**Radical Notation:**

\[ \sqrt[n]{y} \]

- the expression above is read “the \(n^{th}\) root of \(y\)” where \(y\) is the radicand, \(\sqrt{}\) is the radical sign, and \(n\) is the index or root
- if no index is denoted, it is understood to be an index of 2 (a square root)
  \[ \sqrt{y} = \sqrt[2]{y} \]
- the definition of \(\sqrt[n]{y}\) is the value that must be taken to the power of \(n\) to produce \(y\)
  - the expression \(\sqrt{25}\) represents the value that is taken to the power of 2 to produce 25
    - since 5 taken to the power of 2 is 25, the square root of 25 is 5
    - \(\sqrt{25} = 5\) because \(5^2 = 25\)
  - the expression \(\sqrt[3]{-64}\) represents the value that is taken to the power of 3 to produce \(-64\)
    - since \(-4\) taken to the power of 3 is \(-64\), the cubed root of \(-64\) is \(-4\)
    - \(\sqrt[3]{-64} = -4\) because \((-4)^3 = -64\)

**Example 1:** Evaluate each expression.

a. \(\sqrt{16}\)  

Why?

b. \(\sqrt[4]{16}\)  

Why?

c. \(\sqrt{64}\)  

Why?

d. \(\sqrt[3]{64}\)  

Why?

e. \(\sqrt[3]{27}\)  

Why?

f. \(\sqrt{81}\)
**Even Roots:**
- a radical with an index of 2, 4, 6, ...
- the radicand of a radical with an even root must be positive or zero (no negative values)
  o the even root of a positive number is a positive number
    ▪ \( \sqrt[4]{16} = 2 \) because \( 2^4 = 16 \)
  o the even root of zero is zero
    ▪ \( \sqrt{0} = 0 \) because \( 0^2 = 0 \)
  o the even root of a negative number does not exist with real numbers because no real number (negative, positive, or zero) can be taken to an even power and produce a negative value
    ▪ \((-4)^2 = (-4) \cdot (-4)\)
      \[ = 16 \]
    ▪ \((-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3)\)
      \[ = 81 \]
    ▪ \((-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2)\)
      \[ = 64 \]
    ▪ a negative base taken to an **even** exponent will **ALWAYS** result in a positive value, so that is why the **even** root of a negative number does not exist with real numbers

**Odd Roots:**
- a radical with an index of 3, 5, 7, ...
- the radicand of a radical with an odd root can be any real number (negative, positive, or zero)
  o the odd root of a positive number is a positive number
    ▪ \( \sqrt[5]{32} = 2 \) because \( 2^5 = 32 \)
  o the odd root of zero is zero
    ▪ \( \sqrt[3]{0} = 0 \) because \( 0^3 = 0 \)
  o the odd root of a negative number is a negative number
    ▪ \( \sqrt[3]{-64} = -4 \) because \( (-4)^3 = -64 \)
  o a negative base taken to an **odd** exponent will **ALWAYS** result in a negative value, so that is why the **odd** root of a negative number does exist with real numbers, and is negative
**Example 2:** Evaluate each expression; if a solution does not exist in real numbers, write DNE.

a. \( \sqrt{36} \)  
b. \( \sqrt{-81} \)  
c. \( -\sqrt{25} \)  

Why?  
Why?  
Why?  

d. \( 3\sqrt{8} \)  
e. \( 3\sqrt{-8} \)  
f. \( -3\sqrt{125} \)  

Why?  
Why?  
Why?  

g. \( 4\sqrt{81} \)  
h. \( 6\sqrt{-\pi} \)  
i. \( -8\sqrt{1} \)  

Why?  
Why?  
Why?  

j. \( 5\sqrt{0} \)  
k. \( 3\sqrt{-1} \)  
l. \( -9\sqrt{1} \)  

Why?  
Why?  
Why?
Answers to Examples:
1a. 4; 1b. 2; 1c. 8; 1d. 4; 1e. 3; 1f. 9; 2a. 6;
2b. DNE; 2c. −5; 2d. 2; 2e. −2;
2f. −5; 2g. 3; 2h. DNE; 2i. −1; 2j. 0; 2k. −1; 2l. −1;