**Variation:**
- how one quantity changes (varies) in relation to another quantity

**Direct Variation:**
- a dependent variable moves in the same direction as an independent variable
- described by formulas of the form \( y = kx^n \), where \( n > 0 \)
  - when \( x \) increases, \( y \) increases
  - when \( x \) decreases, \( y \) decreases

**Joint Variation:**
- when a quantity varies directly as the product of two or more variables
- described by formulas such as \( y = kxz \), \( y = kx^2z \), \( y = kxz^2 \), \( \ldots \),
where the variable \( y \) is dependent on the product of more than one independent variable (such as \( x \) and \( z \)) as well as a constant of variation \( k \)
  - when the independent variables increase, the dependent variable \( y \) also increases
  - when the independent variables decrease, the dependent variable \( y \) also decreases

- Examples:
  - the area of a triangle varies jointly as the base and the height
    - \( A = k \cdot b \cdot h \); the constant of proportionality \( k = \frac{1}{2} \)
  - the volume of a cone varies jointly as the square of the radius and the height
    - \( V = k \cdot r^2 \cdot h \); the constant of variation \( k = \frac{\pi}{3} \)
**Example 1:** Express the following statement as a formula that involves the given variables and a constant of proportionality $k$, and then determine the value of $k$ from the given conditions.

$y$ varies jointly with the square of $x$ and the cube root of $z$. If $x = 5$ and $z = 8$, then $y = 25$.

**Example 2:** Express the following statement as a formula that involves the given variables and a constant of proportionality $k$, and then determine the value of $k$ from the given conditions.

$r$ is directly proportional to the product of $s$ and $v$. If $s = 2$ and $v = 3$, then $r = 40$. 
**Example 3:** The area of a trapezoid \((A)\) is jointly proportional to its height and the sum of the lengths of the parallel sides.

a. Express the previous statement as a formula, using the trapezoid below on the left and a constant of variation \(k\).

b. Find the constant of variation \(k\), if the area of the trapezoid on the right is 49 \(in^2\).
Answers to Examples:
1. \( y = kx^2 \cdot 3\sqrt{z} \); \( k = \frac{1}{2} \); 2. \( r = ksv \); \( k = \frac{20}{3} \);
3a. \( A = k \cdot h(a + b) \); 3b. \( k = \frac{1}{2} \);