**Example 1:** Complete the input/output table for the function \( f(x) = 2^x \), and use the ordered pairs to sketch the graph of the function. After graphing, list the domain, range, zeros, positive/negative intervals, increasing/decreasing intervals, and the intercepts.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( f(x) = 2^x )</td>
</tr>
<tr>
<td>( x \to -\infty )</td>
<td>( f(x) \to 0 )</td>
</tr>
<tr>
<td>-5</td>
<td>( f(-5) = 2^{-5} = \frac{1}{32} = 0.03125 )</td>
</tr>
<tr>
<td>-4</td>
<td>( f(-4) = 2^{-4} = \frac{1}{16} = 0.0625 )</td>
</tr>
<tr>
<td>-3</td>
<td>( f(-3) = 2^{-3} = \frac{1}{8} = 0.125 )</td>
</tr>
<tr>
<td>-2</td>
<td>( f(-2) = 2^{-2} = \frac{1}{4} = 0.25 )</td>
</tr>
<tr>
<td>-1</td>
<td>( f(-1) = 2^{-1} = \frac{1}{2} = 0.5 )</td>
</tr>
<tr>
<td>0</td>
<td>( f(0) = 2^0 = 1 )</td>
</tr>
<tr>
<td>1</td>
<td>( f(1) = 2^1 = 2 )</td>
</tr>
<tr>
<td>2</td>
<td>( f(2) = 2^2 = 4 )</td>
</tr>
<tr>
<td>3</td>
<td>( f(3) = 2^3 = 8 )</td>
</tr>
<tr>
<td>4</td>
<td>( f(4) = 2^4 = 16 )</td>
</tr>
<tr>
<td>5</td>
<td>( f(5) = 2^5 = 32 )</td>
</tr>
<tr>
<td>( x \to \infty )</td>
<td>( f(x) \to \infty )</td>
</tr>
</tbody>
</table>

Notice that the graph of \( f(x) = 2^x \) is increasing throughout its domain. When a function is always increasing or always decreasing that function is one-to-one, and it will have an inverse function. All exponential functions are one-to-one, therefore all exponential functions have an inverse (we’ll discuss this further in Lessons 31 & 32).

Domain: \((-\infty, \infty)\)

Range: \((0, \infty)\)

Zeros: **NONE**

\( f(x) \neq 0 \)

Positive intervals:

\( f(x) > 0 \) when \( x \) is \((-\infty, \infty)\)

Negative intervals:

\( f(x) < 0 \) when \( x \) is **NONE**

**Increasing intervals:**

\( f(x) \) is rising when \( x \) is \((-\infty, \infty)\)

Decreasing intervals:

\( f(x) \) is falling when \( x \) is **NONE**

Intercepts:

\( x \) – intercept: **NONE**

\( y \) – intercept: \((0, 1)\)
Horizontal Asymptote:
- a horizontal line \((y = \#)\) that the graph of a function approaches when the inputs are large positive numbers \((x \to \infty)\) or large negative numbers \((x \to -\infty)\)
  - in the case of \(f(x) = 2^x\), as the inputs get smaller and smaller \((x \to -\infty)\), the outputs get closer and closer to zero \((f(x) \to 0)\), so the graph has a horizontal asymptote at \(y = 0\)
  - the graph of every exponential function will have a horizontal asymptote
- in the next example (Example 2) I denoted the horizontal asymptote with a dotted line to make it easier to identify

Example 2: Re-write the function \(g(x) = -2^x\) in terms of \(f(x) = 2^x\). Then find its \(y\)-intercept and sketch its graph using transformations and the \(y\)-intercept. Enter exact answers only, no approximations.

Re – write \(g(x)\) in terms of \(f(x)\):

\[g(x) = \]

\(y\) – intercept:

\[g(0) = \]

\((0, \quad )\)
Example 3: Re-write each of the following functions in terms of $f(x) = 2^x$, then transformation the graph of $f$. Also, find the $y$-intercept for each function and enter exact answers only, no approximations.

a. $h(x) = 2^{-x}$

$h(x) =$

$y$ – intercept:

b. $j(x) = 2^x - 1$

$j(x) =$

$y$ – intercept:

c. $k(x) = 3(2^x)$

$k(x) =$

$y$ – intercept:
Example 4: Re-write each of the following functions in terms of \( f(x) = 2^x \), then transformation the graph of \( f \). Also, find the \( y \)-intercept for each function and enter exact answers only, no approximations.

a. \( m(x) = 2^{x-3} \)

\( m(x) = \)

\( y \)-intercept:

b. \( n(x) = -2^x + 1 \)

\( n(x) = \)

\( y \)-intercept:

c. \( p(x) = 2^{\frac{x}{2}} \)

\( p(x) = \)

\( y \)-intercept:
Answers to Examples:

2. \( g(x) = -f(x) \), \( y \)-intercept: \((0, -1)\);

3a. \( h(x) = f(-x) \), \( y \)-intercept: \((0, 1)\);

3b. \( j(x) = f(x) - 3 \), \( y \)-intercept: \((0, -2)\);

3c. \( k(x) = 3f(x) \), \( y \)-intercept: \((0, 3)\);

4a. \( m(x) = f(x - 3) \), \( y \)-intercept: \(\left(0, \frac{1}{8}\right)\);

4b. \( n(x) = -f(x) + 1 \), \( y \)-intercept: \((0, 0)\);

4c. \( k(x) = f\left(\frac{x}{2}\right) \), \( y \)-intercept: \((0, 1)\);