Dividing a polynomial by a monomial:
- write each term in the numerator as an individual fraction over the denominator

\[
\frac{10a^3b^3 - 15a^2b^2 + 5ab^4}{5ab^2} = \frac{10a^3b^3}{5ab^2} - \frac{15a^2b^2}{5ab^2} + \frac{5ab^4}{5ab^2}
\]

- simplify each of the resulting fractions by cancelling common factors or by using the Quotient Rule for Exponents

\[
\frac{10a^3b^3}{5ab^2} - \frac{15a^2b^2}{5ab^2} + \frac{5ab^4}{5ab^2} = 
\]

**Example 1:** Divide the polynomial by the monomial and express your answer as a polynomial.

\[
\frac{3x^3y^2z - 6x^4y^5z^6}{4xyz}
\]
In the next example, be sure to simplify the numerators completely first before writing individual fractions and dividing. That means you need to remove all the parentheses from the numerators and combine like terms.

**Example 2:** Divide the polynomial by the monomial and express your answer as a polynomial.

a. \[ \frac{(3x^2y)^3 - (5x^6y^3 + 2x^3y^2 - 3x^3y^6)}{2x^3y^2} \] 

b. \[ \frac{(x+y)(x-y) - (x-y)^2}{2y} \]
Example 3: Simplify the following expression completely and express your answer as a polynomial.

\[
(3x^3y^4 - 2x^5y^2 + (xy)^3)^2
\]

\[
\left(\frac{3x^3y^4}{x^3y^2} - \frac{2x^5y^2}{x^3y^2} + \frac{(xy)^3}{x^3y^2}\right)^2
\]

\[
(3y^2 - 2x^2 + y)^2
\]

\[
(3y^2 - 2x^2 + y)(3y^2 - 2x^2 + y)
\]

\[
3y^2(3y^2 - 2x^2 + y) - 2x^2(3y^2 - 2x^2 + y) + y(3y^2 - 2x^2 + y)
\]

\[
9y^4 - 6x^2y^2 + 3y^3 - 6x^2y^2 + 4x^4 - 2x^2y + 3y^3 - 2x^2y + y^2
\]

\[
4x^4 - 12x^2y^2 + 9y^4 - 4x^2y + 6y^3 + y^2
\]

Example 3 combines each topic from today’s lecture (adding, subtracting, multiplying, and dividing polynomials) along with concepts from other lectures as well, such as Exponent Rules. This is a great example of combining topics from within lessons and from past lessons, and this is what I’m referring to when I say students need to be able to synthesize information in this course. Please do not overlook problems like this because they are difficult and/or time consuming; these are the types of problems you’ll need to be able to solve to succeed on Exam #1, as well as to succeed in this course.

Answers to Examples:

1. \[
\frac{3}{4}x^2y - \frac{3}{2}x^3y^4z^5; \quad 2a. \quad \frac{3}{2}y^4 + 11x^3y - 1; \quad 2b. \quad x - y;
\]

2. \[
4x^4 - 12x^2y^2 + 9y^4 - 4x^2y + 6y^3 + y^2;
\]