In the previous set of notes, we found the following:

$$\sqrt{36} = 6$$

Re-writing the radicand 36 as 6^2, we have the following:

$$\sqrt{6^2} = 6$$

How can we re-write a radical (such as a square root) using an exponent? In other words, 6 to the power of 2 to the power of what will result in 6^1, or just 6. In this case, what exponent is the equivalent of a square root?

$$\sqrt{6^2} = (6^2)^x = 6^1$$

Using the Power Rule for Exponents, when a base is taken to a power, and then to another power, the exponents are multiplied. So 2 times what produces 1?

$$2x = 1$$

$$x = \frac{1}{2}$$

So the answer is $\frac{1}{2}$. A square root is equivalent to an exponent of $\frac{1}{2}$.

$$\sqrt{6^2} = (6^2)^{\frac{1}{2}} = 6$$

So a square root is equivalent to a power of $\frac{1}{2}$, which is the reciprocal of the index 2. The same is true for any radical; to express a radical as an exponent, we simply need to take the reciprocal of the index of the radical.

Here are a few more examples of radicals and their exponent equivalents.

$$\sqrt[3]{125} = (125)^{\frac{1}{3}} = 5$$
$$\sqrt[4]{x} = (x)^{\frac{1}{4}}$$
$$\sqrt[5]{y^2} = (y^2)^{\frac{1}{5}} = y^{\frac{2}{5}}$$ (this example uses the Power Rule for Exponents)

Converting a radical $\left(\sqrt[n]{x}\right)$ to an exponent $\left(x^{\frac{1}{n}}\right)$

- to write a radical as a fractional exponent, write the radicand as the base and the **reciprocal of the index** as the exponent
  - the radical expression $\sqrt[4]{x^5}$ is equivalent to $(x^5)^{\frac{1}{4}}$, which simplifies to $x^{\frac{5}{4}}$ by using the Power Rule for Exponents
  - the radical expression $\left(\sqrt[4]{x}\right)^5$ is equivalent to $\left(x^{\frac{1}{4}}\right)^5$, which also simplifies to $x^{\frac{5}{4}}$ by using the Power Rule for Exponents
- be sure that the original radical exists, otherwise the new expression is meaningless

Converting an exponent $\left(x^{\frac{1}{n}}\right)$ to a radical $\left(\sqrt[n]{x}\right)$

- to write a fractional exponent as a radical, write the denominator of the exponent as the index of the radical and the base as the radicand
  - the expression $x^{\frac{3}{5}}$ can be written as a radical in two ways, both of which are equivalent
    - $x^{\frac{3}{5}} = \sqrt[5]{x^3}$
    - $x^{\frac{3}{5}} = \left(\sqrt[5]{x}\right)^3$
  - regardless of which way you choose to write the radical expression, the index is the same (in this case 5)
- again, it is imperative that the radical exists, otherwise the expression is meaningless
  - $\sqrt{-9}$ does not exist with real numbers, so it is meaningless to write it as $(-9)^{\frac{1}{2}}$
  - also, keep in mind the difference between $(-9)^{\frac{1}{2}}$ and $-9^{\frac{1}{2}}$
    - $(-9)^{\frac{1}{2}} =$
    - $-9^{\frac{1}{2}} =$

<u>Example 1:</u> Convert each of the following expressions to radical form, and then simplify completely. Do **<u>NOT</u>** leave negative exponents in your answers. If a solution does not exist in real numbers, write DNE.

a. $27^{\frac{4}{3}}$

b. $16^{-\frac{3}{4}}$

$$\frac{1}{16^{\frac{3}{4}}}$$

$$\frac{1}{\sqrt[4]{16^3}} \quad \text{OR} \quad \frac{1}{\left(\sqrt[4]{16}\right)^3}$$

$$\frac{1}{\sqrt[4]{4096}} \quad \text{OR} \quad \frac{1}{(2)^3}$$

$$\mathbf{\frac{1}{8}}$$

c. $(-9)^{-\frac{3}{2}}$

$$\frac{1}{(-9)^{\frac{3}{2}}}$$

$$\frac{1}{\sqrt[2]{(-9)^3}} \quad \text{OR} \quad \frac{1}{\left(\sqrt[2]{-9}\right)^3}$$

$$\frac{1}{\sqrt[2]{-729}} \quad \text{OR} \quad \frac{1}{\left(\sqrt[2]{-9}\right)^3}$$

Since it′s not possible to to take the even root of a negative value using real numbers, neither of radicals exist.

DNE

d. $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$

e. $-4^{-\frac{3}{2}}+\frac{1}{2}\left(\frac{1}{8}\right)^{\frac{2}{3}}$

$$-1\cdot 4^{-\frac{3}{2}}+\frac{1}{2}\left(\frac{1^{\frac{2}{3}}}{8^{\frac{2}{3}}}\right)$$

$$-1\cdot \frac{1}{4^{\frac{3}{2}}}+\frac{1}{2}\left(\frac{\left(\sqrt[3]{1}\right)^{2}}{\left(\sqrt[3]{8}\right)^{2}}\right)$$

$$-1\cdot \frac{1}{\left(\sqrt{4}\right)^{3}}+\frac{1}{2}\left(\frac{(1)^{2}}{(2)^{2}}\right)$$

$$-1\cdot \frac{1}{(2)^{3}}+\frac{1}{2}\left(\frac{1}{4}\right)$$

$$-1\cdot \frac{1}{8}+\frac{1}{8}$$

$$-\frac{1}{8}+\frac{1}{8}$$

0

f. $-32^{-\frac{2}{5}}+\left(\frac{64}{125}\right)^{-\frac{1}{3}}$

Example 2: Simplify the following expression by converting to radical form and/or by using Exponent Rules. Simplify completely and do **NOT** leave negative exponents in your answers. If a solution does not exist in real numbers, write DNE.

$$(27x^6)^{-\frac{4}{3}}\left(3x^{\frac{4}{3}}\right)$$

The first thing to notice about this expression is that $(27x^6)^{-\frac{4}{3}}$ is a Product to a Power, while $\left(3x^{\frac{4}{3}}\right)$ is just a product.

$$(27x^6)^{-\frac{4}{3}}\left(3x^{\frac{4}{3}}\right)$$

Change a negative exponent to a positive exponent by taking the reciprocal of the expression.

$$\frac{1}{(27x^6)^{\frac{4}{3}}}\left(3x^{\frac{4}{3}}\right)$$

$$\frac{3x^{\frac{4}{3}}}{27^{\frac{4}{3}}(x^6)^{\frac{4}{3}}}$$

To simplify a base (x) to a power (6) to another power $\left(\frac{4}{3}\right)$, we multiply the powers, giving us x^8 in this case.

$$\frac{3x^{\frac{4}{3}}}{\left(\sqrt[3]{27}\right)^4 x^8}$$

$\frac{x^{\frac{4}{3}}}{x^8}$ can be simplified using the Quotient Rule for exponents to get $x^{\frac{4}{3}-8}$, which is $x^{-\frac{20}{3}}$

$$\frac{3x^{\frac{4}{3}-8}}{(3)^4}$$

$$\frac{3x^{-\frac{20}{3}}}{81}$$

Once again, to change a negative exponent to a positive exponent we take the reciprocal of the expression. Since the factor $x^{-\frac{20}{3}}$ is already part of a fraction, we can take it's reciprocal by simply moving it from the numerator to the denominator.

$$\mathbf{\frac{1}{27x^{\frac{20}{3}}}}$$

As you complete the homework and prepare for the exam, keep in mind that Example 2 on the previous page is the type of problem that many students have struggled with on past exams. Example 2 requires you to combine concepts from Lesson 1 (Quotient Rule and Power Rule), Lesson 2 (Product to a Power Rule and Negative Exponents), and Lesson 3 (Fractional Exponents), so be sure you understand each of these concepts individually, and also how to use them together in order to simplify expressions. The expressions in Example 3 will be similar to the expression from Example 2, so please be sure you understand how to simplify each of those expressions as well.

<u>Example 3:</u> Simplify the following expression by converting to radical form and/or by using Exponent Rules. Simplify completely and do **<u>NOT</u>** leave negative exponents in your answers. If a solution does not exist in real numbers, write DNE.

a. $(27x)^{\frac{2}{3}}\left(3x^{\frac{4}{5}}\right)$

b. $\left(-2x^{\frac{3}{2}}\right)\ (64x^{6})^{-\frac{4}{3}}$

c. $-\left(\frac{x^3}{64}\right)^{-\frac{4}{3}}\left(\frac{4^{-\frac{3}{2}}}{x^8}\right)$

$$-\left(\frac{64}{x^3}\right)^{\frac{4}{3}}\left(\frac{1}{4^{\frac{3}{2}}x^8}\right)$$

$$-\left(\frac{64^{\frac{4}{3}}}{(x^3)^{\frac{4}{3}}}\right)\frac{1}{(\sqrt{4})^3\cdot x^8}$$

$$-\left(\frac{(\sqrt[3]{64})^4}{x^4}\right)\frac{1}{(2)^3\cdot x^8}$$

$$-\left(\frac{(4)^4}{x^4}\right)\frac{1}{8x^8}$$

$$-\left(\frac{256}{x^4}\right)\frac{1}{8x^8}$$

$$-\frac{256}{x^4}\cdot\frac{1}{8x^8}$$

$$\mathbf{-\frac{32}{x^{12}}}$$

d. $\left(\frac{x^{12}}{81}\right)^{\frac{3}{4}}\left(-\frac{x^9}{27}\right)^{-\frac{2}{3}}\left(\frac{1}{4}x^{-\frac{4}{5}}\right)$

$$\frac{(x^{12})^{\frac{3}{4}}}{81^{\frac{3}{4}}}\cdot\left(-\frac{27}{x^9}\right)^{\frac{2}{3}}\cdot\left(\frac{1}{4}\cdot\frac{1}{x^{\frac{4}{5}}}\right)$$

$$\frac{x^9}{(\sqrt[4]{81})^3}\cdot\frac{(-27)^{\frac{2}{3}}}{(x^9)^{\frac{2}{3}}}\cdot\frac{1}{4x^{\frac{4}{5}}}$$

$$\frac{x^9}{27}\cdot\frac{(\sqrt[3]{-27})^2}{x^6}\cdot\frac{1}{4x^{\frac{4}{5}}}$$

$$\frac{x^9}{27}\cdot\frac{9}{x^6}\cdot\frac{1}{4x^{\frac{4}{5}}}$$

$$\frac{9x^9}{27x^6\cdot 4x^{\frac{4}{5}}}$$

$$\frac{x^3}{3\cdot 4x^{\frac{4}{5}}}$$

$$\frac{x^{3-\frac{4}{5}}}{12}$$

$$\mathbf{\frac{1}{12}x^{\frac{11}{5}}}$$

Once again, the expressions from Examples 2 and 3 can be difficult to simplify completely, so be sure to spend time working on problems like these not only in the homework, but also as you prepare for Exam #1. If you need assistance understanding how to simplify these types of expressions, please let me know.

Answers to Examples:

1a. 81 ; *1b.* $\frac{1}{8}$; *1c.* DNE ; *1d.* $\frac{9}{4}$; *1e.* 0 ; *1f.* 1 ; *2a.* $\frac{1}{27x^{\frac{20}{3}}}$;

3a. $27x^{\frac{22}{15}}$; *3b.* $-\frac{1}{128x^{\frac{13}{2}}}$; *3c.* $-\frac{32}{x^{12}}$; *3d.* $\frac{1}{12}x^{\frac{11}{5}}$;