In this lesson we will discuss one-to-one functions. Before covering what a one-to-one function is, I will first review the definition of a function.

**Function:**
- a connection between sets in which each element of the first set corresponds with exactly one element of the second set (each input produces exactly one output)
  - an example of a function is the set of students at Purdue and their student identification numbers
    - each student has exactly one student identification number and each student identification number corresponds with exactly one student (one-to-one function)
- the graph of a function must pass the vertical line test
  - a vertical line (which represents an input) can only intersect the graph of a function once

An example of a function is \( f(x) = x^2 \). Remember that to find a function value we simply replace \( x \) with a number or an expression. To find \( f(2) \), we replace \( x \) with 2.

\[
f(2) = 2^2
\]

\[
f(2) = 4
\]

To find \( f(-2) \), we replace \( x \) with \(-2\).

\[
f(-2) = (-2)^2
\]

\[
f(-2) = 4
\]

Notice that for the function \( f \), both inputs (2 and \(-2\)) produce the same output (4); \( f \) is a function because each input results in exactly one output, however it is **not a one-to-one function** because each output is not the result of exactly one input. The output 4 is the result of two different inputs 2 and \(-2\).
One-to-one function:
- a function in which each output is the result of exactly one input
  o every input has exactly one output (this makes it a function) and every output is the result of exactly one input (this makes it one-to-one)

Showing algebraically that one output is the result of more than one input as I did earlier with $f(2) = 4$ and $f(-2) = 4$ is one way to determine whether a function is one-to-one or not. Another option to determine whether a function is one-to-one or not is to use its graph; for that, we use the horizontal line test.

Horizontal line test:
- a horizontal line (which represents an output) can only intersect the graph of a function once in order for that function to be one-to-one

The vertical line test is used to determine whether a graph represents a function, and the horizontal line test is used to determine whether a function is one-to-one or not.

Example 1: Given the following graph, determine whether this is the graph of function. If it is, then determine whether this is the graph of a one-to-one function.

Keep in mind that each of these problems in LON-CAPA will always ask if the graph is a function first, before asking if it’s a one-to-one function. So you should never answer No then Yes, because that would mean that you have a graph that is not a function, but is somehow a one-to-one function. Anytime you answer No to the first part, you should always answer no to the second part as well. A graph must first be a function, before it can be a one-to-one function.
**Example 2:** Given the following graph, determine whether this is the graph of function. If it is, determine whether it is a one-to-one function.

![Graph](image1)

**Example 3:** Given the following graph, determine whether this is the graph of function. If it is, determine whether it is a one-to-one function.

![Graph](image2)
Example 4: Given the following graph, determine whether this is the graph of function. If it is, determine whether it is a one-to-one function.

Example 5: Given the following graph, determine whether this is the graph of function. If it is, determine whether it is a one-to-one function.
**Example 6:** Given the following graph, determine whether this is the graph of function. If it is, determine whether it is a one-to-one function.

![Graph 1](image1.png)

**Example 7:** Given the following graph, determine whether this is the graph of function. If it is, determine whether it is a one-to-one function.

![Graph 2](image2.png)
Once again, keep in mind that each of these problems in LON-CAPA will always ask if the graph is a function first, before asking if it’s a one-to-one function. And once again, you should never answer No/Yes, because that would mean that you have a graph that is not a function, but is somehow a one-to-one function. Anytime you answer no to the first part, you should automatically answer no to the second part. A graph must first be a function, before it can be a one-to-one function.
Another tool for determining what is (what is not) a one-to-one function is the **Theorem on Increasing/Decreasing Functions**.

**Theorem on Increasing/Decreasing Functions:**
- a function that is strictly increasing or strictly decreasing throughout its domain is one-to-one (no turning points)
  - all linear functions are one-to-one because they are either always increasing or always decreasing
  - the graph of a quadratic function is parabola which is both increasing and decreasing, so it is not one-to-one
    - as shown before, the way we make it a one-to-one function is by restricting its domain so it’s either always increasing or always decreasing

Basically as long as the graph of a function has no turning points, it is the graph of a one-to-one function.

In the next set of notes we’ll be covering Inverse Functions, and how to find an inverse. Keep in mind that **ONLY** one-to-one function have an inverse, so that is why it is important to understand and identify one-to-one functions.