**Example 1:** Complete the input/output table for the function \( f(x) = \log_2(x) \), and use the ordered pairs to sketch the graph of the function. After graphing, list the domain, range, zeros, positive/negative intervals, increasing/decreasing intervals, and the intercepts.

<table>
<thead>
<tr>
<th>Inputs (these are powers of the base 2)</th>
<th>Outputs (these are exponents that produce the powers of the base 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( f(x) = \log_2(x) )</td>
</tr>
<tr>
<td>( x \to 0 )</td>
<td>( f(x) \to -\infty )</td>
</tr>
<tr>
<td>( \frac{1}{16} )</td>
<td>(-4)</td>
</tr>
<tr>
<td>( \frac{1}{8} )</td>
<td>(-3)</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>(-2)</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>(-1)</td>
</tr>
<tr>
<td>( 1 )</td>
<td>(0)</td>
</tr>
<tr>
<td>( 2 )</td>
<td>(1)</td>
</tr>
<tr>
<td>( 4 )</td>
<td>(2)</td>
</tr>
<tr>
<td>( 8 )</td>
<td>(3)</td>
</tr>
<tr>
<td>( 16 )</td>
<td>(4)</td>
</tr>
<tr>
<td>( x \to \infty )</td>
<td>( f(x) \to \infty )</td>
</tr>
</tbody>
</table>

Notice that the graph of \( f(x) = \log_2(x) \) is increasing throughout its domain. When a function is always increasing or always decreasing that function is one-to-one, and it will have an inverse function. The inverse of a logarithmic function is an exponential function.

**Domain:** \((0, \infty)\)

**Range:** \((\infty, \infty)\)

**Zeros:**
\( f(x) = 0 \) when \( x = 1 \)

**Positive intervals:**
\( f(x) > 0 \) when \( x \) is \((1, \infty)\)

**Negative intervals:**
\( f(x) < 0 \) when \( x \) is \((0, 1)\)

**Increasing intervals:**
\( f(x) \) is rising when \( x \) is \((0, \infty)\)

**Decreasing intervals:**
\( f(x) \) is falling when \( x \) is **NONE**

**Intercepts:**
- **x** – intercept: \((1, 0)\)
- **y** – intercept: **NONE**
**Vertical Asymptote:**
- a vertical line \((x = \#)\) that the graph of a function approaches, but never touches or crosses, when the inputs approach an undefined value \((x \to \#)\, where \# \text{ is a value that is not part of the domain})
  - in the case of \(f(x) = \log_2(x)\), as the inputs get closer and closer to zero \((x \to 0)\), the outputs get smaller and smaller \((f(x) \to -\infty)\), so the graph has a vertical asymptote at \(x = 0\)
  - the graph of every logarithmic function will have a vertical asymptote \((x = \#)\)
- in a later example (Example 3) I will denoted the vertical asymptote with a dotted line to make it easier to identify

**Example 2:** Re-write the following function in terms of \(f(x) = \log_2(x)\). Then find its \(x\)-intercept and sketch its graph using transformations and the \(x\)-intercept.

\[
a. \quad g(x) = \log_2(x + 2)
\]

Re – write \(g(x)\) in terms of \(f(x)\):

\[
\begin{aligned}
g(x) &= \\
\end{aligned}
\]

\(x\) – intercept:

\[
0 = \log_2(x + 2)
\]

\((\quad , 0)\)

When solving problems like these on the homework assignment, you can use the transformation and the \(x\)-intercept to get the graph, or you can simply use transformations only to get the graph, and then identify the \(x\)-intercept from the graph.
**Example 3:** Re-write each of the following functions in terms of \( f(x) = \log_2(x) \), then match the transformation with the appropriate graph. Also, find the \( x \)-intercepts of each function

a. \( h(x) = -\log_2(x) \)

\[ h(x) = \]

\( x \)-intercept:

\[ 0 = -\log_2(x) \quad \rightarrow \quad ( ,0) \]

b. \( j(x) = \log_2(-x) \)

\[ j(x) = \]

\( x \)-intercept:

\[ 0 = \log_2(-x) \quad \rightarrow \quad ( ,0) \]
c. \( k(x) = (\log_2(x)) - 2 \)

\[ k(x) = \]

\[ x \text{ – intercept:} \]

\[ 0 = (\log_2(x)) - 2 \quad \rightarrow \]

( ,0)

d. \( m(x) = 2 \log_2(x) \)

\[ m(x) = \]

\[ x \text{ – intercept:} \]

\[ 0 = 2 \log_2(x) \quad \rightarrow \]

( ,0)

e. \( n(x) = \log_2(2x) \)

\[ n(x) = \]

\[ x \text{ – intercept:} \]

\[ 0 = \log_2(2x) \quad \rightarrow \]

( ,0)
Answers to Examples:
2. \( g(x) = f(x + 2), x - \text{intercept: } (-1, 0) \);
3a. \( h(x) = -f(x), x - \text{intercept: } (1, 0) \);
3b. \( j(x) = f(-x), x - \text{intercept: } (-1, 0) \);
3c. \( k(x) = f(x) - 2, x - \text{intercept: } (4, 0) \);
3d. \( m(x) = 2f(x), x - \text{intercept: } (1, 0) \);
3e. \( n(x) = f(2x), x - \text{intercept: } \left(\frac{1}{2}, 0\right) \);