Variation:

- how one quantity changes (varies) in relation to another quantity

Direct Variation:

- a dependent variable moves in the same direction as an independent variable
- described by formulas of the form $y = kx^n$, where $n > 0$
  - when x increases, y increases
  - when x decreases, y decreases

Joint Variation:

- when a quantity varies directly as the product of two or more variables
  - joint variation is simply an extension of direct variation
  - formulas will still be products, but there will be more than one independent variable
- described by formulas such as $y = kxz$, $y = kx^2z$, $y = kxz^2$, …, where the variable y is dependent on the product of more than one independent variable (such as x and z) as well as a constant of variation k
  - when the independent variables increase, the dependent variable y also increases
  - when the independent variables decrease, the dependent variable y also decreases

- Examples of joint variation:
  - the area of a triangle varies jointly as the base and the height
    - the formula is $A = k \cdot b \cdot h$, where A is the area, b is the base, h is the height, and k is the constant of variation
    - for this formula, the constant of variation $k = \frac{1}{2}$
  - the volume of a cone varies jointly as the square of the radius and the height
    - the formula is $V = k \cdot r^2 \cdot h$, where V is the volume, r is the radius, h is the height, and k is the constant of variation
    - for this formula, the constant of variation $k = \frac{\pi}{3}$

Example 1: Express the following statement as a formula that involves the given variables and a constant of proportionality k, and then determine the value of k from the given conditions.

y varies jointly with the square of x and the cube root of z. If $x = 5$ and $z = 8$, then $y = 25$.

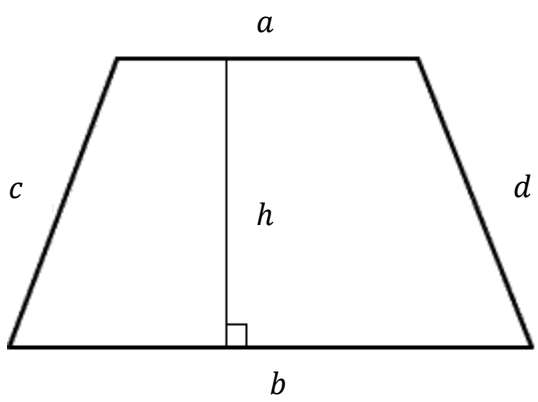
Example 2: Express the following statement as a formula that involves the given variables and a constant of proportionality k, and then determine the value of k from the given conditions.

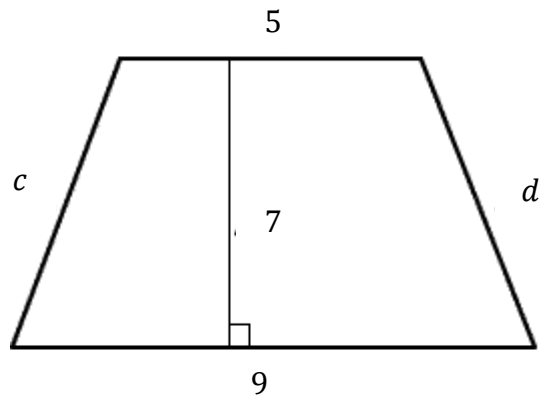
r is directly proportional to the product of s and v. If $s = 2$ and $v = 3$, then $r = 40$.

<u>Example 3:</u> The area of a trapezoid (A) is jointly proportional to its height and the sum of the lengths of the parallel sides.

a. Express the previous statement as a formula, using the trapezoid below on the left and a constant of variation k.

b. Find the constant of variation k, if the area of the trapezoid on the right is $49\ in^2$.





Answers to Examples:

1. $y = kx^2 \cdot \sqrt[3]{z}$; $k = \frac{1}{2}$; *2.* $r = ksv$; $k = \frac{20}{3}$;

3a. $A = k \cdot h(a + b)$; *3b.* $k = \frac{1}{2}$;