**Example 1:** Complete the input/output table for the function \( f(x) = \log(x) \), and use the ordered pairs to sketch the graph of the function. After graphing, list the domain, range, zeros, positive/negative intervals, increasing/decreasing intervals, and the intercepts.

<table>
<thead>
<tr>
<th>Inputs (these are powers of the base 10)</th>
<th>Outputs (these are exponents that produce the powers of the base 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( f(x) = \log(x) )</td>
</tr>
<tr>
<td>( x \rightarrow 0 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{1000} )</td>
<td></td>
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<tr>
<td>( \frac{1}{100} )</td>
<td></td>
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<tr>
<td>( \frac{1}{10} )</td>
<td></td>
</tr>
<tr>
<td>( 1 )</td>
<td></td>
</tr>
<tr>
<td>( 10 )</td>
<td></td>
</tr>
<tr>
<td>( 100 )</td>
<td></td>
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<tr>
<td>( 1000 )</td>
<td></td>
</tr>
<tr>
<td>( x \rightarrow \infty )</td>
<td></td>
</tr>
</tbody>
</table>

\( f(x) \) Outputs

- **Domain:** \((0, \infty)\)
- **Range:** \((-\infty, \infty)\)
- **Zeros:** \(f(x) = 0\) when \(x = 1\)
- **Positive intervals:** \(f(x) > 0\) when \(x\) is \((1, \infty)\)
- **Negative intervals:** \(f(x) < 0\) when \(x\) is \((0, 1)\)
- **Increasing intervals:** \(f(x)\) is rising when \(x\) is \((0, \infty)\)
- **Decreasing intervals:** \(f(x)\) is falling when \(x\) is **NONE**

**Intercepts:**
- \(x\) – intercept: \((1, 0)\)
- \(y\) – intercept: **NONE**
All of this information (domain, range, zeros, etc.) is the same for any logarithmic function \( f(x) = \log_a(x) \) that has NOT been transformed. As shown before, transformations will change some or all of this information based on what type of transformation is performed.

Since the domain, range, zeros, positive/negative intervals, increasing/decreasing intervals, intercepts, and asymptote will be the same for all logarithmic functions of the form \( f(x) = \log_a(x) \), we do not need to repeat this exercise for the natural logarithmic function \( g(x) = \ln(x) \). Remember that the natural logarithmic function \( \ln(x) \) is \( \log_e(x) \), and \( e \) is the natural number 2.71828 ....; therefore it will have the same domain, range, zeros, etc. as the \( f(x) = \log(x) \). I will however include its graph, along with the graph of \( f(x) = \log_2(x) \) for comparison.
Example 2: Re-write the function \( j(x) = -\log(x) \) in terms of \( f(x) = \log(x) \). Then find the \( x \)-intercept of \( j \) and find its graph by transforming the graph of the original function \( f \). Enter exact answers only (no approximations) for the \( x \)-intercept.

\[
j(x) =
\]

\( x \)-intercept:

( ,0)

When solving problems like this in LON-CAPA, you can identify the transformation and find the \( x \)-intercept first, and then use both in order to transform the graph of the original function. Or you can simply transform the original graph first, and then identify the \( x \)-intercept from the graph, like I did above. If you want to identify the \( x \)-intercept first, before getting the graph of the new function, you’ll have to find the \( x \)-intercept algebraically:

\[
x \text{- intercept of } j(x) = -\log(x):
\]

\[
0 = -\log_{10}(x)
\]

0 = \( \log_{10}(x) \) \hspace{1cm} \text{I multiplied both sides of the equation by } -1 \text{ to change the sign of the logarithm.}

10^0 = x \hspace{1cm} \text{Then I converted from log form to exponential form in order to isolate } x.

1 = x

Finding the \( x \)-intercept algebraically was optional on this problem because were able to find it graphically as well. That will not always be the case, as we’ll see on the next example.
Example 3: Re-write the function \( k(x) = \log(x) + 2 \) in terms of \( f(x) = \log(x) \). Then find the \( x \)-intercept of \( k \) and find its graph by transforming the graph of the original function \( f \). Enter exact answers only (no approximations) for the \( x \)-intercept.

\[
k(x) = f(x) + 2
\]

To transform the graph of the function \( f \), we need to shift it up 2 units. Doing so will mean the old \( x \)-intercept of \((1, 0)\) will become \((1, 2)\). The new \( x \)-intercept of the function \( k \) might not be identifiable from its graph, so it may be necessary to find the \( x \)-intercept algebraically.

\( x \)-intercept of \( k(x) = \log(x) + 2 \):

\[
0 = \log_{10}(x) + 2
\]

\[
-2 = \log_{10}(x)
\]

I subtracted 2 from both sides of the equation to isolate the logarithm.

\[
10^{-2} = x
\]

Then I converted from log form to exponential form in order to isolate \( x \).

\[
\frac{1}{100} = x
\]

\((0.01, 0)\)

This is an example of a problem where finding the \( x \)-intercept graphically may not be an option (at least on paper). Be aware that something like this may come up in LON-CAPA on the homework and/or on an exam.
**Example 4:** Re-write each of the following functions in terms of $f(x) = \log(x)$. Then find the $x$-intercept of each new function and find its graph by transforming the graph of the original function $f$. Enter exact answers only (no approximations) for the $x$-intercept.

a. $m(x) = \log(2x)$

$m(x) = $

$x$ – intercept:

$0 = \log_{10}(2x) \rightarrow$

( ,0)

---

b. $r(x) = \log(x - 4)$

$r(x) =$

$x$ – intercept:

$0 = \log_{10}(x - 4) \rightarrow$

( ,0)
**Example 5:** Re-write each of the following functions in terms of \( g(x) = \ln(x) \). Then find the \( x \)-intercept of each new function and find its graph by transforming the graph of the original function \( g \). Enter exact answers only (no approximations) for the \( x \)-intercept.

a. \( n(x) = \ln(-x) \)

\[
n(x) =
\]

\( x \)-intercept:

\[
0 = \log_e(-x) \rightarrow
\]

\((0,0)\)

b. \( p(x) = \ln(x-2) \)

\[
p(x) =
\]

\( x \)-intercept:

\[
0 = \log_e(x-2) \rightarrow
\]

\((3,0)\)

c. \( q(x) = 2 \ln(x) \)

\[
q(x) =
\]

\( x \)-intercept:

\[
0 = 2 \log_e(x) \rightarrow
\]

\((1,0)\)
Once again, when solving problems like these on the homework assignment, you can identify the transformation and find the \( x \)-intercept first, and then use both in order to transform the graph of the original function. Or you can simply transform the original graph first, and then identify the \( x \)-intercept from the graph. However keep in mind there could be instances where the \( x \)-intercept cannot be found graphically, so you should be prepared to find the \( x \)-intercept algebraically in those cases.

Answers to Examples:

2. \( j(x) = -f(x), x - \text{intercept: } (1, 0); \)

3. \( k(x) = f(x) + 2, x - \text{intercept: } \left( \frac{1}{100}, 0 \right); \)

4a. \( m(x) = f(2x), x - \text{intercept: } \left( \frac{1}{2}, 0 \right); \)

4b. \( r(x) = f(x - 4), x - \text{intercept: } (5, 0); \)

5a. \( n(x) = g(-x), x - \text{intercept: } (-1, 0); \)

5b. \( p(x) = g(x - 2), x - \text{intercept: } (3, 0); \)

5c. \( q(x) = 2g(x), x - \text{intercept: } (1, 0); \)