In addition to polynomial form \((f(x) = ax^2 + bx + c)\), quadratic functions can also be expressed in what is known as **standard form**.

**Standard Form of a Quadratic Function:**
- \(f(x) = a(x - h)^2 + k\)
- a way of expressing a quadratic function as a perfect square
  - one benefit of standard form over polynomial form is that the vertex \((h, k)\) is easily identifiable
    - the vertex is \((h, k)\) where \(h\) is the \(x\)-coordinate of the vertex \(\left(\frac{-b}{2a}\right)\) and \(k\) is the \(y\)-coordinate of the vertex \(f \left(\frac{-b}{2a}\right)\).
  - another benefit of standard form is that graphing the quadratic function can be completed using transformations of the parent function \(x^2\) (more on this in the next lesson)

A quadratic function in polynomial form \((f(x) = ax^2 + bx + c)\) can be converted to standard form \((f(x) = a(x - h)^2 + k)\) by completing the square or by simply plugging in the vertex and the leading coefficient. To find the vertex of a quadratic function in polynomial form, it is often easiest to use the formula \(\frac{-b}{2a}\) to find the \(x\)-coordinate \((h)\) and \(f \left(\frac{-b}{2a}\right)\) to find the \(y\)-coordinate \((k)\). **The leading coefficient \(a\) is the same in both polynomial form and standard form.** Using the completing the square method is not recommended because it is more difficult, but completing the square is how the \(\frac{-b}{2a}\) formula is derived.

**Steps for Converting a Quadratic Function to Standard Form:**
1. find the vertex of the quadratic function
2. identify the leading coefficient \(a\) of the quadratic function
3. plug the vertex and the leading coefficient \(a\) into the function \(f(x) = a(x - h)^2 + k\), the \(x\)-coordinate of the vertex is \(h\), the \(y\)-coordinate of the vertex is \(k\)
Example 1: Find the vertex $(h, k)$ for each of the following quadratic functions, and then express the quadratic functions in standard form $(f(x) = a(x - h)^2 + k)$.

a. $f(x) = x^2 - 6x + 11$

$$x = \frac{-b}{2a}$$

$$x = \frac{-(6)}{2(1)}$$

$$x = \frac{6}{2}$$

$$x = 3$$

$$f(3) = (3)^2 - 6(3) + 11$$

$$f(3) = 9 - 18 + 11$$

$$f(3) = 2$$

Vertex: (3, 2)

b. $g(x) = -x^2 - 4x - 5$

$$x = \frac{-b}{2a}$$

$$x = \frac{-(4)}{2(-1)}$$

$$x = \frac{4}{2}$$

$$x = -2$$

$$g(-2) = -(-2)^2 - 4(-2) - 5$$

$$g(-2) = -(4) + 8 - 5$$

$$g(-2) = -4 + 8 - 5$$

$$g(-2) = -1$$

Vertex: (-2, -1)

To express a quadratic function in standard form, we simply find the vertex $(h, k)$ and identify the leading coefficient $a$, and then plug them into the function $f(x) = a(x - h)^2 + k$.

$a = 1$

$$f(x) = a(x - h)^2 + k$$

$$f(x) = 1(x - 3)^2 + 2$$

$$f(x) = (x - 3)^2 + 2$$

$a = -1$

$$g(x) = a(x - h)^2 + k$$

$$g(x) = -1(x - (-2))^2 + (-1)$$

$$g(x) = -(x - (-2))^2 + (-1)$$

$$g(x) = -(x + 2)^2 - 1$$
Example 2: Find the vertex \((h, k)\) of each of the following quadratic functions, and then express the quadratic functions in standard form \((f(x) = a(x - h)^2 + k)\).

a. \(h(x) = 2x^2 - 16x + 35\) 
   b. \(j(x) = -3x^2 - 6x - 5\)

c. \(k(x) = 3x^2 - x\) 
   d. \(m(x) = -4x^2 + 4x\)
e. \( n(x) = 3(x + 2)(x - 10) \)

\[
n(x) = 3(x^2 - 8x - 20)
\]

\[
n(x) = 3x^2 - 24x - 60
\]

\[
x = \frac{-(-24)}{2(3)}
\]

\[
x = \frac{24}{6}
\]

\[
x = 4
\]

To find the function value for \( x = 4 \), you can back substitute to the original function \( n(x) = 3(x + 2)(x - 10) \), or to the function in polynomial form \( n(x) = 3x^2 - 24x - 60 \); it makes no difference.

\[
n(4) = 3(4 + 2)(4 - 10)
\]

\[
n(x) = 3(6)(-6)
\]

\[
f(3) = -108
\]

Vertex: \((3, -108)\)

Once again we find the vertex \((h, k)\) and identify the leading coefficient \(a\), and then plug them into the function.

\[
a = 3
\]

\[
f(x) = a(x - h)^2 + k
\]

\[
f(x) = 3(x - 3)^2 + (-108)
\]

\[
f(x) = 3(x - 3)^2 - 108
\]
When converting a quadratic function from polynomial form to standard form, the vertex must be found first, while the leading coefficient $a$ is already given in whatever form the quadratic function is given. When find the standard form of a quadratic function from a graph or information about a graph (as we’ll see in the next lesson), the value of the leading coefficient $a$ will need to be found first, while the vertex will be given.

**Answers to Examples:**

1a. $V(3, 2), f(x) = (x - 3)^2 + 2$ ;

1b. $V(-2, -1), g(x) = -(x + 2)^2 - 1$ ;

2a. $V(4, 3), h(x) = 2(x - 4)^2 + 3$ ;

2b. $V(-1, -2), j(x) = -3(x + 1)^2 - 2$ ;

2c. $V\left(\frac{1}{6}, -\frac{1}{12}\right), k(x) = 3 \left(x - \frac{1}{6}\right)^2 - \frac{1}{12}$ ;

2d. $V\left(\frac{1}{2}, 1\right), m(x) = -4 \left(x - \frac{1}{2}\right)^2 + 1$ ;

2e. $V(4, -108), n(x) = 3(x - 4)^2 - 108$ ;

2f. $V(3, 25), p(x) = -(x - 3)^2 + 25$ ;