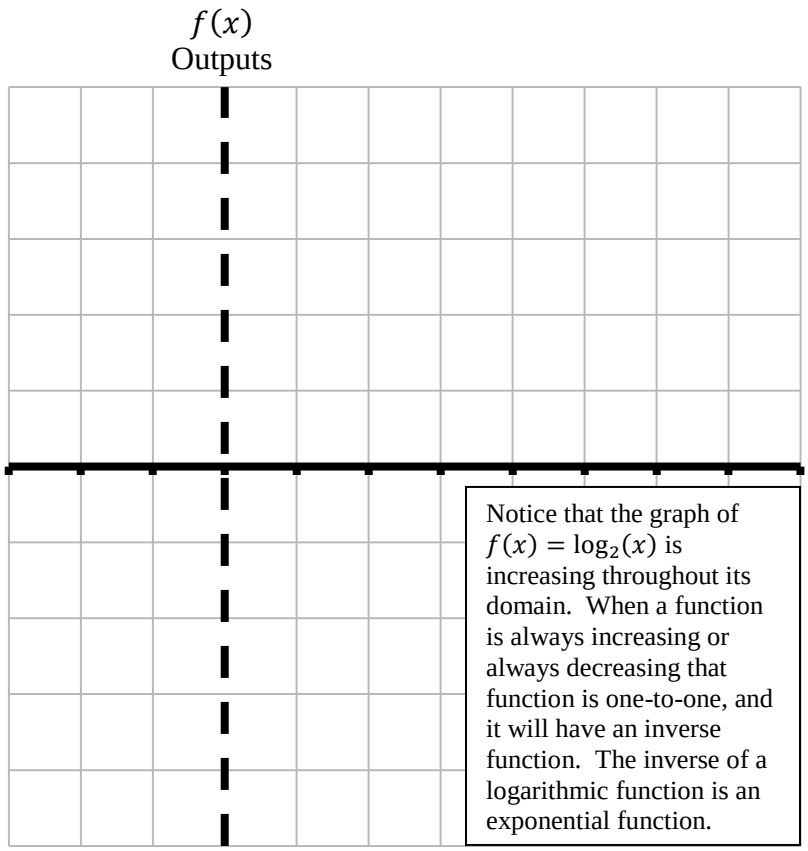
Example 1: Complete the input/output table for the function $f(x) = \log_2(x)$, and use the ordered pairs to sketch the graph of the function. After graphing, list the domain, range, zeros, positive/negative intervals, increasing/decreasing intervals, and the intercepts.

Inputs (these are powers of the base 2)	**Outputs** (these are exponents that produce the powers of the base 2)
x	$f(x) = \log_2(x)$
$x \to 0$	
$\frac{1}{16}$	
$\frac{1}{8}$	
$\frac{1}{4}$	
$\frac{1}{2}$	
1	
2	
4	
8	
16	
$x \to \infty$	



Notice that the graph of $f(x) = \log_2(x)$ is increasing throughout its domain. When a function is always increasing or always decreasing that function is one-to-one, and it will have an inverse function. The inverse of a logarithmic function is an exponential function.

Domain: $(\mathbf{0}, \infty)$

Range: $(-\infty, \infty)$

Zeros:
$f(x) = 0$ when $x = \mathbf{1}$

Positive intervals:
$f(x) > 0$ when x is $(\mathbf{1}, \infty)$

Negative intervals:
$f(x) < 0$ when x is $(\mathbf{0}, \mathbf{1})$

Increasing intervals:
$\boldsymbol{f(x)}$ **is rising when** $\boldsymbol{x}$ **is** $(\mathbf{0}, \infty)$

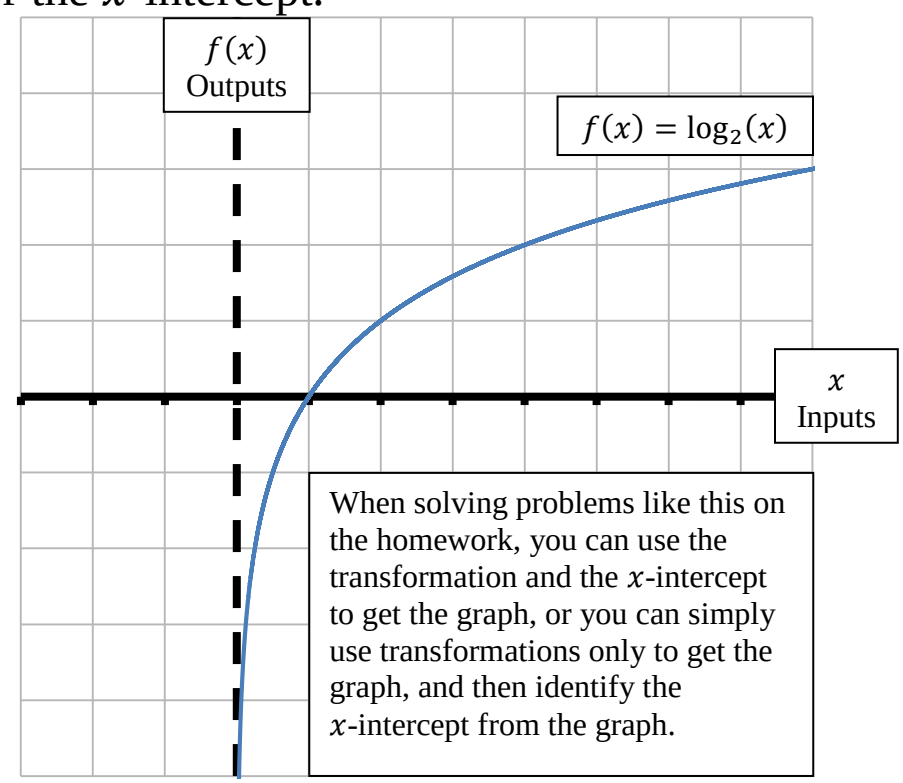
Decreasing intervals:
$f(x)$ is falling when x is **NONE**

Intercepts:
$x-$ intercept: $(\mathbf{1}, \mathbf{0})$
$y-$ intercept: **NONE**

Vertical Asymptote:

- a vertical line ($x = \#$) that the graph of a function approaches, but never touches or crosses, when the inputs approach an undefined value ($x \to \#$, where # is a value that is not part of the domain)
  - in the case of $f(x) = \log_2(x)$, as the inputs get closer and closer to zero ($x \to 0$), the outputs get smaller and smaller ($f(x) \to -\infty$), so the graph has a vertical asymptote at $x = 0$
  - the graph of every logarithmic function will have a vertical asymptote ($x = \#$)
- in a later example (Example 2) I will denoted the vertical asymptote with a dotted line to make it easier to identify

Example 2: Re-write the function $g(x) = \log_2(x+2)$ in terms of $f(x) = \log_2(x)$. Then find the x-intercept of g and find its graph by transforming the graph of the original function f. Enter exact answers only (no approximations) for the x-intercept.



When solving problems like this on the homework, you can use the transformation and the x-intercept to get the graph, or you can simply use transformations only to get the graph, and then identify the x-intercept from the graph.

Re − write $g(x)$
<u>in terms of $f(x)$:</u>

$\boldsymbol{g(x) =}$

$\boldsymbol{x - \text{intercept}}$:

$0 = \log_2(x+2)$

$2^0 = x + 2$

$1 = x + 2$

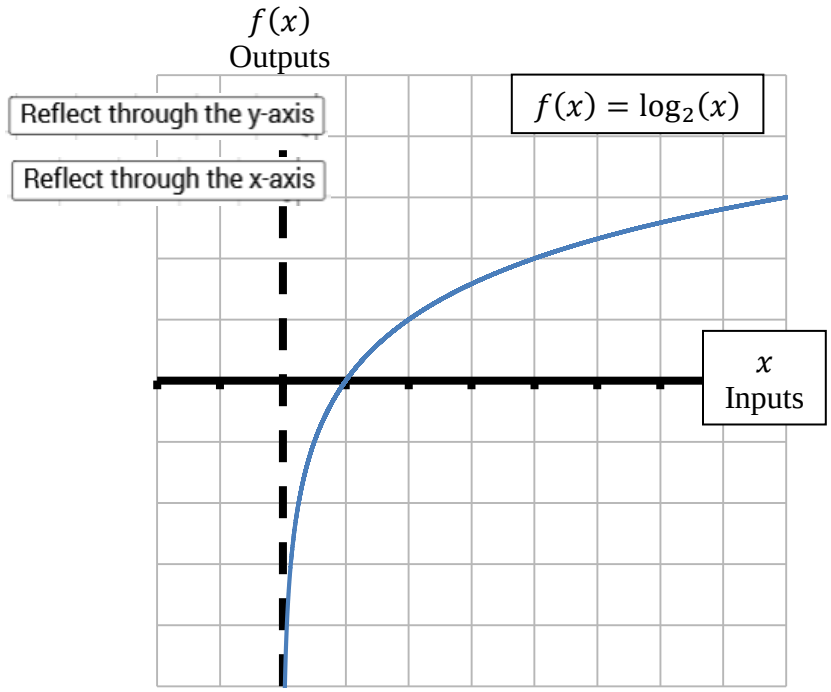
$-1 = x$

$\boldsymbol{(-1, 0)}$

To find the x-intercept algebraically, remember that a logarithm is an exponent, so anything equal to a logarithm is also an exponent. In this example, the logarithm $\log_2(x+2)$ represents the exponent that makes the base 2 equal to the argument $x + 2$. Since 0 is equal to $\log_2(x+2)$, 0 is the exponent that makes 2 equal to $x + 2$.

<u>Example 3:</u> Re-write each of the following functions in terms of $f(x) = \log_2(x)$, then match the transformation with the appropriate graph. Also, find the x-intercepts of each function.

a. $h(x) = -\log_2(x)$



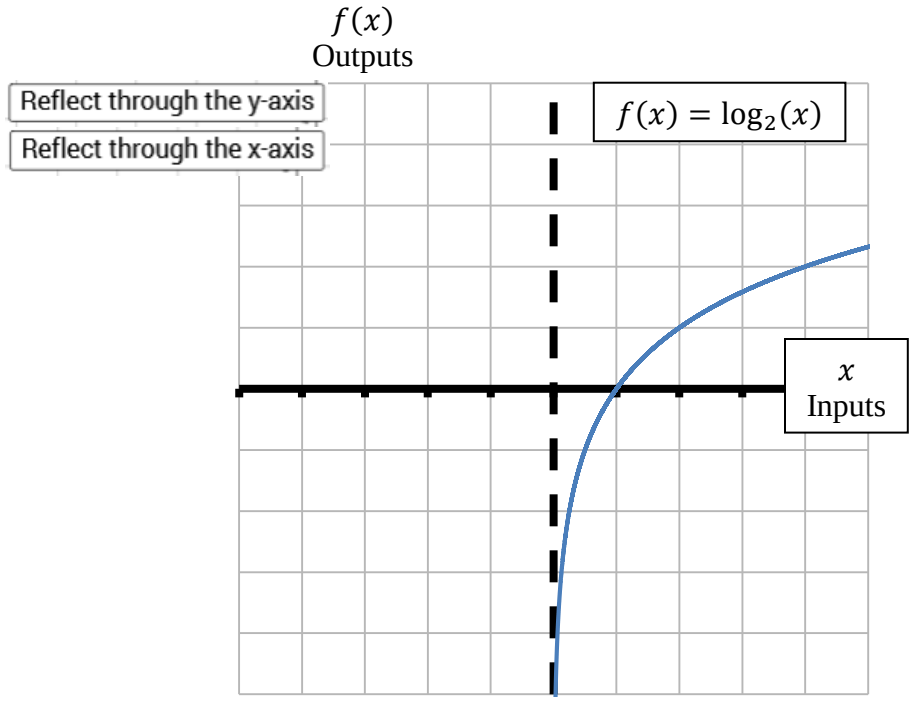
$\boldsymbol{h(x) =}$

$\boldsymbol{x - \text{intercept}}$:

$0 = -\log_2(x) \quad \rightarrow$

$(\quad ,0)$

b. $j(x) = \log_2(-x)$



$\boldsymbol{j(x) =}$

$\boldsymbol{x - \text{intercept}}$:

$0 = \log_2(-x) \quad \rightarrow$

$(\quad ,0)$

LON-CAPA Problem :

Given the function $f(x) =$, along with it's graph below, complete the following:

a. Express the new function $h(x) =$ in terms of the original function f.

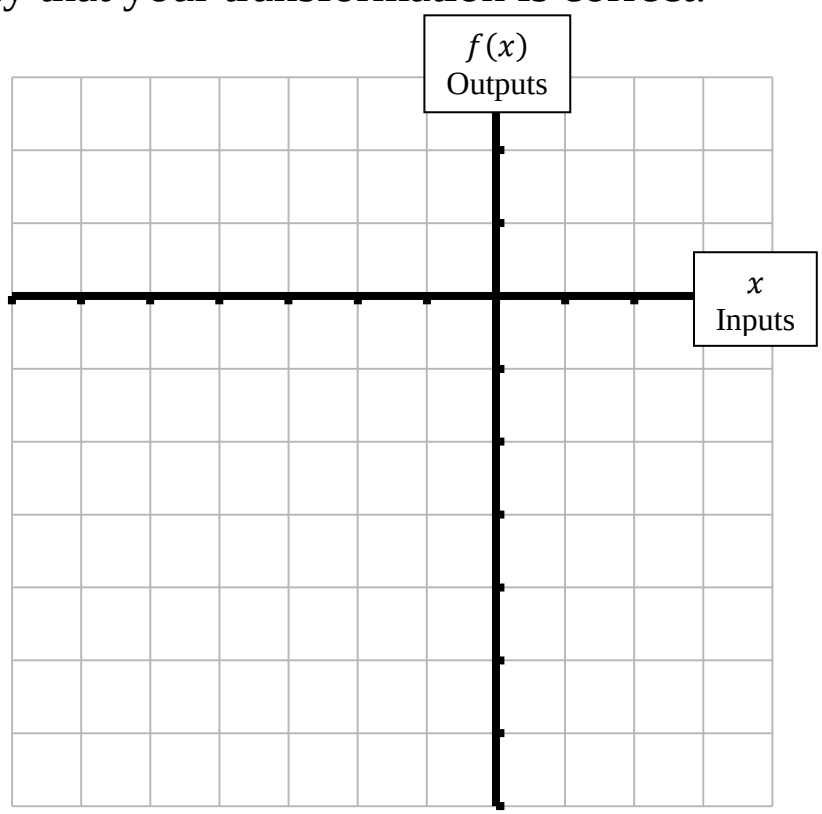
$\boldsymbol{h(x) =}$

b. Find the x-intercept of the function $h(x) =$ and enter your answer as an ordered pair (x, y). Enter exact answers only, no approximations.

$\boldsymbol{x - \text{intercept } (x, y) =}$

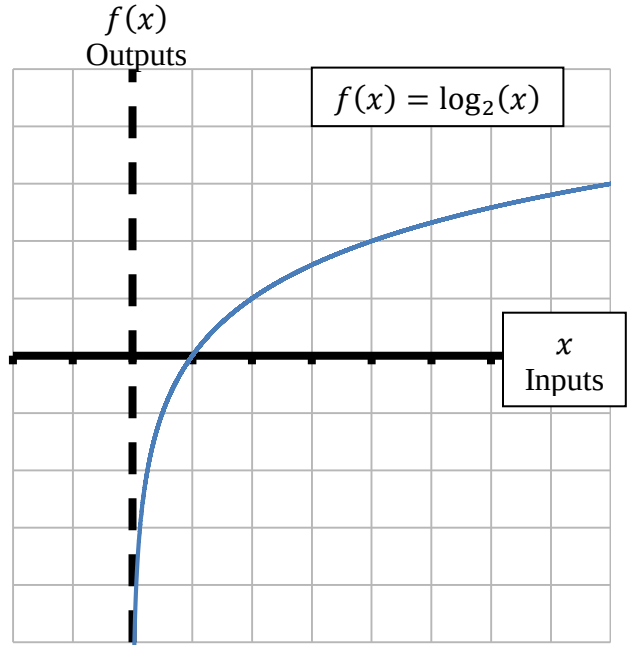
c. Transform the graph of f to get the graph of $h(x) =$. Use the x-intercept of h to verify that your transformation is correct.

Keep in mind that even though the point you're given on the graph maybe the x-intercept, after transforming the point it may no longer be an intercept. Use the answer from part a. to determine how to transform the point that you are given on the graph. After transforming the graph, use your answer from part b. to verify that the x-intercept on the new graph is correct.



<u>Example 4:</u> Re-write each of the following functions in terms of $f(x) = \log_2(x)$, then match the transformation with the appropriate graph. Also, find the x-intercepts of each function.

a. $k(x) = (\log_2(x)) - 2$



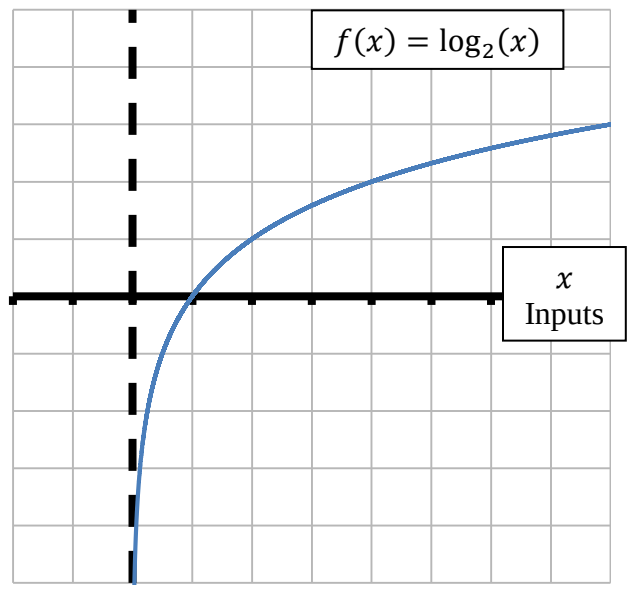
$\boldsymbol{k(x)} =$

$\boldsymbol{x - \text{intercept}}$:

$0 = (\log_2(x)) - 2 \quad \rightarrow$

$(\quad ,0)$

b. $n(x) = \log_2(2x)$



$\boldsymbol{n(x)} =$

$\boldsymbol{x - \text{intercept}}$:

$0 = \log_2(2x) \quad \rightarrow$

$(\quad ,0)$

Answers to Examples:

2. $g(x) = f(x+2), x-\text{intercept: } (-1, 0)$;

3a. $h(x) = -f(x), x-\text{intercept: } (1, 0)$;

3b. $j(x) = f(-x), x-\text{intercept: } (-1, 0)$;

4a. $k(x) = f(x) - 2\ , x-\text{intercept: } (4, 0)$;

4b. $n(x) = f(2x), x-\text{intercept: } \left(\frac{1}{2}, 0\right)$;