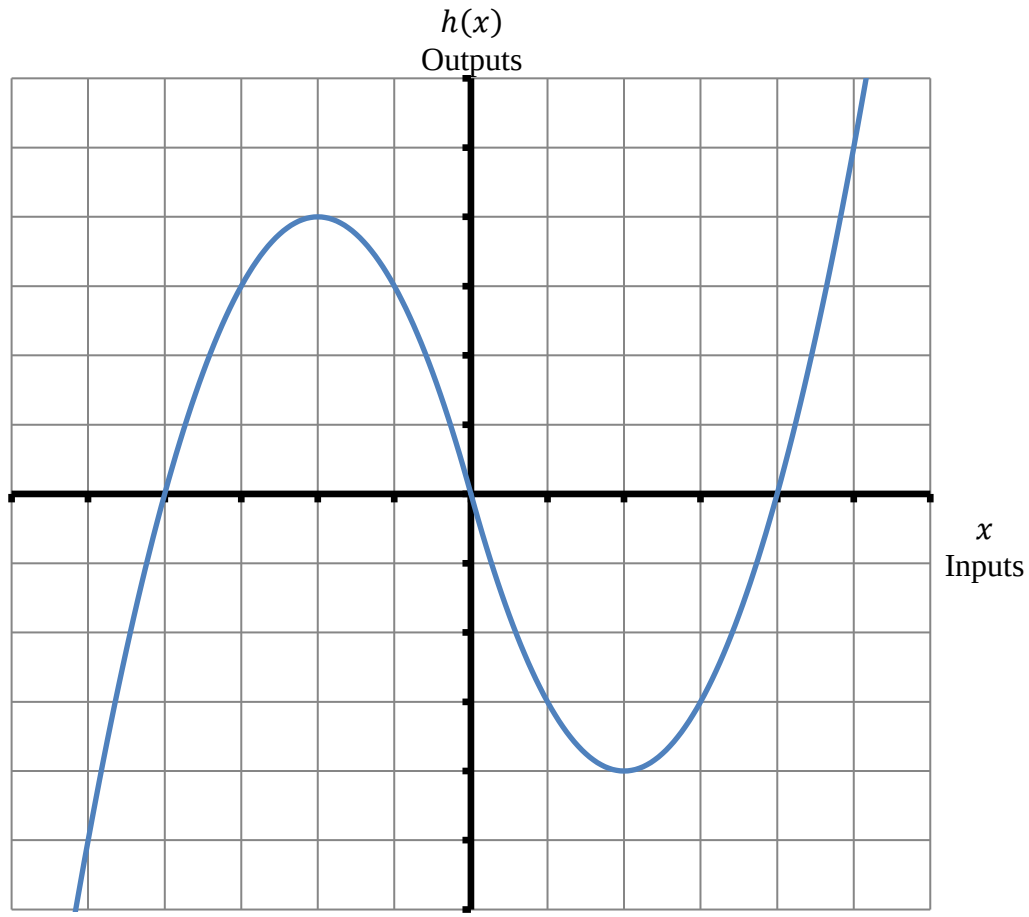
$$(-6, 0), (-2, 0), (2, 0), (0, 2)$$

These are the actual ordered pairs that represent the points where the graph touches or crosses the x-axis or the y-axis.

<u>Things to remember about graphs of functions:</u>

1. The domain is the set of inputs, and it is found by determining how far to the left and right a graph moves (do not simply look at where the graph touches the x-axis)
2. The range is the set of outputs, and it is found by determining how far down and up the graph goes
3. The zeros are the inputs that produce an output of zero; these are just the x-values where a graph touches or crosses the x-axis
4. Every element of the domain will be a zero, or will be part of the negative interval or the positive interval
5. Intercepts are points on a graph, and should always be listed as ordered pairs.

<u>Example 3:</u> Use the graph of the function h given below to find the following.



a. Find the domain of h and list your answer in interval notation

Domain: $(-\infty, \infty)$

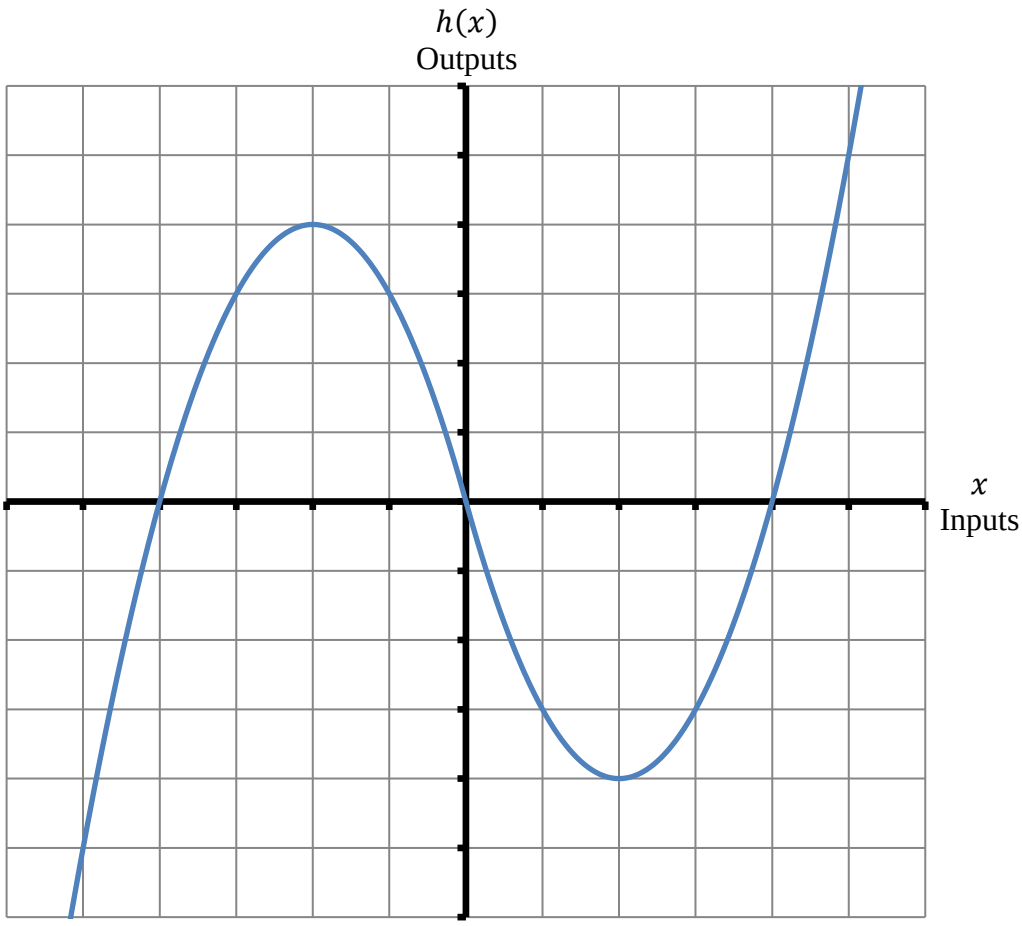
Since the graph of this function goes to the left and to the right without stopping, the domain of the function (the set of inputs) goes on to negative infinity and to positive infinity.

b. Find the range of h and list your answer in interval notation

Range: $(-\infty, \infty)$

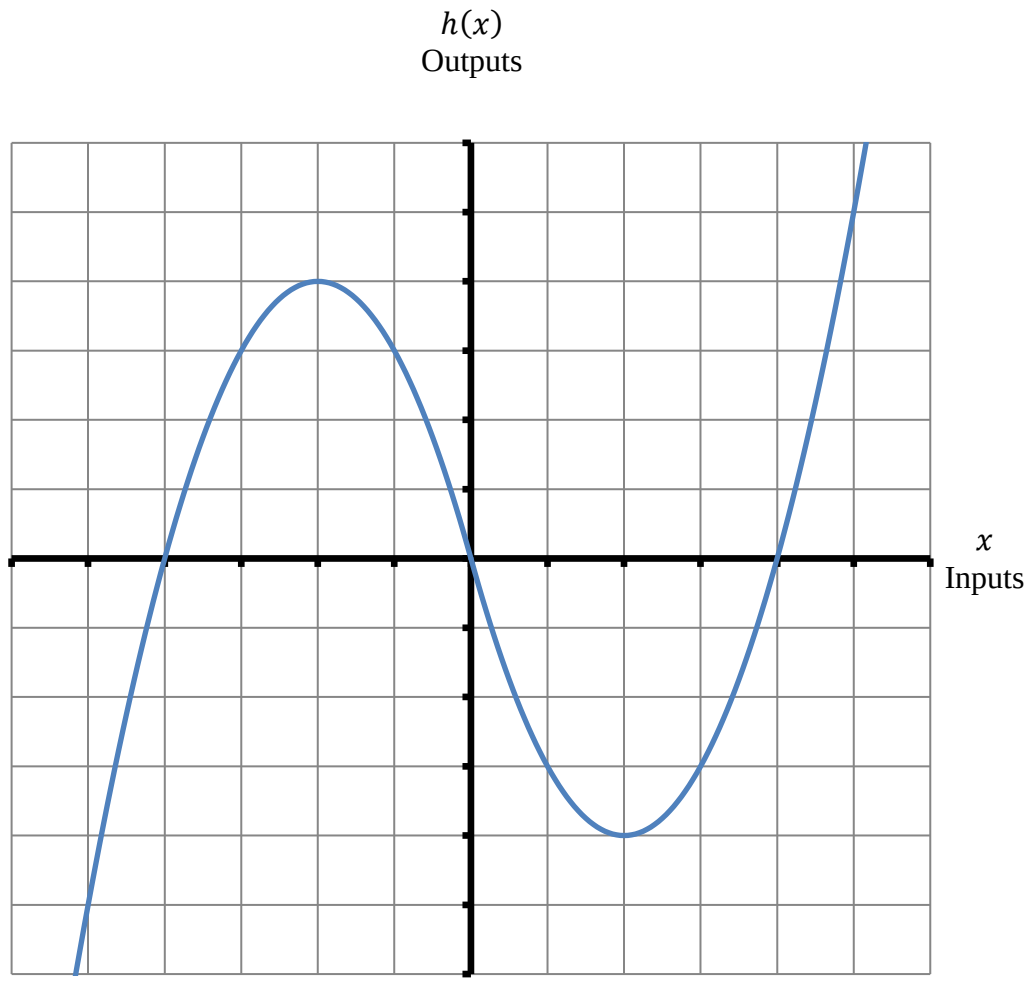
Since the graph of this function goes to the left and to the right without stopping, the domain of the function (the set of inputs) goes on to negative infinity and to positive infinity.

c. Find $h(-5)$ d. Find $h(0)$ e. Find $h(3)$



f. Find all the x-values, such that $h(x) = 5$

g. Find the zeros of the function ($h(x) = 0$ when $x =$?)



h. Find all the x-values, such that $h(x) > 0$ and list your answers in interval notation

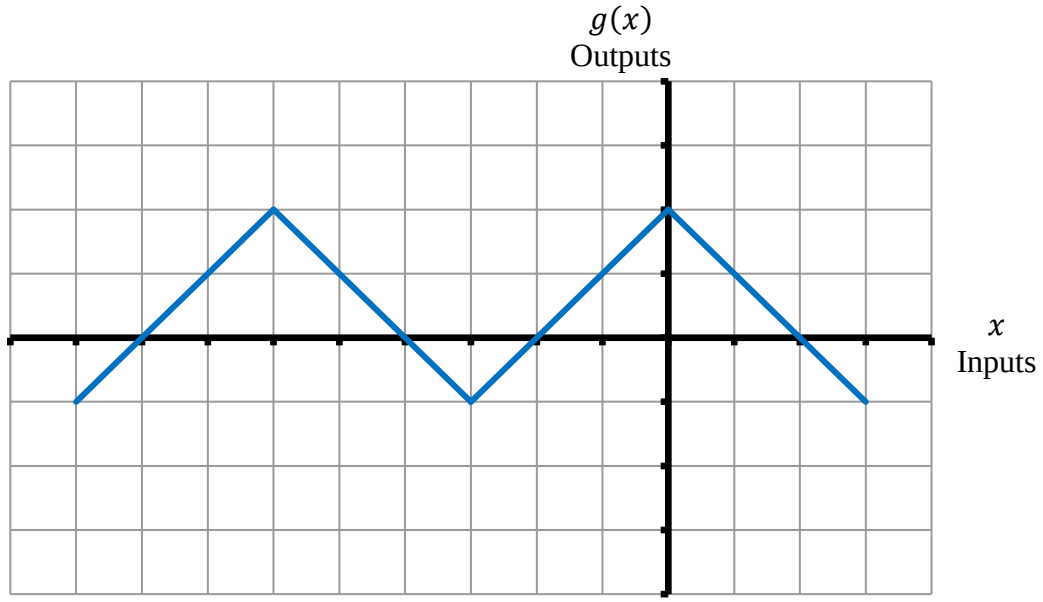
Where does the function lie above the x-axis?

i. Find all the x-values, such that $h(x) < 0$ and list your answers in interval notation

Where does the function lie below the x-axis?

j. Find the x-intercepts and the y-intercepts of the function, and list your answers as ordered pairs

Example 4: Use the graph of the function g given below to find the following. In each case, think about whether you are looking for an input or an output.



a. Find the domain of g and list your answer in interval notation

$[-9, 3]$

b. Find the range of g and list your answer in interval notation

$[-1,2]$

c. Find $g(-7)$

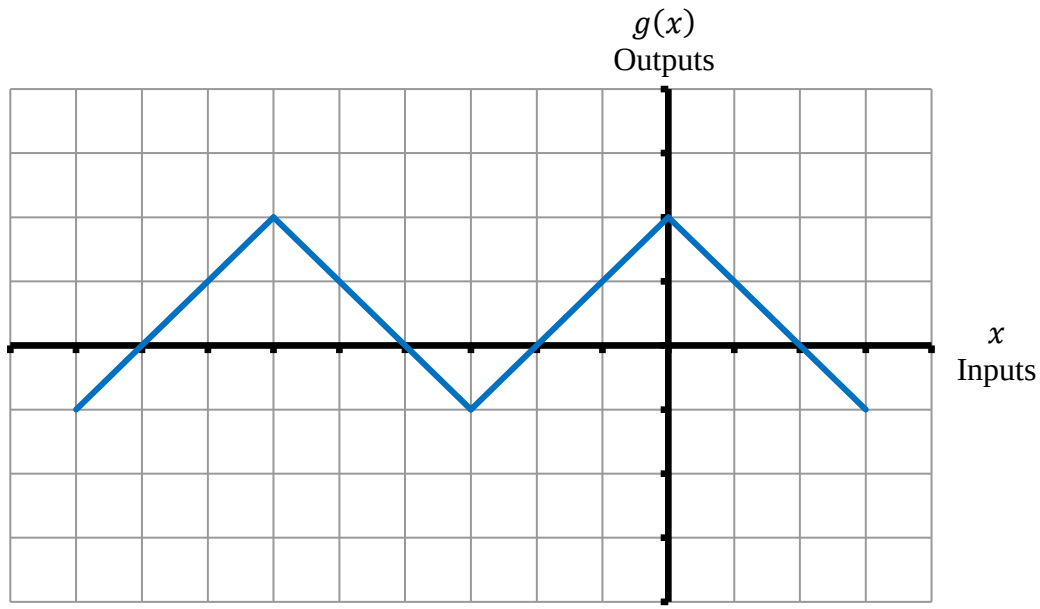
$g(-7) = 1$

d. Find $g(0)$

$g(0) = 2$

e. Find $g(3)$

$g(3) = -1$



f. Find all the x-values, such that $g(x) = -1$

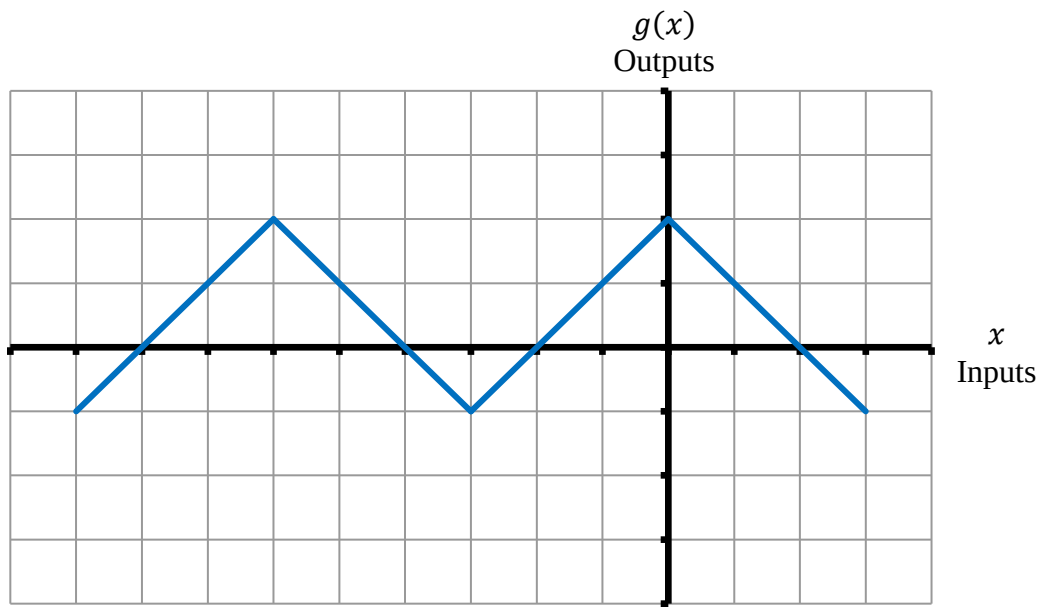
$x = -9, -3, 3$, because $g(-9) = -1$, $g(-3) = -1$, and $g(3) = -1$

g. Find all the x-values, such that $g(x) = 3$

There is no x-value that will make $g(x) = 3$, so NONE.

h. Find the zeros of the function ($g(x) = 0$ when $x =$?)

$x = -8, -4, -2, 2$, because
$g(-8) = 0$, $g(-4) = 0$, $g(-2) = 0$ and $g(2) = 0$



i. Find all the x-values, such that $g(x) > 0$ and list your answers in interval notation

$g(x) > 0$ when x is part of the interval $(-8, -4) \cup (-2, 2)$

j. Find all the x-values, such that $g(x) < 0$ and list your answers in interval notation

$g(x) < 0$ when x is part of the interval $[-9, -8) \cup (-4, -2) \cup (2, 3]$

-9 and 3 both have square brackets because they actually produce negative outputs

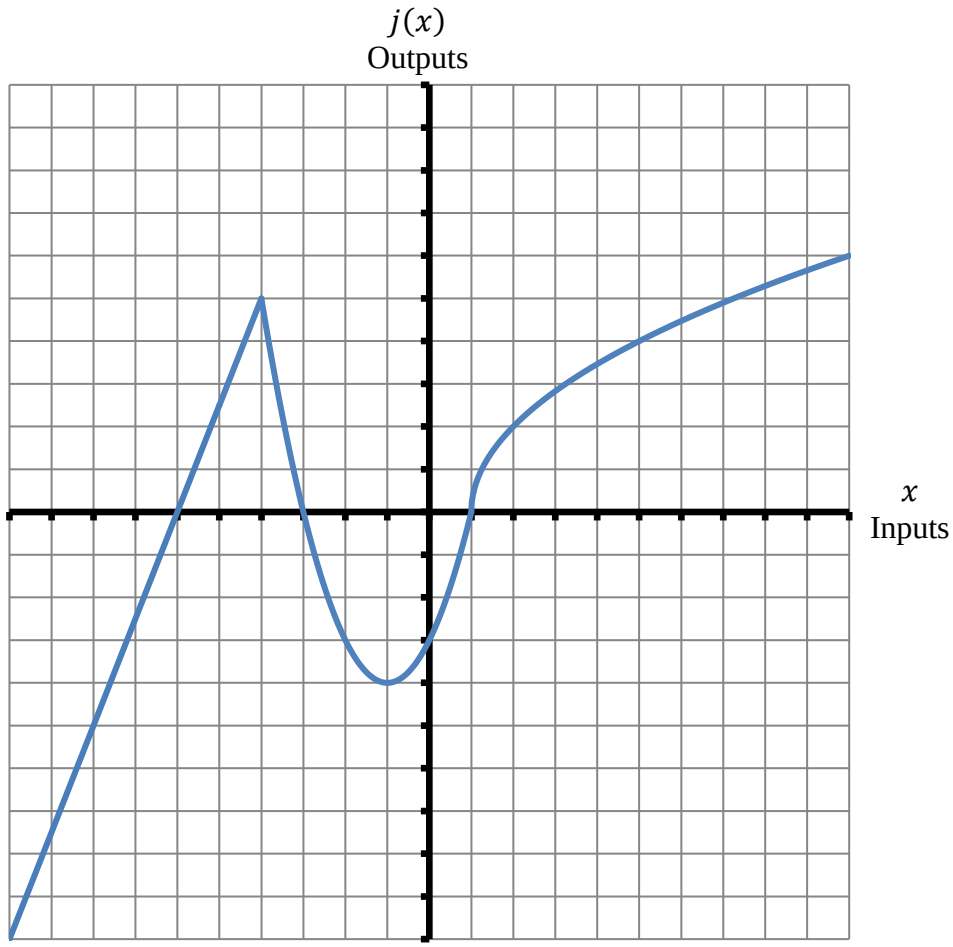
$g(-9) = -1$ and $g(3) = -1$

k. Find the x-intercepts and the y-intercepts of the function, and list your answers as ordered pairs

$x-$ intercepts: $(-8, 0), (-4, 0), (-2, 0), (2, 0)$

$y-$ intercept: $(0, 2)$

<u>Example 5:</u> Use the graph of the function j given below to find the following.



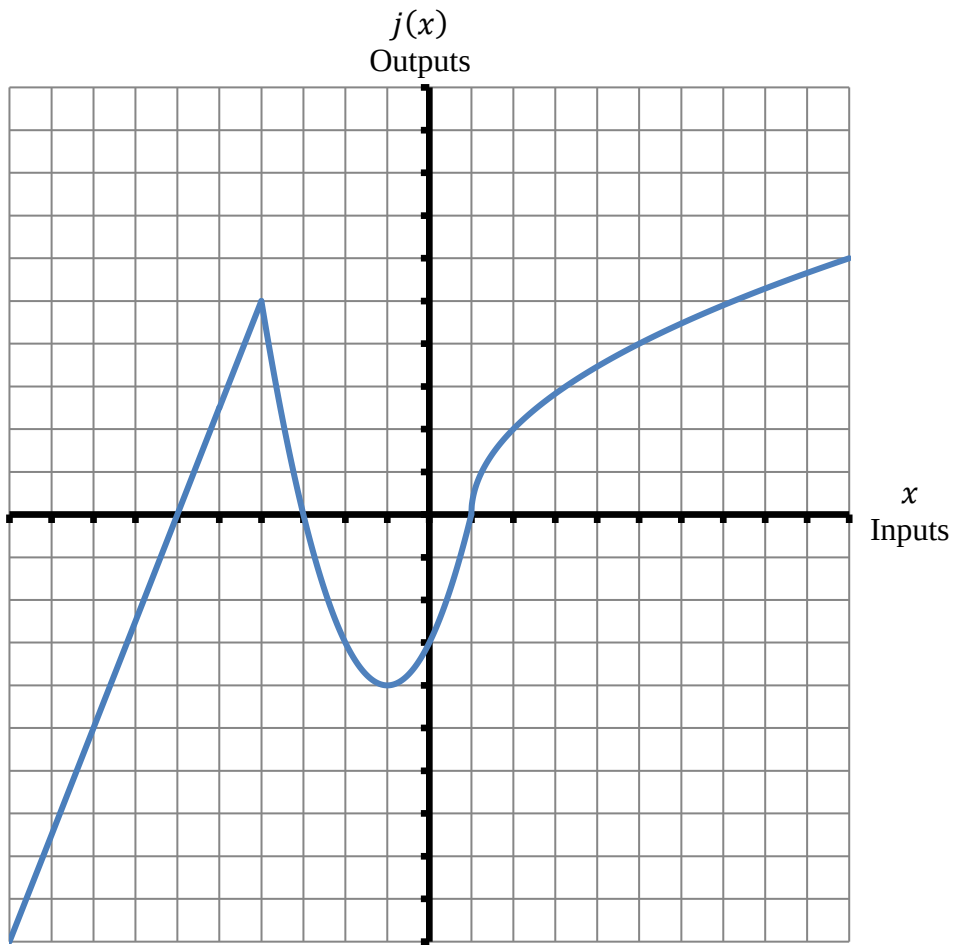
a. Find the domain of j and list your answer in interval notation

b. Find the range of j and list your answer in interval notation

c. Find $j(-4)$

d. Find $j(0)$

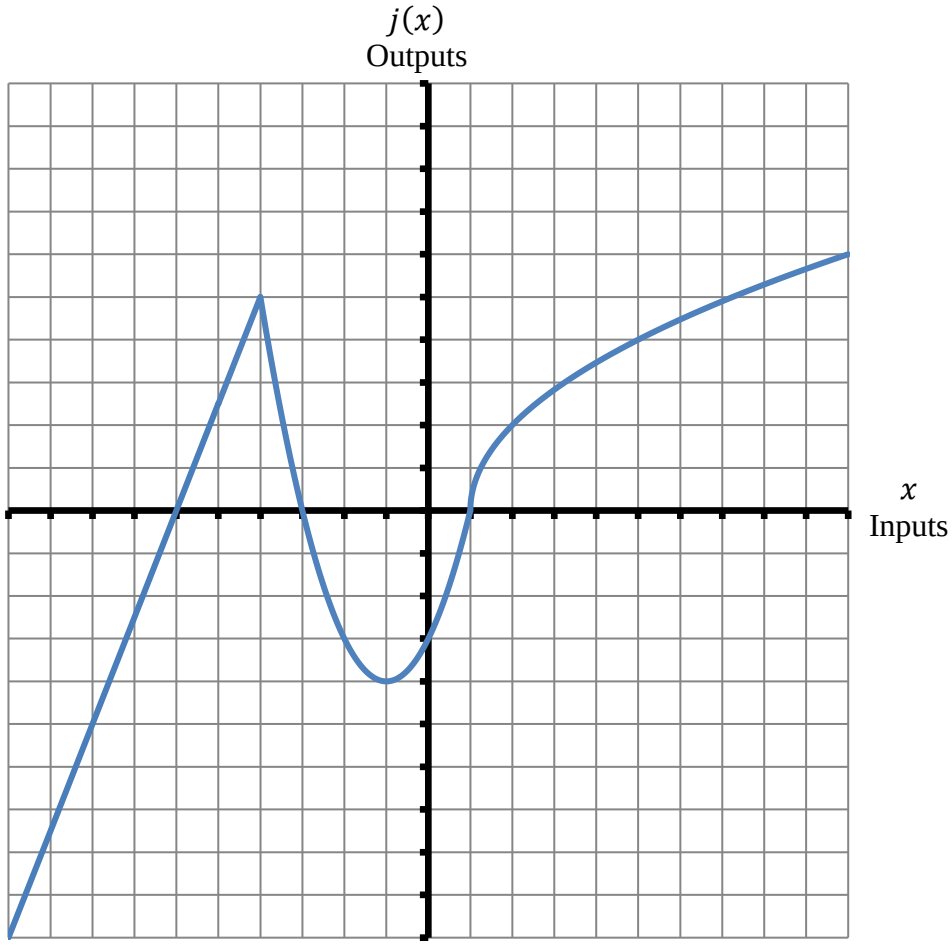
e. Find $j(5)$



f. Find all the x-values, such that $j(x) = -5$

g. Find the zeros of the function ($j(x) = 0$ when $x =$?)

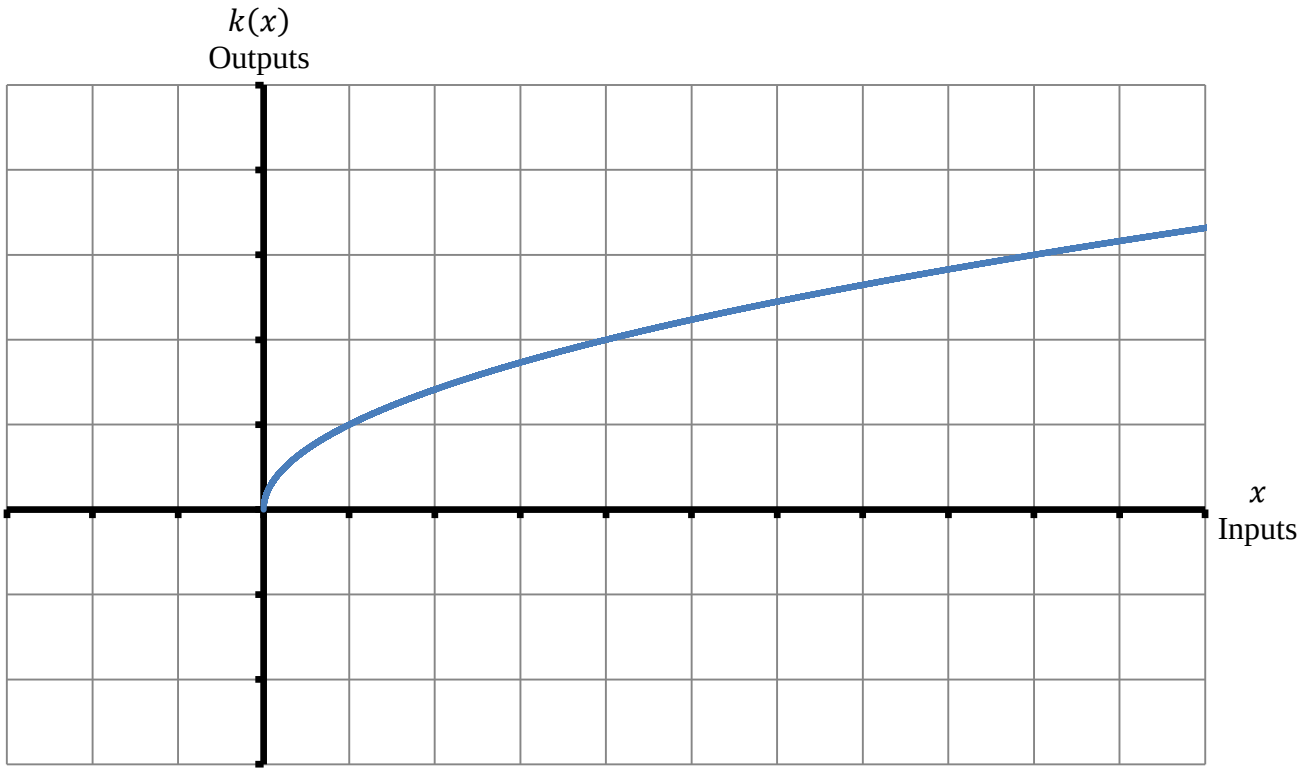
h. Find all the x-values, such that $j(x) > 0$ and list your answers in interval notation



i. Find all the x-values, such that $j(x) < 0$ and list your answers in interval notation

j. Find the x-intercepts and the y-intercepts of the function, and list your answers as ordered pairs

<u>Example 6:</u> Use the graph of the function k given below to find the following.



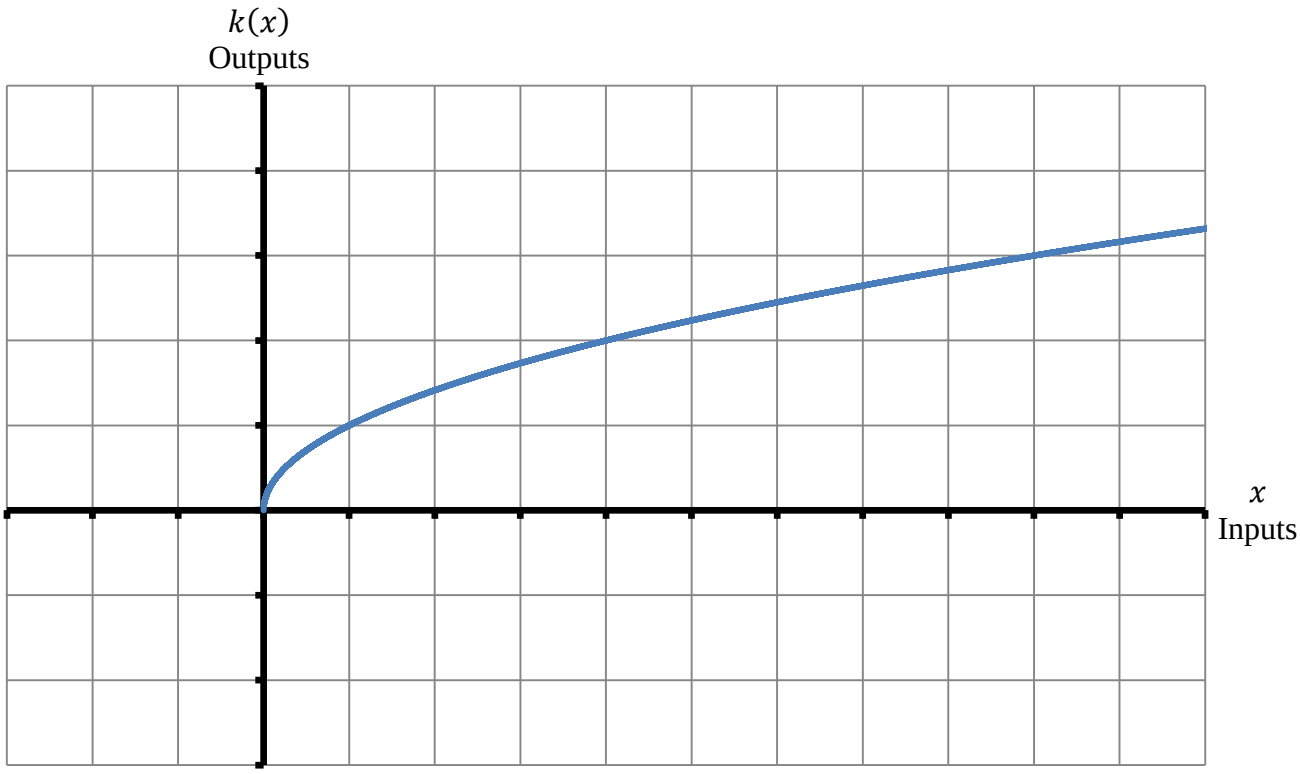
a. Find the domain of k and list your answer in interval notation

b. Find the range of k and list your answer in interval notation

c. Find $k(-1)$

d. Find $k(0)$

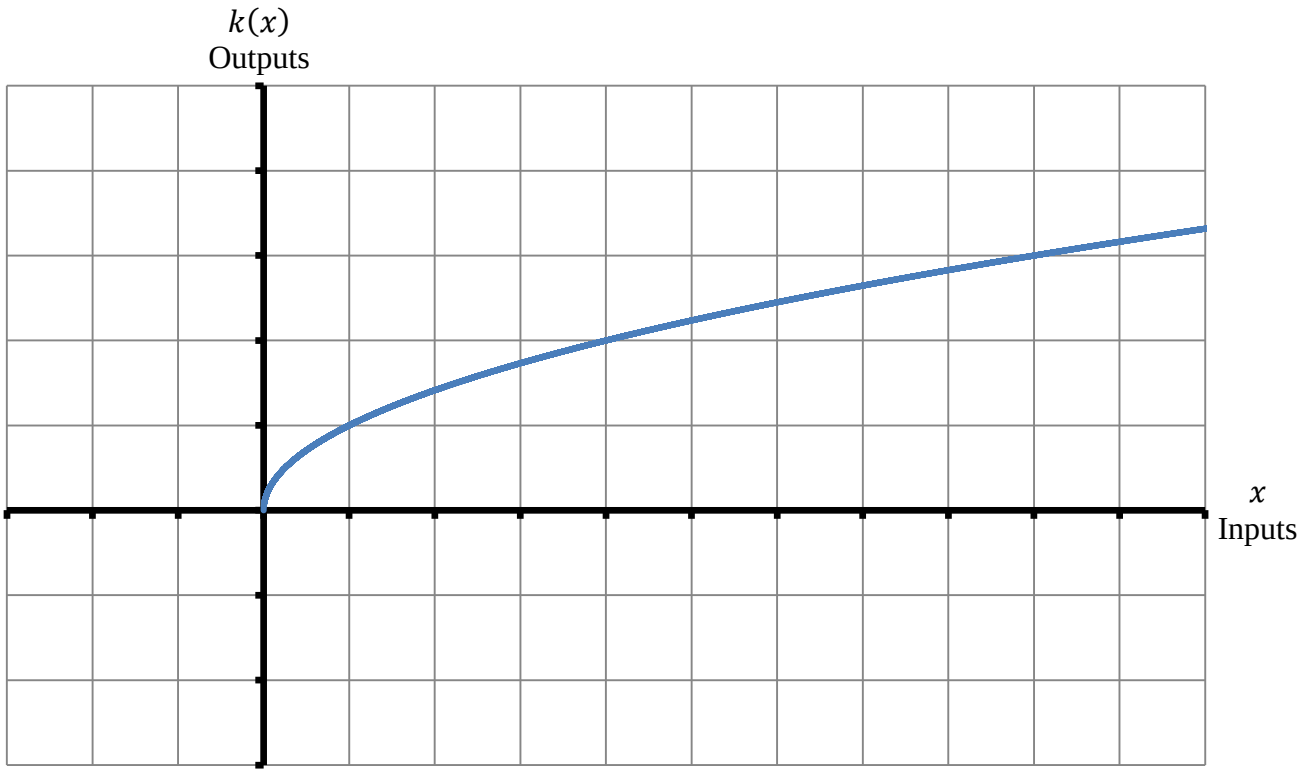
e. Find $k(4)$



f. Find all the x-values, such that $k(x) = 3$

g. Find the zeros of the function ($k(x) = 0$ when $x =$?)

h. Find all the x-values, such that $k(x) > 0$ and list your answers in interval notation



i. Find all the x-values, such that $k(x) < 0$ and list your answers in interval notation

j. Find the x-intercepts and the y-intercepts of the function, and list your answers as ordered pairs

Answers to Examples:

1a. $[-6,-2]$; *1b.* $[-2,2]$; *1c.* $f(-4)=-2$; *1d.* $f(0)=2$;
1e. $f(2)=0$; *1f.* $f(x)=-2$ when $x=-4$;
1g. $f(x)=2$ when $x=0$; *1h.* $f(x)=0$ when $x=-6,-2,2$;

2a. $f(x)=0$ when $x=-6,-2,2$;
2b. $f(x)>0$ when x is part of the interval $(-2,2)$;
2c. $f(x)<0$ when x is part of the interval $(-6,-2)$;
2d. $x-$ intercepts: $(-6,0),(-2,0),(2,0)$; $y-$ intercept: $(0,2)$;

3a. $(-\infty,\infty)$; *3b.* $(-\infty,\infty)$; *3c.* $f(-5)=-5$; *3d.* $f(0)=0$;
3e. $f(3)=-3$; *3f.* $f(x)=5$ when $x=5$;
3g. $f(x)=0$ when $x=-4,0,4$;
3h. $f(x)>0$ when x is part of the interval $(-4,0)\cup(4,\infty)$;
3i. $f(x)<0$ when x is part of the interval $(-\infty,-4)\cup(0,4)$;
3j. $(-4,0),(0,0),(4,0)$;

4a. $[-9,3]$; *4b.* $[-1,2]$; *4c.* $g(-7)=1$; *4d.* $g(0)=2$;
4e. $g(3)=-1$; *4f.* $g(x)=-1$ when $x=-9,-3,3$;
4g. $g(x)\neq 3$;
4h. $g(x)=0$ when $x=-8,-4,-2,2$;
4i. $g(x)<0$ when x is part of the interval $[-9,-8)\cup(-4,-2)\cup(2,3]$;
4j. $(-8,0),(-4,0),(-2,0)\ (2,0)\ (0,2)$;

5a. $(-\infty,\infty)$; *5b.* $(-\infty,\infty)$; *5c.* $j(-4)=5$; *5d.* $j(0)=-3$;
5e. $j(5)=4$; *5f.* $j(x)=-5$ when $x=-8$;
5g. $j(x)=0$ when $x=-6,-3,1$;
5h. $j(x)>0$ when x is part of the interval $(-6,-3)\cup(1,\infty)$;
5i. $j(x)<0$ when x is part of the interval $(-\infty,-6)\cup(-3,1)$;
5j. $(-6,0),(-3,0),(1,0),(0,-3)$;

6a. $[0,\infty)$; *6b.* $[0,\infty)$; *6c.* $k(-1)\ is\ undefined$; *6d.* $k(0)=0$;
6e. $k(4)=2$; *6f.* $k(x)=3$ when $x=9$;
6g. $k(x)=0$ when $x=0$;
6h. $k(x)>0$ for every value of x in its domain $(0,\infty)$;
6i. $k(x)$ is never less than 0 ; *6j.* $(0,0)$;