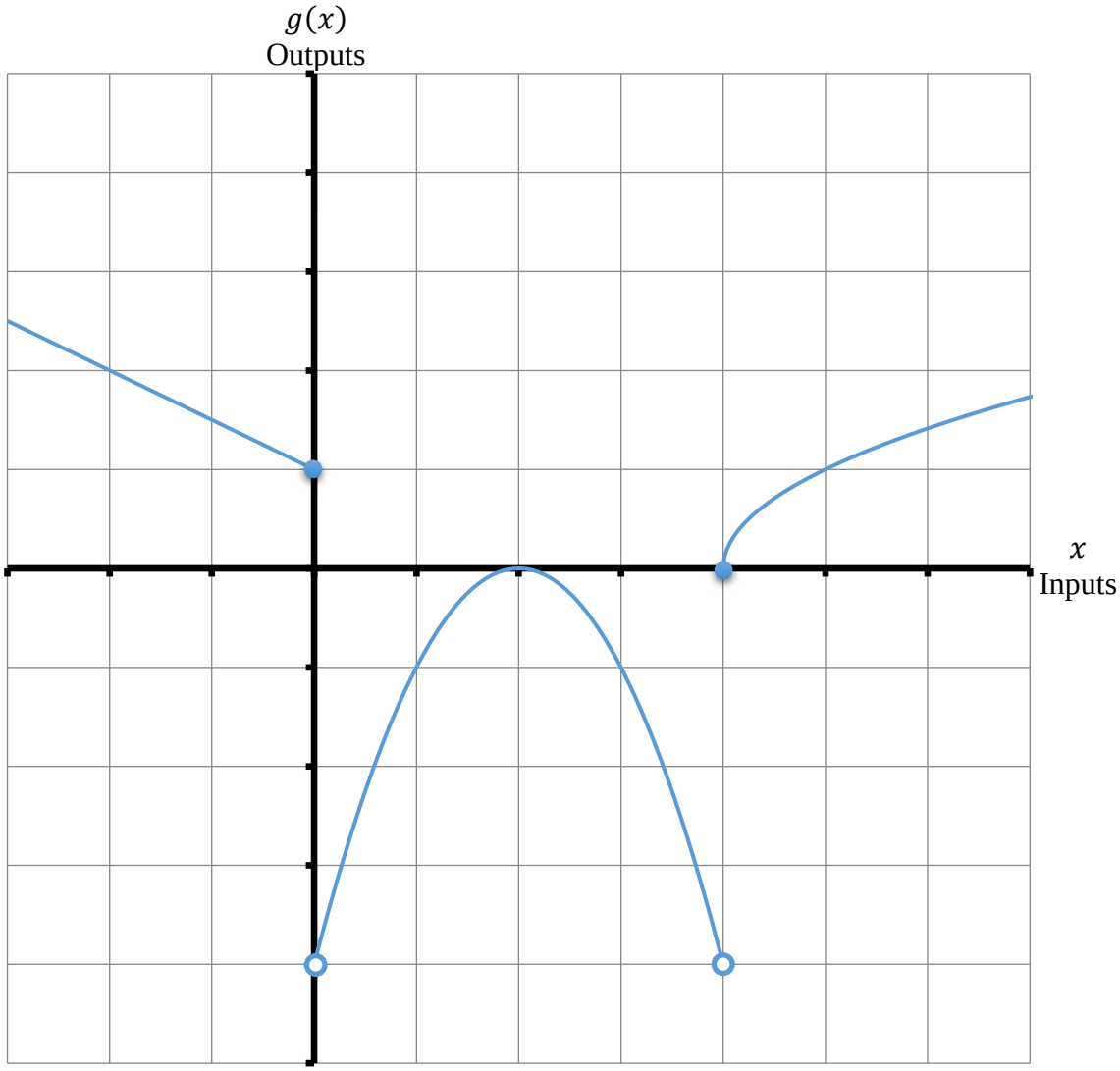
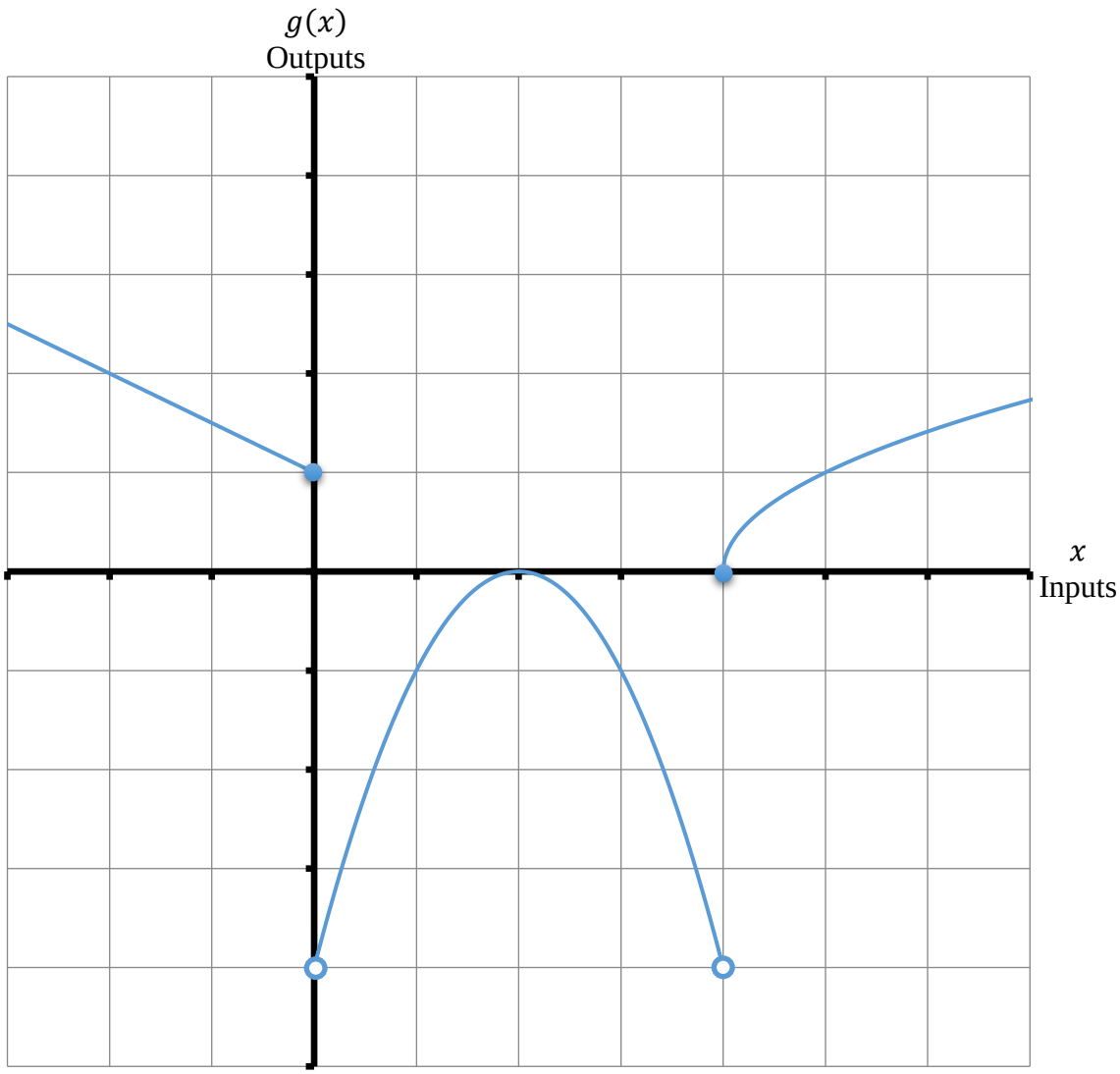
Based on the graph of a piecewise-defined function, we can often answer questions about the domain and range of the function, as well as the zeros, the intervals where the function is positive, negative, increasing, and decreasing, and the intercepts.

<u>Example 1:</u> Given the graph of the piecewise-defined function g below:

$$g(x) = \begin{cases} -\frac{1}{2}x + 1 & \text{if} \quad x \leq 0 \\ -(x-2)^2 & \text{if } 0 < x < 4 \\ \sqrt{x-4} & \text{if} \quad x \geq 4 \end{cases}$$





a. list the domain and range in interval notation

Domain: $(-\infty, \infty)$

Range: $(-\mathbf{4}, \infty)$

Going from left to right, every input is included for the domain.

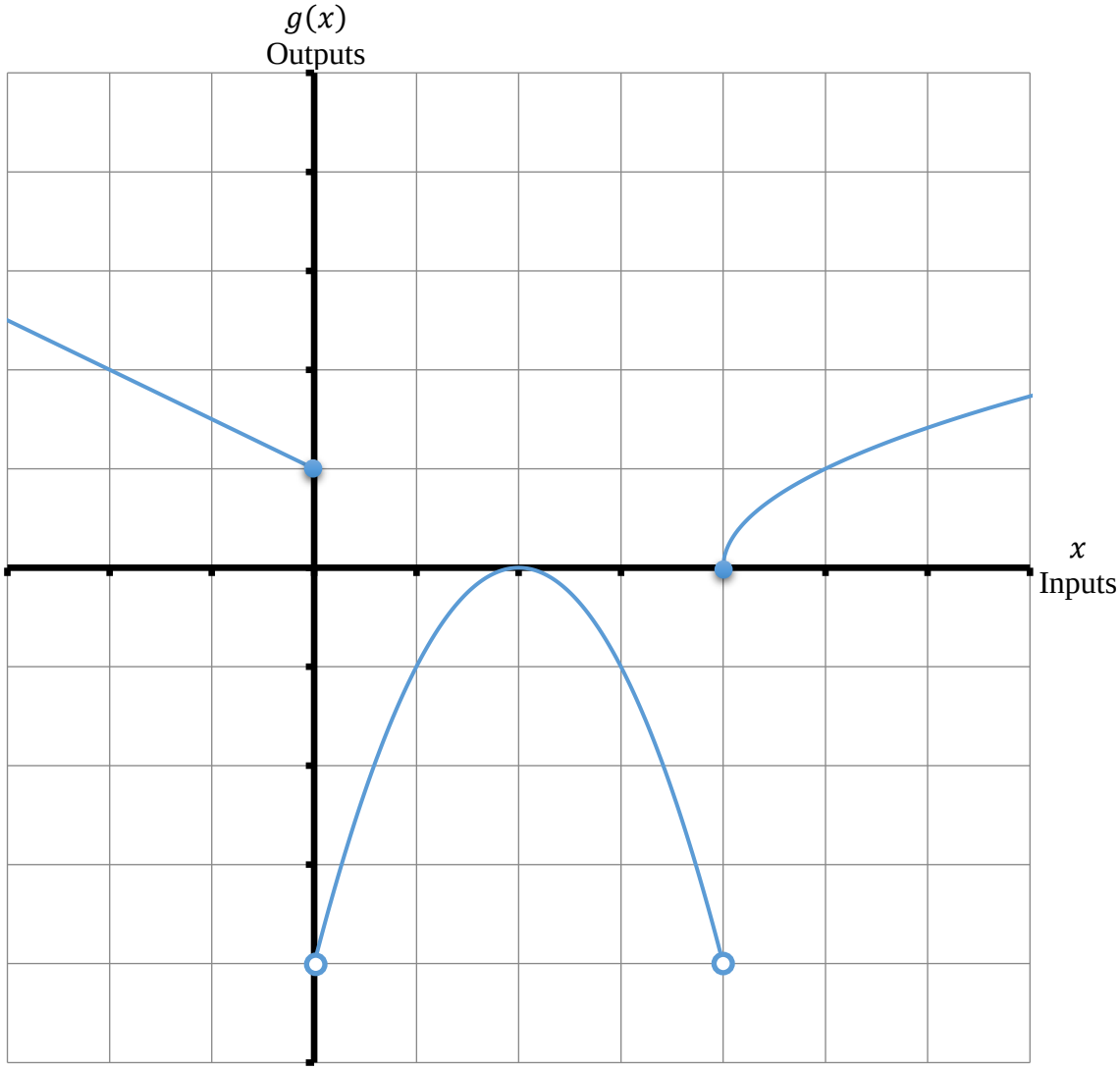
Vertically however, the graph goes up to positive infinity and down to only -4. The open points $(0, -4)$ and $(4, -4)$ tell us that -4 is not included in the range.

b. find the zeros of the function

Zeros: $g(x) = 0$ when $x = \mathbf{2, 4}$

To find the zeros of a function graphically, simply list the x-coordinates of the points where a graph touches or crosses the x-axis. If those x-coordinates are not exact integer values, you will need to find the zeros algebraically.

On this problem, both zeros are integer values (2 and 4), so there's not need to find the zeros algebraically. However if that was not the case, we would take whichever pieces of the function were touching or crossing the x-axis, set them equal to zero, and solve for x to find the zeros.



c. list the intervals where the function is positive or negative

Positive Intervals: $g(x) > 0$ when x is

$(-\infty, \mathbf{0}] \cup (\mathbf{4}, \infty)$

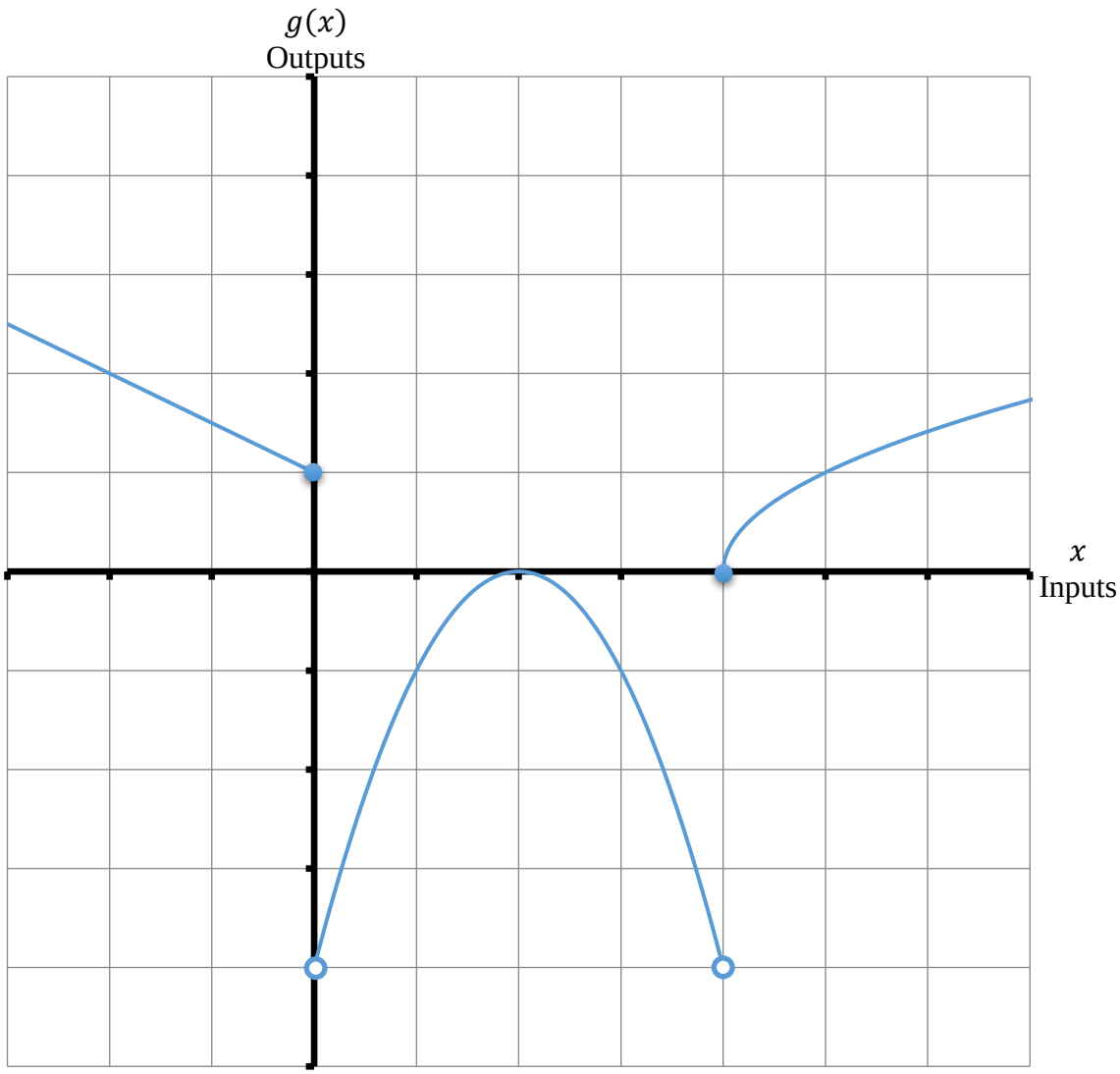
Negative Intervals: $g(x) < 0$ when x is

$(\mathbf{0}, \mathbf{2}) \cup (\mathbf{2}, \mathbf{4})$

The first piece of the graph and the third piece of the graph lie completely above the x-axis, with the exception of the point $(4, 0)$, which is where the third piece begins.

The second piece of the graph lies completely below the x-axis, with the exception of the point $(2, 0)$.

Notice that zero in the positive interval has a square bracket on it because when zero is the input, the output is positive.



d. list the intervals where the function is increasing or decreasing

Increasing Intervals: $g(x)$ is increasing when x is

$$(\mathbf{0}, \mathbf{2}) \cup [\mathbf{4}, \infty)$$

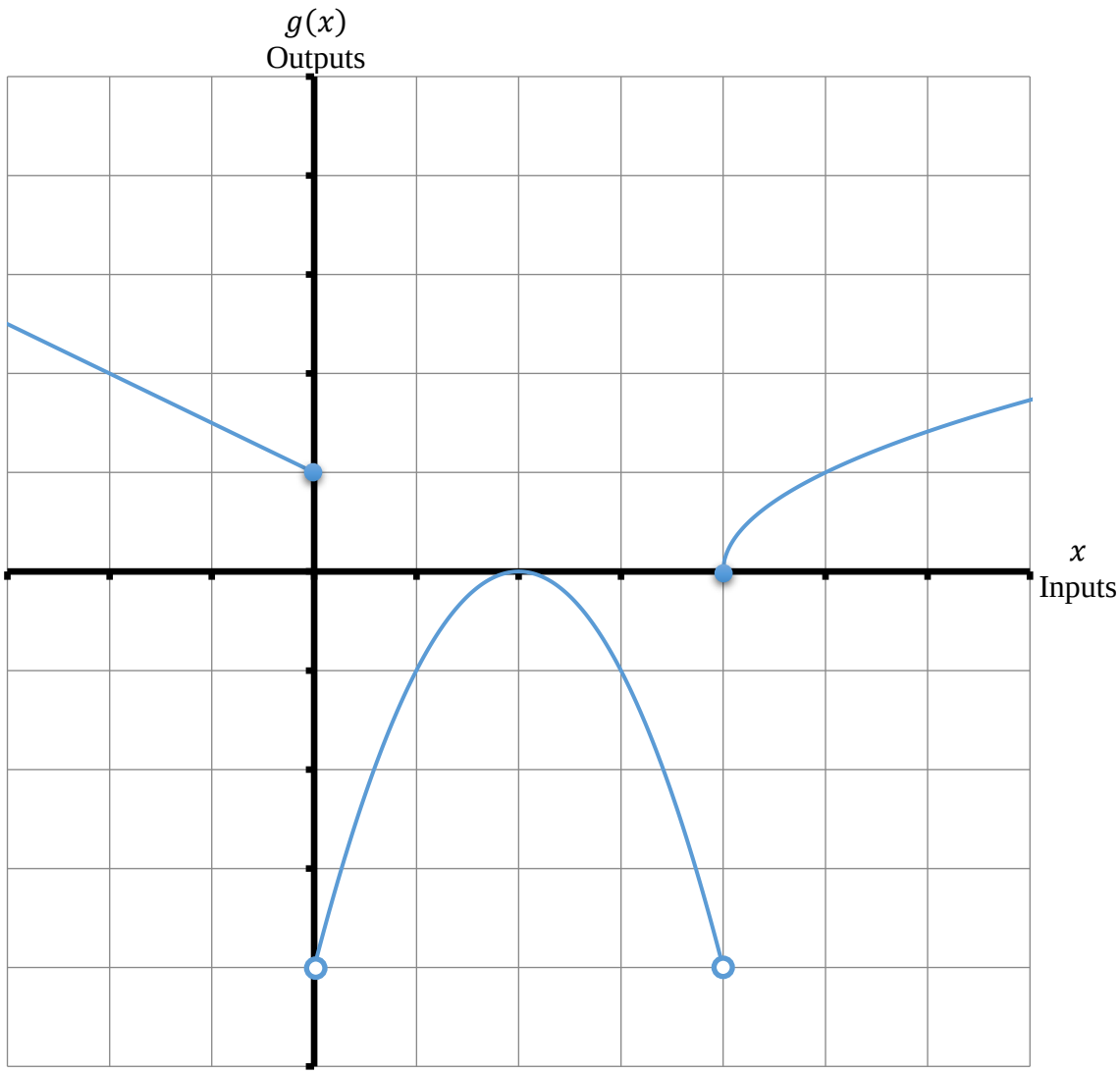
Decreasing Intervals: $g(x)$ is decreasing when x is

$$(-\infty, \mathbf{0}] \cup (\mathbf{2}, \mathbf{4})$$

The first piece of the graph is decreasing, and its domain is $(-\infty, 0]$, so that will make up the first part of the decreasing interval.

The second piece of the graph is a parabola, with the left half increasing from $(0, 2)$ and the right half decreasing from $(2, 4)$.

The third piece of the graph is increasing, and its domain is $[4, \infty)$, so that will make up the last part of the increasing interval.



e. list the intercepts

x-intercept(s):

$\mathbf{(2, 0), (4, 0)}$

y-intercept:

$\mathbf{(0, 1)}$

Notice that while there appear to be two points that lie on the y-axis, the lower of the two points $(0, -4)$ is an open point, so it is technically not part of the graph. Therefore the only y-intercept is the point $(0, 1)$.

<u>Example 2:</u> Given the graph of the piecewise-defined function h below:

a. list the domain and range in interval notation

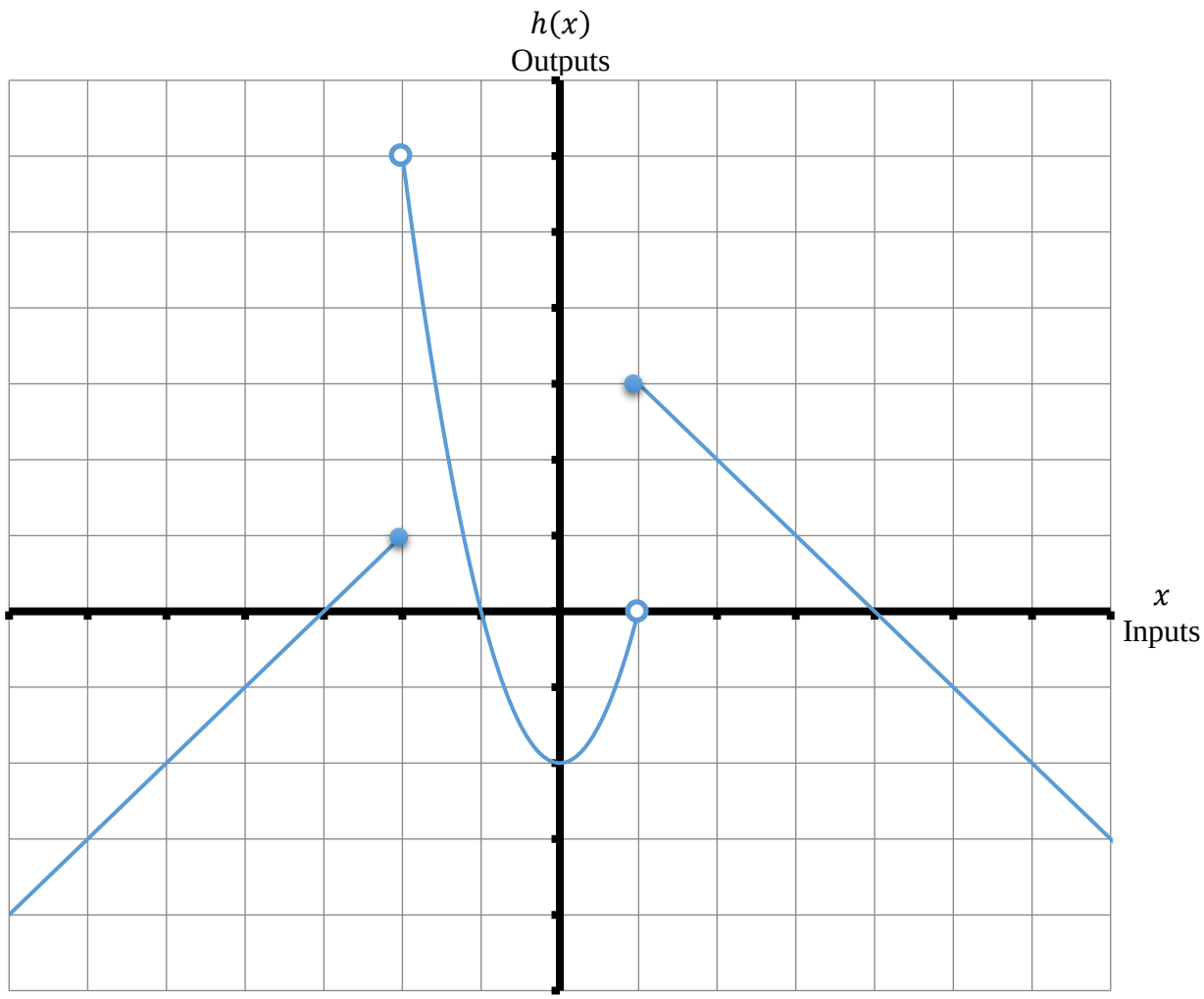
b. find the zeros of the function

c. list the intervals where the function is positive and negative

d. list the intervals where the function is increasing and decreasing

e. list the intercepts

$$h(x) = \begin{cases} x+3 & \text{if } x \le -2 \\ 2x^2 - 2 & \text{if } -2 < x < 1 \\ 4 - x & \text{if } x \ge 1 \end{cases}$$



<u>Example 3:</u> Sketch the graph of the piecewise-defined function h, then:

a. list the domain and range in interval notation

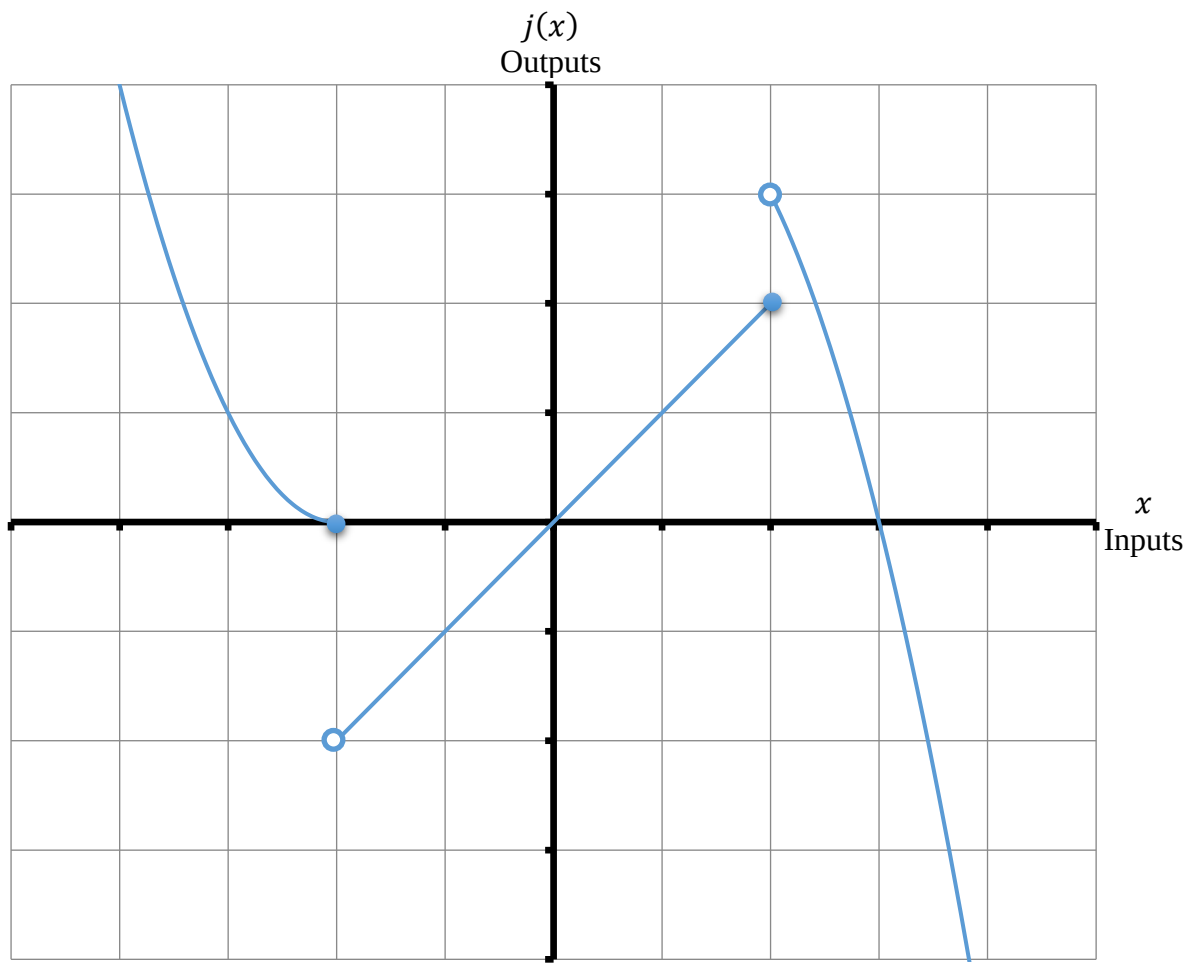
b. find the zeros of the function

c. list the intervals where the function is positive and negative

d. list the intervals where the function is increasing and decreasing

e. list the intercepts

$$j(x) = \begin{cases} (x+2)^2 & \text{if } x \leq -2 \\ x & \text{if } -2 < x \leq 2 \\ -(x-1)^2 + 4 & \text{if } x > 2 \end{cases}$$



Answers to Examples:

1a. $D: (-\infty, \infty), R: (-4, \infty)$; *1b.* $g(x) = 0$ when $x = 2, 4$;

1c. $g(x) > 0: (-\infty, 0] \cup (4, \infty), g(x) < 0: (0, 2) \cup (2, 4)$;

1d. $\uparrow: (0, 2) \cup [4, \infty), \downarrow: (-\infty, 0] \cup (2, 4)$;

1e. $x - int: (2, 0), (4, 0)$; $y - int: (0, 1)$;

2a. $D: (-\infty, \infty), R: (-\infty, 6)$; *2b.* $h(x) = 0$ when $x = -3, -1, 4$;

2c. $h(x) > 0: (-3, -1) \cup [1, 4)$

$h(x) < 0: (-\infty, -3) \cup (-1, 1) \cup (4, \infty)$;

2d. $\uparrow: (-\infty, -2] \cup (0, 1), \downarrow: (-2, 0) \cup [1, \infty)$;

2e. $x - int: (-3, 0), (-1, 0)(4, 0), y - int: (0, -2)$;

3a. $D: (-\infty, \infty), R: (-\infty, \infty)$; *3b.* $g(x) = 0$ when $x = -2, 0, 3$;

3c. $g(x) > 0: (-\infty, -2) \cup (0, 3)$, $g(x) < 0: (-2, 0) \cup (3, \infty)$;

3d. $\uparrow: (-2, 2], \downarrow: (-\infty, -2] \cup (2, \infty)$;

3e. $x - int: (-2, 0), (0, 0), (3, 0)$; $y - int: (0, 0)$;