Radical Notation:

\[ \sqrt[n]{y} \]

- the expression above is read “the \( n^{th} \) root of \( y \)”, where \( y \) is the radicand, \( \sqrt{} \) is the radical sign, and \( n \) is the index or root
- if no index is denoted, it is understood to be an index of 2 (a square root)
  \[ \sqrt{y} = \sqrt[n]{y} \]
- the definition of \( \sqrt[n]{y} \) is the value that must be taken to the power of \( n \) to produce \( y \)
  \[ \sqrt[n]{y} \] represents the value that is taken to the power of \( n \) to produce \( y \)
  \[ \sqrt{25} = 5 \text{ because } 5^2 = 25 \]
  \[ \sqrt[3]{-64} = -4 \text{ because } (-4)^3 = -64 \]

**Example 1:** Evaluate each expression.

a. \( \sqrt{16} \)  
   Why?

b. \( \sqrt[4]{16} \)  
   Why?

c. \( \sqrt{64} \)  
   Why?

d. \( \sqrt[3]{64} \)  
   Why?

e. \( \sqrt[3]{27} \)  
   Why?

f. \( \sqrt{81} \)  
   Why?

Because \( 4^3 = 64 \)  
Because \( 3^3 = 27 \)  
Because \( 9^2 = 81 \)
**Even Roots:**
- a radical with an index of 2, 4, 6, ...
- the radicand of a radical with an even root must be positive or zero (no negative values)
  - the even root of a positive number is a positive number
    - $\sqrt[4]{16} = 2$ because $2^4 = 16$
  - the even root of zero is zero
    - $\sqrt{0} = 0$ because $0^2 = 0$
  - the even root of a negative number does not exist with real numbers because no real number (negative, positive, or zero) can be taken to an even power and produce a negative value
    - $(-4)^2 = (-4) \cdot (-4) = 16$
    - $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$
    - $(-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 64$
    - a negative base taken to an even exponent will **ALWAYS** result in a positive value, so that is why the even root of a negative number does not exist with real numbers

**Odd Roots:**
- a radical with an index of 3, 5, 7, ...
- the radicand of a radical with an odd root can be any real number (negative, positive, or zero)
  - the odd root of a positive number is a positive number
    - $\sqrt[5]{32} = 2$ because $2^5 = 32$
  - the odd root of zero is zero
    - $\sqrt[3]{0} = 0$ because $0^3 = 0$
  - the odd root of a negative number is a negative number
    - $\sqrt[3]{-64} = -4$ because $(-4)^3 = -64$
  - a negative base taken to an odd exponent will **ALWAYS** result in a negative value, so that is why the odd root of a negative number does exist with real numbers, and is negative
**Example 2:** Evaluate each expression; if a solution does not exist in real numbers, write DNE.

a. \( \sqrt{36} \)  

b. \( \sqrt{-81} \)  

c. \( -\sqrt{25} \)

Why?  
Why?  
Why?

d. \( 3\sqrt{8} \)  

e. \( 3\sqrt{-8} \)  

f. \( -3\sqrt{125} \)

Why?  
Why?  
Why?

g. \( 4\sqrt{81} \)  

h. \( 6\sqrt{-\pi} \)  

i. \( -8\sqrt{1} \)

3  

DNE  

-1

Why?  
Why?  
Why?

**Because** \( 3^4 = 81 \)  

Because no real number to the power of 6 will be negative  

Because **1^8 = 1** and then it’s negated

j. \( 5\sqrt{0} \)  

k. \( 7\sqrt{-1} \)  

l. \( -9\sqrt{1} \)

0  

-1  

-1

Why?  
Why?  
Why?

**Because** \( 0^5 = 0 \)  

Because \( (-1)^7 = -1 \)  

Because **1^9 = 1** and then it’s negated
Answers to Examples:
1a. 4; 1b. 2; 1c. 8; 1d. 4; 1e. 3; 1f. 9; 2a. 6;
2b. DNE; 2c. −5; 2d. 2; 2e. −2;
2f. −5; 2g. 3; 2h. DNE; 2i. −1; 2j. 0; 2k. −1; 2l. −1;