Radical Notation:

\( n\sqrt{y} \)

- the expression above is read “the \( n^{th} \) root of \( y \)”, where \( y \) is the
  radicand, \( \sqrt{\phantom{y}} \) is the radical sign, and \( n \) is the index or root
- if no index is denoted, it is understood to be an index of 2 (a square root)
  \( \sqrt{y} = \sqrt[2]{y} \)
- the definition of \( n\sqrt{y} \) is the value that must be taken to the power of \( n \)
  to produce \( y \)
  \( \sqrt{25} \) represents the value that is taken to the
  power of 2 to produce 25
  ▪ since 5 taken to the power of 2 is 25, the square root of 25
    is 5
  ▪ \( \sqrt{25} = 5 \) because \( 5^2 = 25 \)

\( \sqrt[3]{-64} \) represents the value that is taken to the
power of 3 to produce \(-64 \)
  ▪ since \(-4 \) taken to the power of 3 is \(-64 \), the cubed root of
    \(-64 \) is \(-4 \)
  ▪ \( \sqrt[3]{-64} = -4 \) because \((-4)^3 = -64 \)

Example 1: Evaluate each expression.

a. \( \sqrt{16} \)  
   b. \( \sqrt[4]{16} \)  
   c. \( \sqrt[6]{64} \)

Why?

Why?

Why?

d. \( \sqrt[3]{64} \)  
   e. \( \sqrt[3]{27} \)  
   f. \( \sqrt{81} \)

Why?

Why?

Why?
**Even Roots:**
- a radical with an index of 2, 4, 6, ...
- the radicand of a radical with an even root must be positive or zero (no negative values)
  - the even root of a positive number is a positive number
    ▪ \( \sqrt[2]{16} = 2 \) because \( 2^4 = 16 \)
  - the even root of zero is zero
    ▪ \( \sqrt{0} = 0 \) because \( 0^2 = 0 \)
  - the even root of a negative number does not exist with real numbers because no real number (negative, positive, or zero) can be taken to an even power and produce a negative value
    ▪ \((-4)^2 = (-4) \cdot (-4) = 16\)
    ▪ \((-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81\)
    ▪ \((-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 64\)
    ▪ a negative base taken to an even exponent will **ALWAYS** result in a positive value, so that is why the even root of a negative number does not exist with real numbers

**Odd Roots:**
- a radical with an index of 3, 5, 7, ...
- the radicand of a radical with an odd root can be any real number (negative, positive, or zero)
  - the odd root of a positive number is a positive number
    ▪ \( \sqrt[5]{32} = 2 \) because \( 2^5 = 32 \)
  - the odd root of zero is zero
    ▪ \( \sqrt[3]{0} = 0 \) because \( 0^3 = 0 \)
  - the odd root of a negative number is a negative number
    ▪ \( \sqrt[3]{-64} = -4 \) because \( (-4)^3 = -64 \)
  - a negative base taken to an odd exponent will **ALWAYS** result in a negative value, so that is why the odd root of a negative number does exist with real numbers, and is negative
Example 2: Evaluate each expression; if a solution does not exist in real numbers, write DNE.

a. $\sqrt{36}$  
b. $\sqrt{-81}$  
c. $-\sqrt{25}$  

d. $\sqrt[3]{8}$  
e. $\sqrt[3]{-8}$  
f. $-\sqrt[3]{125}$  

g. $\sqrt[4]{81}$  
h. $\sqrt[6]{-\pi}$  
i. $-\sqrt[8]{1}$  

j. $\sqrt[5]{0}$  
k. $\sqrt[3]{-1}$  
l. $-\sqrt[9]{1}$  

Why?  
Why?  
Why?
Answers to Examples:

1a. 4; 1b. 2; 1c. 8; 1d. 4; 1e. 3; 1f. 9; 2a. 6;
2b. DNE; 2c. −5; 2d. 2; 2e. −2;
2f. −5; 2g. 3; 2h. DNE; 2i. −1; 2j. 0; 2k. −1; 2l. −1;