Based on the graph of a piecewise-defined function, we can often answer questions about the domain and range of the function, as well as the zeros, the intervals where the function is positive, negative, increasing, and decreasing, and the intercepts.

**Example 1:** Given the graph of the piecewise-defined function $g$ below:

$$g(x) = \begin{cases} \frac{-1}{2}x + 1 & \text{if } x \leq 0 \\ -(x - 2)^2 & \text{if } 0 < x < 4 \\ \sqrt{x - 4} & \text{if } x \geq 4 \end{cases}$$
a. list the domain and range in interval notation

Domain:

Range:

b. find the zeros of the function

Zeros: $g(x) = 0$ when $x =$

To find the zeros of a function graphically, simply list the $x$-coordinates of the points where a graph touches or crosses the $x$-axis. If those $x$-coordinates are not exact integer values, you will need to find the zeros algebraically.

On this problem, both zeros are integer values (2 and 4), so there’s not need to find the zeros algebraically. However if that was not the case, we would take whichever pieces of the function were touching or crossing the $x$-axis, set them equal to zero, and solve for $x$ to find the zeros.
c. list the intervals where the function is positive or negative

Positive Intervals: $g(x) > 0$ when $x$ is

Negative Intervals: $g(x) < 0$ when $x$ is
d. list the intervals where the function is increasing or decreasing

Increasing Intervals: $g(x)$ is increasing when $x$ is

Decreasing Intervals: $g(x)$ is decreasing when $x$ is
e. list the intercepts

$x$-intercept(s):

$y$-intercept:
Example 2: Given the graph of the piecewise-defined function $h$ below:

a. list the domain and range in interval notation
b. find the zeros of the function
c. list the intervals where the function is positive and negative
d. list the intervals where the function is increasing and decreasing
e. list the intercepts

$$h(x) = \begin{cases} 
  x + 3 & \text{if } x \leq -2 \\
  2x^2 - 2 & \text{if } -2 < x < 1 \\
  4 - x & \text{if } x \geq 1
\end{cases}$$
Example 3: Sketch the graph of the piecewise-defined function \( h \), then:

a. list the domain and range in interval notation
b. find the zeros of the function
c. list the intervals where the function is positive and negative
d. list the intervals where the function is increasing and decreasing
e. list the intercepts

\[
j(x) = \begin{cases} 
(x + 2)^2 & \text{if } x \leq -2 \\
x & \text{if } -2 < x \leq 2 \\
-(x - 1)^2 + 4 & \text{if } x > 2 
\end{cases}
\]
Answers to Examples:
1a. $D: (-\infty, \infty), R: (-4, \infty)$; 1b. $g(x) = 0$ when $x = 2, 4$;
1c. $g(x) > 0: (-\infty, 0] \cup (4, \infty), g(x) < 0: (0, 2) \cup (2, 4)$;
1d. $\uparrow: (0, 2) \cup [4, \infty), \downarrow: (-\infty, 0] \cup (2, 4)$;
1e. $x - \text{int}: (2, 0), (4, 0); y - \text{int}: (0, 1)$;
2a. $D: (-\infty, \infty), R: (-\infty, 6)$; 2b. $h(x) = 0$ when $x = -3, -1, 4$;
2c. $h(x) > 0: (-3, -1) \cup [1, 4)$
    $h(x) < 0: (-\infty, -3) \cup (-1, 1) \cup (4, \infty)$;
2d. $\uparrow: (-\infty, -3] \cup (0, 1), \downarrow: (-3, 0) \cup [1, \infty)$;
2e. $x - \text{int}: (-3, 0), (-1, 0)(4, 0), y - \text{int}: (0, -2)$;
3a. $D: (-\infty, \infty), R: (-\infty, \infty)$; 3b. $g(x) = 0$ when $x = -2, 0, 3$;
3c. $g(x) > 0: (-\infty, -2) \cup (0, 3), g(x) < 0: (-2, 0) \cup (3, \infty)$;
3d. $\uparrow: (-2, 2], \downarrow: (-\infty, -2] \cup (2, \infty)$;
3e. $x - \text{int}: (-2, 0), (0, 0), (3, 0); y - \text{int}: (0, 0)$;