In addition to polynomial form \((f(x) = ax^2 + bx + c)\), quadratic functions can also be expressed in what is known as \textbf{standard form}.

\textbf{Standard Form of a Quadratic Function:}
- \(f(x) = a(x - h)^2 + k\)
- a way of expressing a quadratic function as a perfect square
  - one benefit of standard form over polynomial form is that the vertex \((h, k)\) is easily identifiable
    - the vertex is \((h, k)\), where \(h\) is the \(x\)-coordinate of the vertex \(\left(\frac{-b}{2a}\right)\) and \(k\) is the \(y\)-coordinate of the vertex \(f \left( \frac{-b}{2a} \right)\).
  - another benefit of standard form is that graphing the quadratic function can be completed using transformations of the parent function \(x^2\) (more on this in the next lesson)

A quadratic function in polynomial form \((f(x) = ax^2 + bx + c)\) can be converted to standard form \((f(x) = a(x - h)^2 + k)\) by completing the square or by simply plugging in the vertex and the leading coefficient. To find the vertex of a quadratic function in polynomial form, it is often easiest to use the formula \(\frac{-b}{2a}\) to find the \(x\)-coordinate \((h)\) and \(f \left( \frac{-b}{2a} \right)\) to find the \(y\)-coordinate \((k)\). \textbf{The leading coefficient} \(a\) \textbf{is the same in both polynomial form and standard form}. Using the completing the square method is not recommended because it is more difficult, but completing the square is how the \(\frac{-b}{2a}\) formula is derived.

\textbf{Steps for Converting a Quadratic Function to Standard Form:}
1. find the vertex of the quadratic function
2. identify the leading coefficient \(a\) of the quadratic function
3. plug the vertex and the leading coefficient \(a\) into the function \(f(x) = a(x - h)^2 + k\), the \(x\)-coordinate of the vertex is \(h\), the \(y\)-coordinate of the vertex is \(k\)

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Lesson 23
Finding a Quadratic Function Algebraically
**Example 1:** Find the vertex \((h, k)\) for each of the following quadratic functions, and then express the quadratic functions in standard form \(f(x) = a(x - h)^2 + k\).

a. \(f(x) = x^2 - 6x + 11\)

\[
x = \frac{-b}{2a}
\]
\[
x = \frac{-(6)}{2(1)}
\]
\[
x = \frac{6}{2}
\]
\[
x = 3
\]
\[
f(3) = (3)^2 - 6(3) + 11
\]
\[
f(3) = 9 - 18 + 11
\]
\[
f(3) = 2
\]

Vertex: \((3, 2)\)

b. \(g(x) = -x^2 - 4x - 5\)

\[
x = \frac{-b}{2a}
\]
\[
x = \frac{-(-4)}{2(-1)}
\]
\[
x = \frac{4}{-2}
\]
\[
x = -2
\]
\[
g(-2) = -(-2)^2 - 4(-2) - 5
\]
\[
g(-2) = -(4) + 8 - 5
\]
\[
g(-2) = -4 + 8 - 5
\]
\[
g(-2) = -1
\]

Vertex: \((-2, -1)\)

To express a quadratic function in standard form, we simply find the vertex \((h, k)\) and identify the leading coefficient \(a\), and then plug them into the function \(f(x) = a(x - h)^2 + k\).

a. \(f(x) = a(x - h)^2 + k\)  
   \[
a = 1
\]
   \[
f(x) = 1(x - 3)^2 + 2
\]
   \[
f(x) = (x - 3)^2 + 2
\]

b. \(g(x) = a(x - h)^2 + k\)  
   \[
a = -1
\]
   \[
g(x) = -1(x - (-2))^2 + (-1)
\]
   \[
g(x) = -(x + 2)^2 + (-1)
\]
   \[
g(x) = -1(x + 2)^2 - 1
\]
**Example 2:** Find the vertex \((h, k)\) of each of the following quadratic functions, and then express the quadratic functions in standard form \((f(x) = a(x - h)^2 + k)\).

a. \(h(x) = 2x^2 - 16x + 35\)  
b. \(j(x) = -3x^2 - 6x - 5\)

c. \(k(x) = 3x^2 - x\)  
d. \(m(x) = -4x^2 + 4x\)
e. \( n(x) = 3(x + 2)(x - 10) \)

\[ n(x) = 3(x^2 - 8x - 20) \]

\[ n(x) = 3x^2 - 24x - 60 \]

\[ x = \frac{-(-24)}{2(3)} \]

\[ x = \frac{24}{6} \]

\[ x = 4 \]

To find the function value for \( x = 4 \), you can back substitute to the original function \( n(x) = 3(x + 2)(x - 10) \), or to the function in polynomial form \( n(x) = 3x^2 - 24x - 60 \); it makes no difference.

\[ n(4) = 3(4 + 2)(4 - 10) \]

\[ n(x) = 3(6)(-6) \]

\[ f(3) = -108 \]

Vertex: \((3, -108)\)

Once again we find the vertex \((h, k)\) and identify the leading coefficient \(a\), and then plug them into the function.

\[ a = 3 \]

\[ f(x) = a(x - h)^2 + k \]

\[ f(x) = 3(x - 3)^2 + (-108) \]

\[ f(x) = 3(x - 3)^2 - 108 \]
When converting a quadratic function from polynomial form to standard form, the vertex must be found first, while the leading coefficient $a$ is already given in whatever form the quadratic function is given. When find the standard form of a quadratic function from a graph or information about a graph (as we’ll see in the next lesson), the value of the leading coefficient $a$ will need to be found first, while the vertex will be given.

**Answers to Examples:**

1a. $V(3, 2), f(x) = (x - 3)^2 + 2$;  
1b. $V(-2, -1), g(x) = -(x + 2)^2 - 1$;  
2a. $V(4, 3), h(x) = 2(x - 4)^2 + 3$;  
2b. $V(-1, -2), j(x) = -3(x + 1)^2 - 2$;  
2c. $V\left(\frac{1}{6}, -\frac{1}{12}\right), k(x) = 3\left(x - \frac{1}{6}\right)^2 - \frac{1}{12}$;  
2d. $V\left(\frac{1}{2}, 1\right), m(x) = -4\left(x - \frac{1}{2}\right)^2 + 1$;  
2e. $V(4, -108), n(x) = 3(x - 4)^2 - 108$;  
2f. $V(3, 25), p(x) = -(x - 3)^2 + 25$;